

# Anomalies in Physics or Physics of anomalies?

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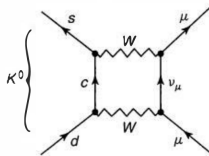
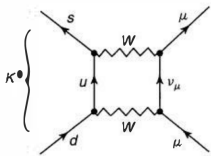
IFJ FCAL workshop  
May 10, 2018

# Outline

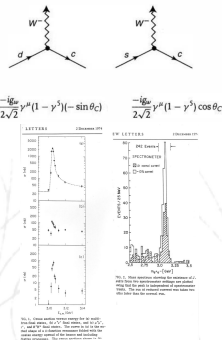
1. Why flavour is important.
2. The flavour anomalies:
  - $R(D^*)$
  - $R_K$  and  $R_{K^*}$
  - $P'_5$
3. Global fits results.
4. Conclusions.

# Why Flavour is important?

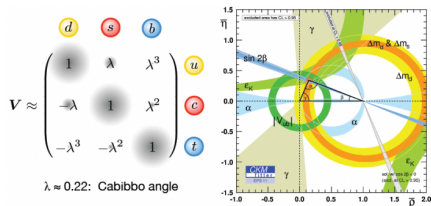
# A lesson from history - GIM mechanism



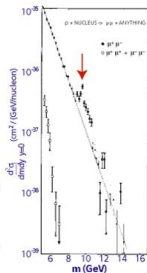
- Cabibbo angle was successful in explaining dozens of decay rates in the 1960s.
- There was, however, one that was not observed by experiments:  $K^0 \rightarrow \mu^- \mu^+$ .
- Glashow, Iliopoulos, Maiani (GIM) mechanism was proposed in the 1970 to fix this problem. The mechanism required the existence of a 4<sup>th</sup> quark.
- At that point most of the people were skeptical about that. Fortunately in 1974 the discovery of the  $J/\psi$  meson silenced the skeptics.



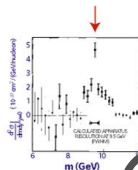
# A lesson from history - CKM matrix



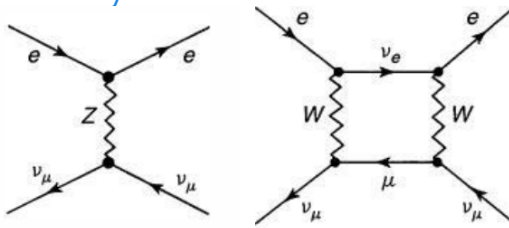
- Similarly, CP violation was discovered in 1960s in the neutral kaons decays.
- $2 \times 2$  Cabbibo matrix could not allow for any CP violation.
- For CP violation to be possible one needs at least a  $3 \times 3$  unitary matrix  $\rightarrow$  Cabibbo-Kobayashi-Maskawa matrix (1973).
- It predicts existence of  $b$  (1977) and  $t$  (1995) quarks.



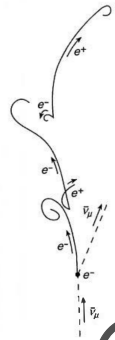
Results published in  
Physical Review Letters  
August 1, 1977



# A lesson from history - Weak neutral current



- Weak neutral currents were first introduced in 1958 by Buldman.
- Later on they were naturally incorporated into unification of weak and electromagnetic interactions.
- 't Hooft proved that the GWS models was renormalizable.
- Everything was there on theory side, only missing piece was the experiment, till 1973.



## Study the CKM matrix

Arises from Higgs Yukawa interactions  
Unitary in the SM, with one CP violating phase.

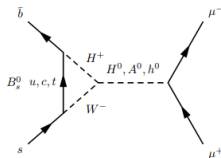
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Test unitarity with many measurements.

Find new sources of CPV  
wru anti-matter!?

## Measure decays of ground state b-hadrons

Properties influenced by virtual particles in NP models  
Compare results to SM predictions  
(need QCD input).



Particularly sensitive to NP models preferring third generation.

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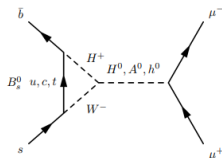
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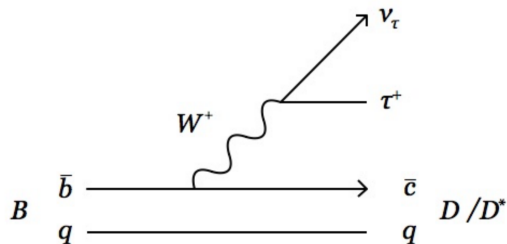


Particularly sensitive to NP models preferring third generation.



# Why semi-leptonic decays?

⇒ A decay is semi-leptonic if its products are part leptons and part hadrons.



$$\frac{d\Gamma}{dq^2}(B \rightarrow D l \nu) \propto$$

$$G_F^2 |V_{cb}|^2 f(q^2)^2$$

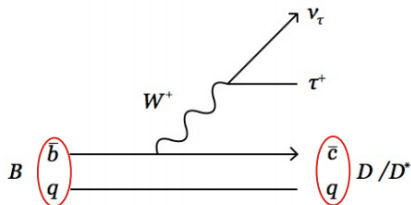
EW                      QCD

⇒ These decays can be factorised into the weak and strong parts, greatly simplifying theoretical calculations.

# Types of semi-leptonic decays

Two types of semi-leptonic b-decay

## Charged current

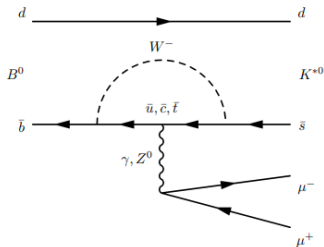


Can proceed via tree level - large  $O(\%)$  branching fractions.

Factorised up to (small) QED corrections.

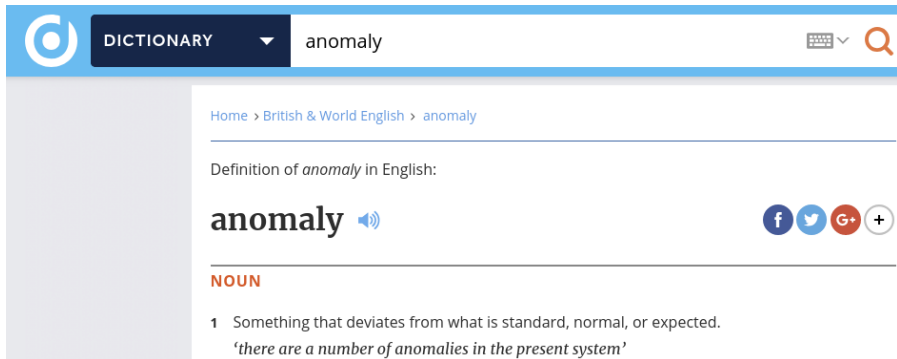
When you factorise, QCD part broken down into form-factors.

## Neutral current



Forbidden at tree level - low  $O(10^{-6})$  branching fractions.


Factorised up to corrections from  $B \rightarrow h(\rightarrow \mu^+ \mu^-)h$  decays.







The screenshot shows the Cambridge Dictionary interface. At the top, there is a blue header with the Cambridge logo on the left, a dark blue navigation bar with the word 'DICTIONARY' and a dropdown arrow, and a search bar containing the word 'anomaly'. To the right of the search bar are icons for a keyboard and a magnifying glass. Below the header, a breadcrumb trail reads 'Home > British & World English > anomaly'. The main content area features the text 'Definition of *anomaly* in English:' followed by the word 'anomaly' in a large, bold font with a speaker icon for audio playback. To the right of the word are social media sharing icons for Facebook, Twitter, Google+, and a plus sign for more options. Below this, the word is categorized as 'NOUN' in orange. A numbered definition follows: '1 Something that deviates from what is standard, normal, or expected. *'there are a number of anomalies in the present system'*'

Home > British & World English > anomaly

Definition of *anomaly* in English:

**anomaly** 

**NOUN**

1 Something that deviates from what is standard, normal, or expected.  
*'there are a number of anomalies in the present system'*

⇒ Today I will talk about three anomalies in  $B$  decays:

- $R(D^*)$
- $R_{K/K^*}$
- $P'_5$

# Anomaly 1

$$R(D^*) = \frac{\mathcal{B}(B \rightarrow D^* \tau \nu)}{\mathcal{B}(B \rightarrow D^* \mu \nu)}$$

# $R(D^*)$

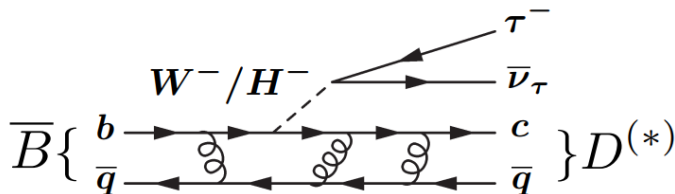
⇒ Large rate of charged current decays allow for measurement in semi-tauonic decays

$$R(D^*) = \frac{\mathcal{B}(B \rightarrow D^* \tau \nu)}{\mathcal{B}(B \rightarrow D^* \mu \nu)}$$

⇒ Form ratio of decays with different lepton generations.

⇒ Cancel QCD uncertainties.

⇒  $R(D^*)$  is sensitive to the NP with strong 3rd generation couplings.



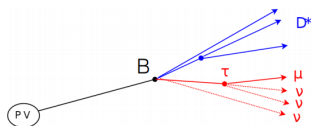
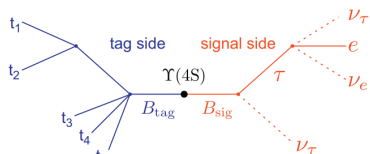
# The Rule of three

	BaBar	Belle	LHCb
#B's produced	O(400M)	O(700M)	O(800B)*
Production mechanism	$\Upsilon(4S) \rightarrow B\bar{B}$	$\Upsilon(4S) \rightarrow B\bar{B}$	$pp \rightarrow gg \rightarrow b\bar{b}$
Publications	Phys.Rev.Lett 109, 101802 (2012) Phys. Rev. D 88, 072012 (2013)	Phys.Rev.D 92, 072014 (2015) arXiv:1603.06711	Phys.Rev.Lett. 115, 111803 (2015)

# Experimental challenges

- ⇒ With the  $\tau \rightarrow \mu\nu\nu$  decay we are missing 3 neutrinos!
- ⇒ No sharp peak in any distributions.

⇒ At B-factories, can control this using tagging technique.



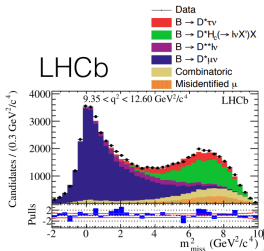
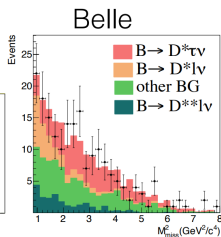
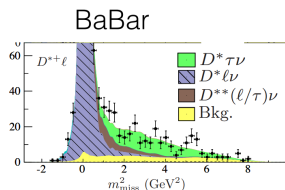
⇒ More difficult at LHCb, compensate using large boost (flight information) and huge B production



# Signal fits

⇒ Three main backgrounds:

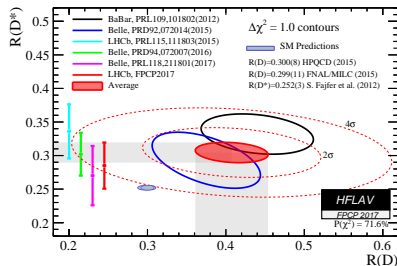
- $B \rightarrow D^* \ell \nu$ .
- $B \rightarrow D^{**} \ell \nu$ .
- $B \rightarrow DD^* X$



⇒ Fit variables which discriminate between the signal and background modes.

# Results

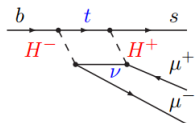
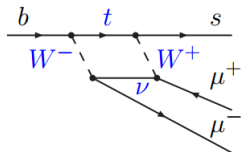
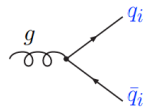
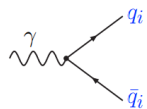
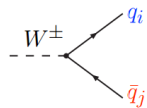
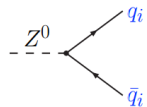
⇒ All experiments see an access w.r.t. to SM prediction:



- ⇒ Theoretical uncertainties negligible.
- ⇒ The ball is on the experimental side.

# Introduction to anomaly 2 & 3

- The SM allows only the charged interactions to change flavour.
  - Other interactions are flavour conserving.
- One can escape this constraint and produce  $b \rightarrow s$  and  $b \rightarrow d$  at loop level.
  - These kind of processes are suppressed in SM  $\rightarrow$  Rare decays.
  - New Physics can enter in the loops.

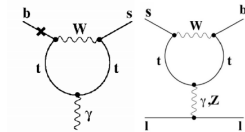


# Analysis of Rare decays

Analysis of FCNC in a model-independent approach, effective Hamiltonian:

$$b \rightarrow s\gamma(^*) : \mathcal{H}_{\Delta F=1}^{SM} \propto \sum_{i=1}^{10} V_{ts}^* V_{tb} \mathcal{C}_i \mathcal{O}_i + \dots$$

- $\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}$
- $\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b) (\bar{\ell}\gamma_\mu \ell)$
- $\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b) (\bar{\ell}\gamma_\mu \gamma_5 \ell), \dots$



- **SM** Wilson coefficients up to NNLO + e.m. corrections at  $\mu_{ref} = 4.8$  GeV [Misiak et al.]:

$$\mathcal{C}_7^{SM} = -0.29, \mathcal{C}_9^{SM} = 4.1, \mathcal{C}_{10}^{SM} = -4.3$$

- **NP** changes short distance  $\mathcal{C}_i - \mathcal{C}_i^{SM} = \mathcal{C}_i^{NP}$  and induce new operators, like

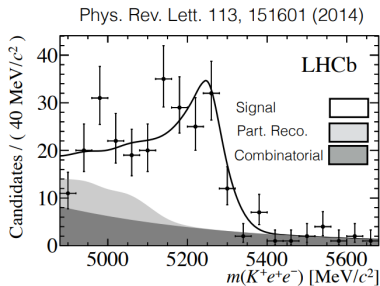
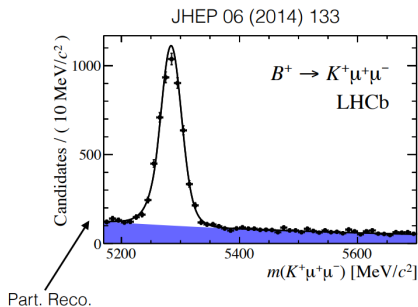
$\mathcal{O}'_{7,9,10} = \mathcal{O}_{7,9,10} (P_L \leftrightarrow P_R) \dots$  also scalars, pseudoscalar, tensor operators...

## Anomaly 2

$$R_{K/K^*} = \frac{\mathcal{B}(B \rightarrow K/K^* \mu\mu)}{\mathcal{B}(B \rightarrow K/K^* ee)}$$

# Measurement at LHCb

- ⇒ Most precise measurements performed at LHCb.
- ⇒ Main challenge is due to electron Bremsstrahlung.

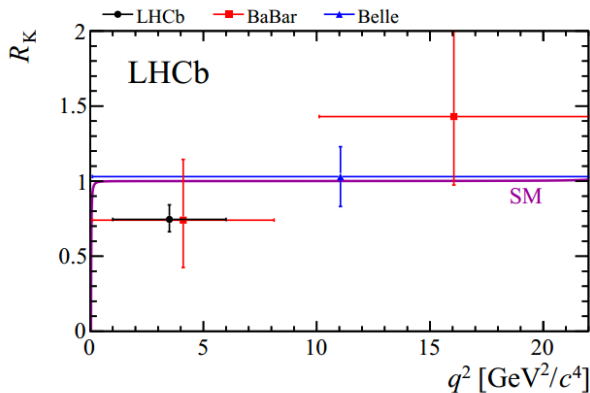


- ⇒ To protect ourselves from electron reconstruction issues we use a double ratio:

$$R_K = \frac{\mathcal{B}(B \rightarrow K \mu \mu) \times \mathcal{B}(B \rightarrow K J/\psi (\rightarrow ee))}{\mathcal{B}(B \rightarrow K ee) \times \mathcal{B}(B \rightarrow K J/\psi (\rightarrow \mu \mu))}$$

# Result

$$R_K = 0.745^{+0.090}_{-0.074}(\text{stat.}) \pm 0.036(\text{syst})$$



$\Rightarrow$  2.6  $\sigma$  away from SM prediction.

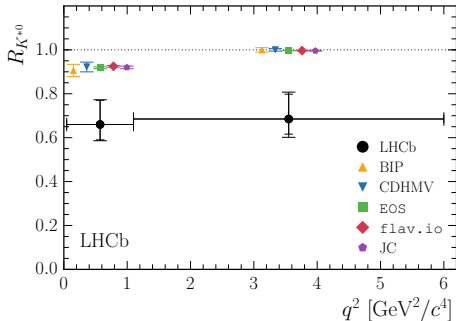
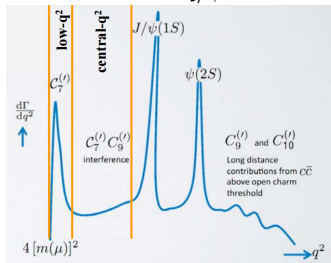
# The continuation - $R_{K^*}$

⇒ The neutral continuation of the  $R_K$  measurement is to measure its partner:

$$R_{K^*} = \frac{\mathcal{B}(B \rightarrow K^* \mu \mu)}{\mathcal{B}(B \rightarrow K^* e e)}$$

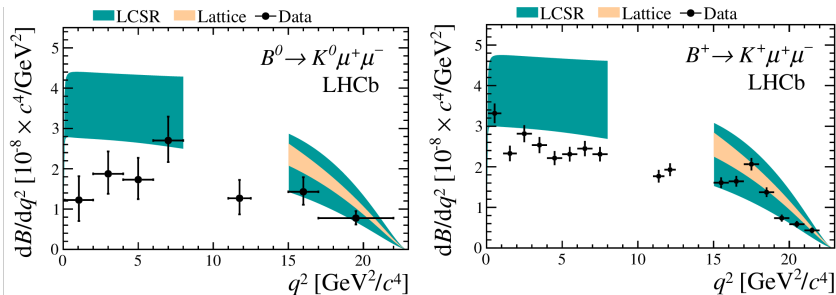
⇒ Measurement performed in two  $q^2$  bins.

⇒ Normalized in double ratio to  $B \rightarrow K^* J/\psi$ .

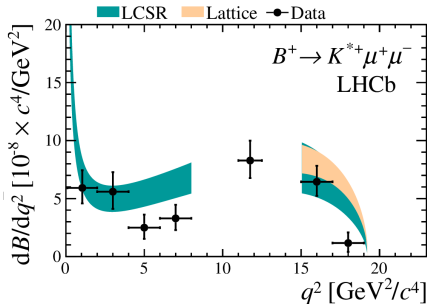




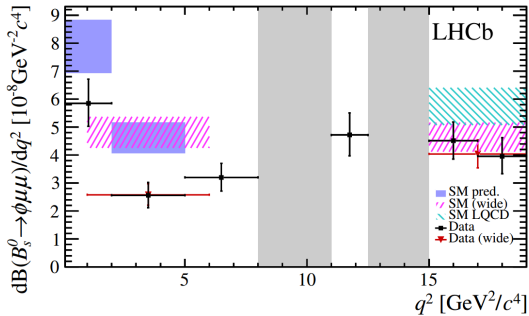
# Branching fraction measurements of $B \rightarrow K^{*\pm} \mu \mu$



- Despite large theoretical errors the results are consistently smaller than SM prediction.



# Branching fraction measurements of $B_s^0 \rightarrow \phi \mu \mu$



- Recent LHCb measurement [JHEPP09 (2015) 179].
- Suppressed by  $\frac{f_s}{f_d}$ .
- Cleaner because of narrow  $\phi$  resonance.
- $3.3 \sigma$  deviation in SM in the  $1 - 6 \text{ GeV}^2$  bin.

## Anomaly 3

$$P'_5 = \sqrt{2} \frac{\Re(A_{\perp}^L A_{\parallel}^{L*} - A_{\perp}^R A_{\parallel}^{R*})}{\sqrt{|A_0|^2 (|A_{\perp}|^2 + |A_0|^2)}}$$

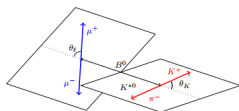
# $B^0 \rightarrow K^* \mu^- \mu^+$ kinematics

$\Rightarrow$  The kinematics of  $B^0 \rightarrow K^* \mu^- \mu^+$  decay is described by three angles  $\theta_l$ ,  $\theta_k$ ,  $\phi$  and invariant mass of the dimuon system ( $q^2$ ).

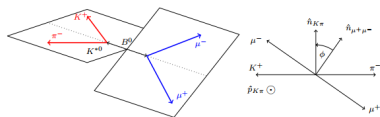
$\Rightarrow \cos \theta_k$ : the angle between the direction of the kaon in the  $K^*$  ( $\overline{K^*}$ ) rest frame and the direction of the  $K^*$  ( $\overline{K^*}$ ) in the  $B^0$  ( $\overline{B^0}$ ) rest frame.

$\Rightarrow \cos \theta_l$ : the angle between the direction of the  $\mu^-$  ( $\mu^+$ ) in the dimuon rest frame and the direction of the dimuon in the  $B^0$  ( $\overline{B^0}$ ) rest frame.

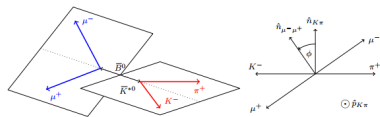
$\Rightarrow \phi$ : the angle between the plane containing the  $\mu^-$  and  $\mu^+$  and the plane containing the kaon and pion from the  $K^*$ .



(a)  $\theta_k$  and  $\theta_l$  definitions for the  $B^0$  decay



(b)  $\phi$  definition for the  $B^0$  decay



(c)  $\phi$  definition for the  $B^0$  decay

## $B^0 \rightarrow K^* \mu^- \mu^+$ kinematics

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$$\begin{aligned} \frac{d^4\Gamma}{dq^2 d\cos\theta_K d\cos\theta_l d\phi} = & \frac{9}{32\pi} \left[ J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_l \right. \\ & + J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\ & + (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\ & \left. + J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi \right], \end{aligned}$$

⇒ This is the most general expression of this kind of decay.

# Transversity amplitudes

⇒ One can link the angular observables to transversity amplitudes

$$J_{1s} = \frac{(2 + \beta_\ell^2)}{4} \left[ |A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2 \right] + \frac{4m_\ell^2}{q} \operatorname{Re} \left( A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*} \right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q} \left[ |A_t|^2 + 2\operatorname{Re}(A_0^L A_0^{R*}) \right] + \beta_\ell^2 |A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4} \left[ |A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2 \right], \quad J_{2c} = -\beta_\ell^2 \left[ |A_0^L|^2 + |A_0^R|^2 \right],$$

$$J_3 = \frac{1}{2} \beta_\ell^2 \left[ |A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2 \right], \quad J_4 = \frac{1}{\sqrt{2}} \beta_\ell^2 \left[ \operatorname{Re}(A_0^L A_\parallel^{L*} + A_0^R A_\parallel^{R*}) \right],$$

$$J_5 = \sqrt{2} \beta_\ell \left[ \operatorname{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \frac{m_\ell}{\sqrt{q^2}} \operatorname{Re}(A_\parallel^L A_S^* + A_\parallel^{R*} A_S) \right],$$

$$J_{6s} = 2\beta_\ell \left[ \operatorname{Re}(A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*}) \right], \quad J_{6c} = 4\beta_\ell \frac{m_\ell}{\sqrt{q^2}} \operatorname{Re}(A_0^L A_S^* + A_0^{R*} A_S),$$

$$J_7 = \sqrt{2} \beta_\ell \left[ \operatorname{Im}(A_0^L A_\parallel^{L*} - A_0^R A_\parallel^{R*}) + \frac{m_\ell}{\sqrt{q^2}} \operatorname{Im}(A_\perp^L A_S^* - A_\perp^{R*} A_S) \right],$$

$$J_8 = \frac{1}{\sqrt{2}} \beta_\ell^2 \left[ \operatorname{Im}(A_0^L A_\perp^{L*} + A_0^R A_\perp^{R*}) \right], \quad J_9 = \beta_\ell^2 \left[ \operatorname{Im}(A_\parallel^{L*} A_\perp^L + A_\parallel^{R*} A_\perp^R) \right],$$

## Link to effective operators

⇒ So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{\perp}^{L,R} = \sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} + C_9^{\text{eff}'}) \mp (C_{10} + C'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp}(E_{K^*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\perp}(E_{K^*})$$

$$A_0^{L,R} = -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_{K^*} \sqrt{\hat{s}}} \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C'_{10}) + 2\hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\parallel}(E_{K^*}),$$

where  $\hat{s} = q^2/m_B^2$ ,  $\hat{m}_i = m_i/m_B$ . The  $\xi_{\parallel,\perp}$  are the form factors.

## Link to effective operators

⇒ So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{\perp}^{L,R} = \sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} + C_9^{\text{eff}'}) \mp (C_{10} + C'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp}(E_{K^*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\perp}(E_{K^*})$$

$$A_0^{L,R} = -\frac{N m_B (1 - \hat{s})^2}{2\hat{m}_{K^*} \sqrt{\hat{s}}} \left[ (C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C'_{10}) + 2\hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{\parallel}(E_{K^*}),$$

where  $\hat{s} = q^2/m_B^2$ ,  $\hat{m}_i = m_i/m_B$ . The  $\xi_{\parallel,\perp}$  are the form factors.

⇒ Now we can construct observables that cancel the  $\xi$  form factors at leading order:

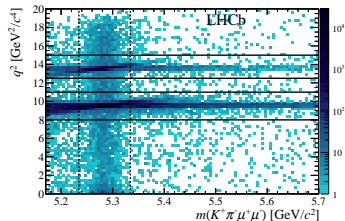
$$P'_5 = \frac{J_5 + \bar{J}_5}{2\sqrt{-(J_2^c + \bar{J}_2^c)(J_2^s + \bar{J}_2^s)}}$$



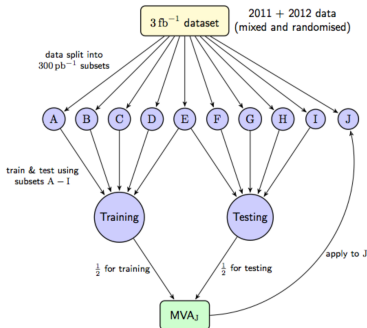
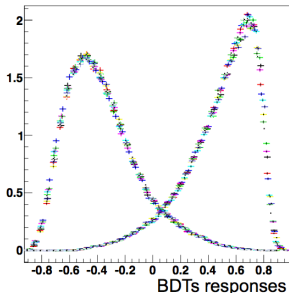
# LHCb measurement of $B_d^0 \rightarrow K^* \mu \mu$

# Multivariate simulation

- PID, kinematics and isolation variables used in a Boosted Decision Tree (BDT) to discriminate signal and background.
- BDT with k-Folding technique.
- Completely data driven.

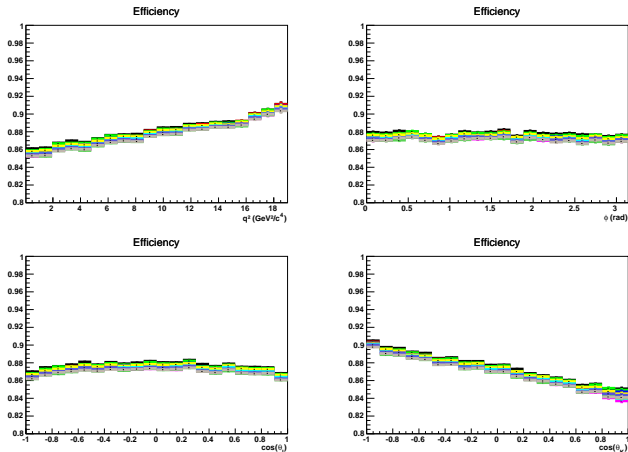


MVA\_baseline\_S



# Multivariate simulation, efficiency

⇒ BDT was also checked in order not to bias our angular distribution:



⇒ The BDT has small impact on our angular observables. We will correct for these effects later on.

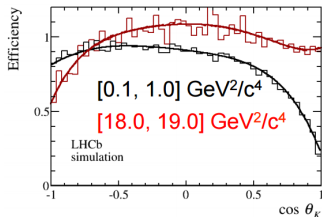
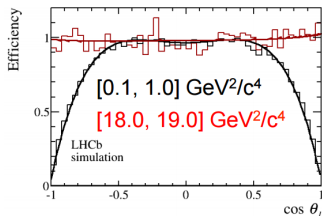
# Detector acceptance

- Detector distorts our angular distribution.
- We need to model this effect.
- 4D function is used:

$$\epsilon(\cos \theta_l, \cos \theta_k, \phi, q^2) = \sum_{ijkl} P_i(\cos \theta_l) P_j(\cos \theta_k) P_k(\phi) P_l(q^2),$$

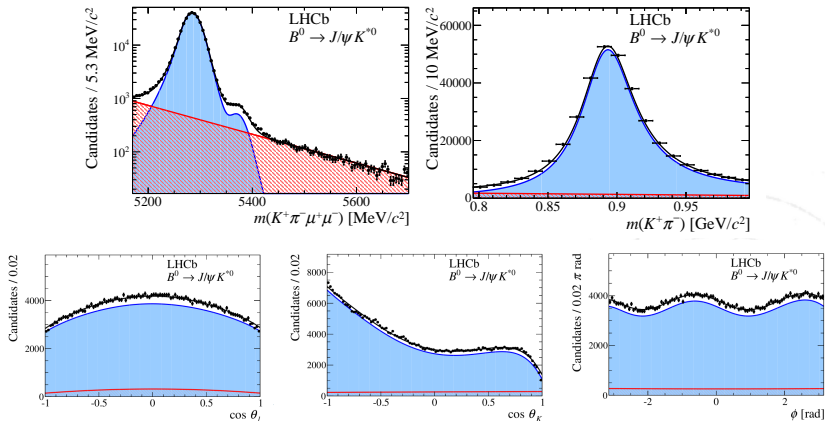
where  $P_i$  is the Legendre polynomial of order  $i$ .

- We use up to 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 5<sup>th</sup> order for the  $\cos \theta_l, \cos \theta_k, \phi, q^2$ .
- The coefficients were determined using Method of Moments, with a huge simulation sample.
- The simulation was done assuming a flat phase space and reweighting the  $q^2$  distribution to make it flat.
- To make this work the  $q^2$  distribution needs to be reweighted to be flat.



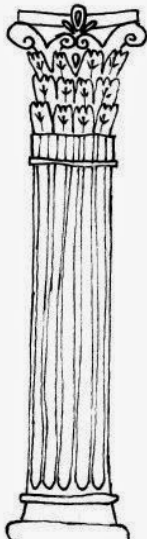
# Control channel

- We tested our unfolding procedure on  $B \rightarrow J/\psi K^*$ .
- The result is in perfect agreement with other experiments and our different analysis of this decay.

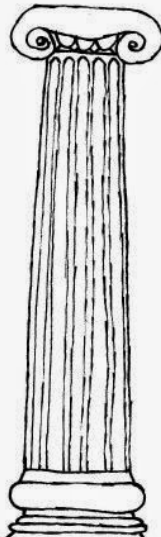


# The columns of New Physics

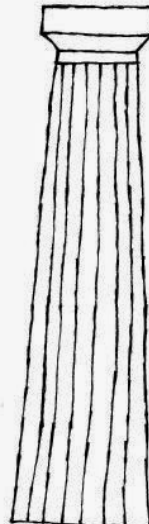
Amplitudes



Maximum likelihood fit



Method of Moments



# The columns of New Physics

## 1. Maximum likelihood fit:

- The most standard way of obtaining the parameters.
- Suffers from convergence problems, under coverages, etc. in low statistics.

## 2. Method of moments:

- Less precise than the likelihood estimator (10 – 15% larger uncertainties).
- Does not suffer from the problems of likelihood fit.

## 3. Amplitude fit:

- Incorporates all the physical symmetries inside the amplitudes! The most precise estimator.
- Has theoretical assumptions inside!

# Maximum likelihood fit - Results

⇒ In the maximum likelihood fit one could weight the events accordingly to the  $\frac{1}{\epsilon(\cos \theta_l, \cos \theta_k, \phi, q^2)}$

⇒ Better alternative is to put the efficiency into the maximum likelihood fit itself:

$$\mathcal{L} = \prod_{i=1}^N \epsilon_i(\Omega_i, q_i^2) \mathcal{P}(\Omega_i, q_i^2) / \int \epsilon(\Omega, q^2) \mathcal{P}(\Omega, q^2) d\Omega dq^2$$

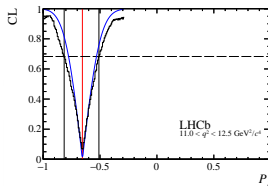
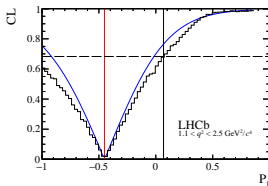
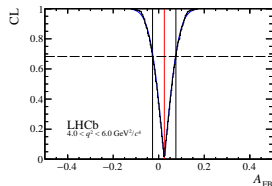
⇒ Only the relative weights matters!

⇒ The Procedure was commissioned with TOY MC study.

⇒ Use Feldmann-Cousins to determine the uncertainties.

⇒ Angular background component is modelled with 2<sup>nd</sup> order Chebyshev polynomials, which was tested on the side-bands.

⇒ S-wave component treated as nuisance parameter.





# Method of moments

⇒ See [Phys.Rev.D91\(2015\)114012](#), F.Beaujean , M.Chrzaszcz, N.Serra, D. van Dyk for details.

⇒ The idea behind Method of Moments is simple: Use orthogonality of spherical harmonics,  $f_j(\vec{\Omega})$  to solve for coefficients within a  $q^2$  bin:

$$\int f_i(\vec{\Omega})f_j(\vec{\Omega}) = \delta_{ij}$$

$$M_i = \int \left( \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \right) \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} f_i(\vec{\Omega}) d\Omega$$

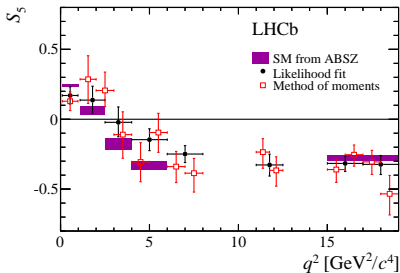
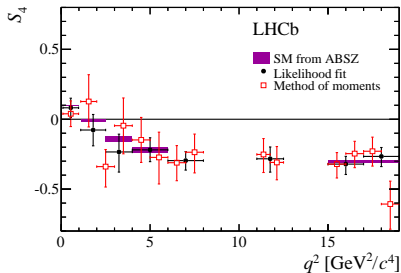
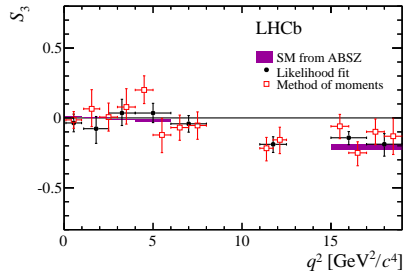
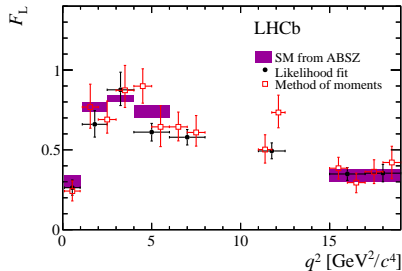
⇒ Don't have true angular distribution but we "sample" it with our data.

⇒ Therefore:  $\int \rightarrow \sum$  and  $M_i \rightarrow \hat{M}_i$

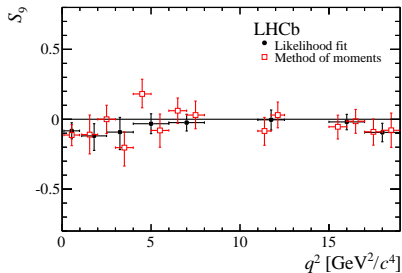
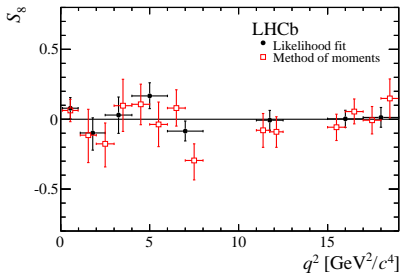
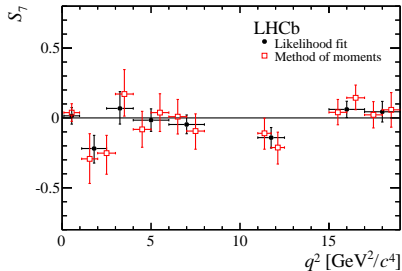
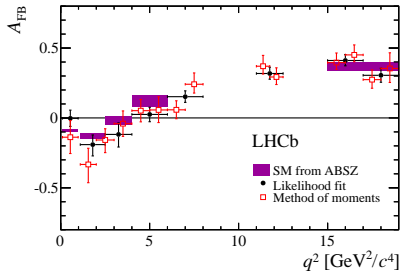
$$\hat{M}_i = \frac{1}{\sum_e \omega_e} \sum_e \omega_e f_i(\vec{\Omega}_e)$$

⇒ The weight  $\omega$  accounts for the efficiency. Again the normalization of weights does not matter.

# Method of moments - results

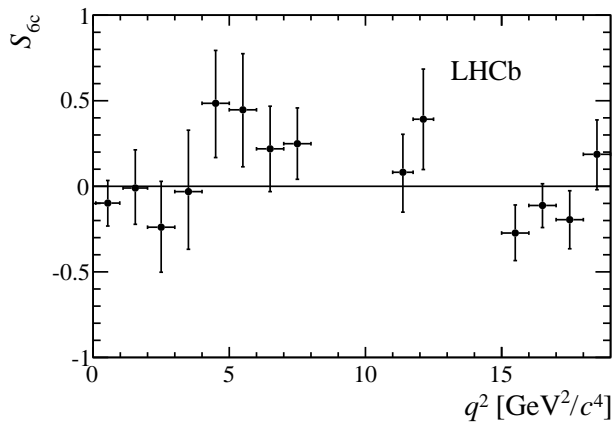


# Method of moments - results



# Method of moments - results

⇒ Method of Moments allowed us to measure for the first time a new observable:



# Compatibility with SM

⇒ Use EOS software package to test compatibility with SM.

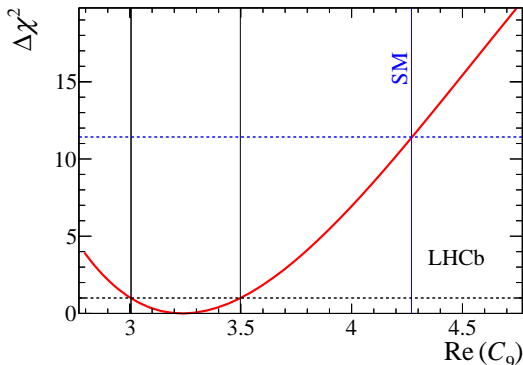
⇒ Perform the  $\chi^2$  fit to the measured:

$$F_L, A_{FB}, S_{3,\dots,9}.$$

⇒ Float a vector coupling:  $\Re(C_9)$ .

⇒ Best fit is found to be  $3.4 \sigma$  away from the SM.

$$\Delta\mathcal{R}(C_9) \equiv \mathcal{R}(C_9)^{\text{fit}} - \mathcal{R}(C_9)^{\text{SM}} = -1.03$$



# Global picture of $P_5'$

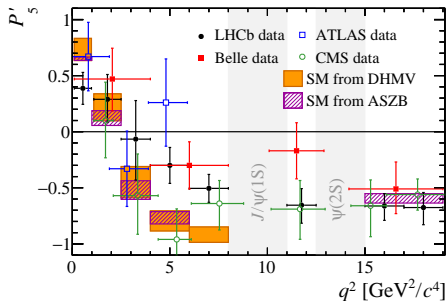
⇒ 2013 LHCb:  
arXiv::1308.1707

⇒ 2015 LHCb:  
arXiv::1512.0444

⇒ 2016 Belle:  
arXiv::1604.04042

⇒ 2017:  
ATLAS-CONF-2017-023  
( $20.5 \text{ fb}^{-1}$ ) and  
CMS-PAS-BPH-15-008  
( $20.8 \text{ fb}^{-1}$ )

⇒ Theory: DHMV: arXiv::1407.8526  
ASZB: arXiv::1411.3161



# Global fit to $b \rightarrow sll$ measurements

# Link the observables

⇒ Fits prepare by S. Descotes-Genon, L. Hofer, J. Matias, J. Virto, [arXiv::1510.04239](https://arxiv.org/abs/1510.04239)

- Inclusive

- $B \rightarrow X_s \gamma$  ( $BR$ ) .....  $c_7^{(\prime)}$
- $B \rightarrow X_s \ell^+ \ell^-$  ( $dBR/dq^2$ ) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$

- Exclusive leptonic

- $B_s \rightarrow \ell^+ \ell^-$  ( $BR$ ) .....  $c_{10}^{(\prime)}$

- Exclusive radiative/semileptonic

- $B \rightarrow K^* \gamma$  ( $BR, S, A_T$ ) .....  $c_7^{(\prime)}$
- $B \rightarrow K \ell^+ \ell^-$  ( $dBR/dq^2$ ) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- **$B \rightarrow K^* \ell^+ \ell^-$**  ( $dBR/dq^2$ , **Optimized Angular Obs.**) ..  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- $B_s \rightarrow \phi \ell^+ \ell^-$  ( $dBR/dq^2$ , Angular Observables) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$  (None so far)
- etc.



# Statistic details

⇒ Frequentist approach:

$$\chi^2(C_i) = [O_{\text{exp}} - O_{\text{th}}(C_i)]_j [\text{Cov}^{-1}]_{jk} [O_{\text{exp}} - O_{\text{th}}(C_i)]_k$$

- $\mathbf{Cov} = \mathbf{Cov}^{\text{exp}} + \mathbf{Cov}^{\text{th}}$ . We have  $\text{Cov}^{\text{exp}}$  for the first time
- Calculate  $\text{Cov}^{\text{th}}$ : correlated multigaussian scan over all nuisance parameters
- $\text{Cov}^{\text{th}}$  depends on  $C_i$ : Must check this dependence

For the Fit:

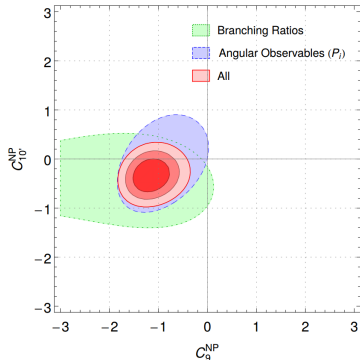
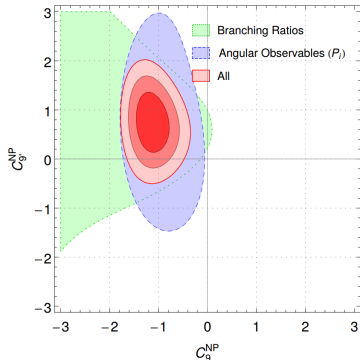
- Minimise  $\chi^2 \rightarrow \chi_{\text{min}}^2 = \chi^2(C_i^0)$  (Best Fit Point =  $C_i^0$ )
- Confidence level regions:  $\chi^2(C_i) - \chi_{\text{min}}^2 < \Delta\chi_{\sigma,n}$

⇒ The results from 1D scans:

Coefficient $C_i^{\text{NP}} = C_i - C_i^{\text{SM}}$	Best fit	$1\sigma$	$3\sigma$	$\text{Pull}_{\text{SM}}$
$C_9^{\text{NP}}$	-1.09	[-1.29, -0.87]	[-1.67, -0.39]	4.5 ←
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.68	[-0.85, -0.50]	[-1.22, -0.18]	4.2 ←
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}}$	-1.06	[-1.25, -0.86]	[-1.60, -0.40]	4.8 ← (no $R_{\text{F}}$ )

# Theory implications

- The data can be explained by modifying the  $C_9$  Wilson coefficient.
- Overall there is around  $4.5 \sigma$  discrepancy wrt. SM.

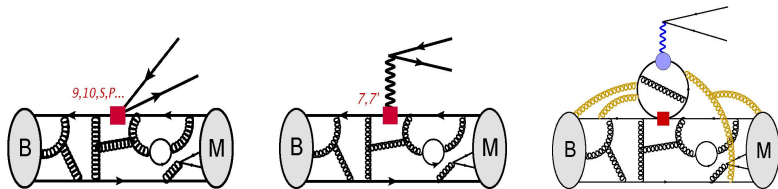


## 2D scans

Coefficient	Best Fit Point	Pull <sub>SM</sub>
$(\mathcal{C}_7^{\text{NP}}, \mathcal{C}_9^{\text{NP}})$	$(-0.00, -1.07)$	<b>4.1</b>
$(\mathcal{C}_9^{\text{NP}}, \mathcal{C}_{10}^{\text{NP}})$	$(-1.08, 0.33)$	<b>4.3</b>
$(\mathcal{C}_9^{\text{NP}}, \mathcal{C}_{7'}^{\text{NP}})$	$(-1.09, 0.02)$	<b>4.2</b>
$(\mathcal{C}_9^{\text{NP}}, \mathcal{C}_{9'}^{\text{NP}})$	$(-1.12, 0.77)$	<b>4.5</b>
$(\mathcal{C}_9^{\text{NP}}, \mathcal{C}_{10'}^{\text{NP}})$	$(-1.17, -0.35)$	<b>4.5</b>
$(\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}, \mathcal{C}_{10}^{\text{NP}} = \mathcal{C}_{10'}^{\text{NP}})$	$(-1.15, 0.34)$	<b>4.7</b>
$(\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}, \mathcal{C}_{10}^{\text{NP}} = -\mathcal{C}_{10'}^{\text{NP}})$	$(-1.06, 0.06)$	<b>4.4</b>
$(\mathcal{C}_9^{\text{NP}} = \mathcal{C}_{9'}^{\text{NP}}, \mathcal{C}_{10}^{\text{NP}} = \mathcal{C}_{10'}^{\text{NP}})$	$(-0.64, -0.21)$	3.9
$(\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}}, \mathcal{C}_{9'}^{\text{NP}} = \mathcal{C}_{10'}^{\text{NP}})$	$(-0.72, 0.29)$	3.8

- $\mathcal{C}_9^{\text{NP}}$  always play a dominant role
- All 2D scenarios above  $4\sigma$  are quite indistinguishable.

# $B \rightarrow K^* \ell \ell$ Amplitudes



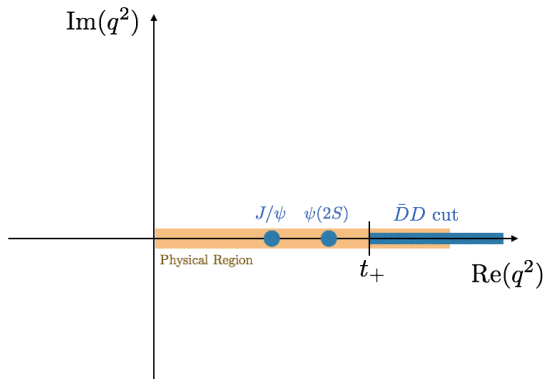
$$A_{\lambda}^{L,R} = N_{\lambda} \left\{ (C_9 \mp C_{10}) \mathcal{F}_{\lambda}(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_{\lambda}^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_{\lambda}(q^2) \right] \right\}$$

- ▶ Local (Form Factors) :  $\mathcal{F}_{\lambda}^{(T)}(q^2) = \langle \bar{M}_{\lambda}(k) | \bar{s} \Gamma_{\lambda}^{(T)} b | \bar{B}(k+q) \rangle$
- ▶ Non-Local :  $\mathcal{H}_{\lambda}(q^2) = i \mathcal{P}_{\mu}^{\lambda} \int d^4x e^{iq \cdot x} \langle \bar{M}_{\lambda}(k) | T \{ \mathcal{J}_{em}^{\mu}(x), \mathcal{C}_i \mathcal{O}(0) \} | \bar{B}(q+k) \rangle$
- ▶ CKM structure :  $\mathcal{H}_{\lambda} = -\frac{\lambda_u}{\lambda_t} \mathcal{H}_{\lambda}^{(u)} - \frac{\lambda_c}{\lambda_t} \mathcal{H}_{\lambda}^{(c)} \Rightarrow \mathcal{O} \sim (\bar{c}b)(\bar{s}c)$

# Analytic structure of $\mathcal{H}_\lambda(q^2)$

[Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305]

Neglecting OZI- and CKM-suppressed contributions :

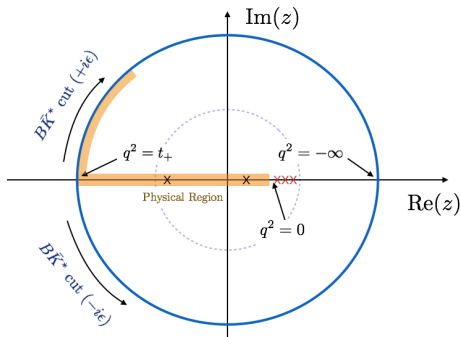
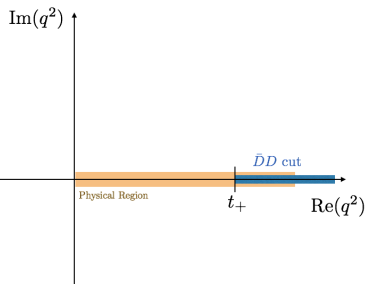


$$\hat{\mathcal{H}}_\lambda(q^2) = (q^2 - M_{J/\psi}^2)(q^2 - M_{\psi(2S)}^2) \mathcal{H}_\lambda(q^2) \quad \text{has no poles.}$$

# Accessing $q^2 > 0$ : $z$ expansion

[Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305]

► Conformal mapping :  $q^2 \mapsto z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$



- $\hat{\mathcal{H}}_\lambda(q^2(z))$  is analytic in  $|z| < 1$
- Taylor expand  $\hat{\mathcal{H}}_\lambda(z)$  around  $z = 0$ .
- Expansion needed for  $|z| < 0.52$  ( $-7 \text{ GeV}^2 \leq q^2 \leq 14 \text{ GeV}^2$ )

# Accessing $q^2 > 0$ : $z$ expansion

[Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305]

## Some details for actual parametrisation :

- ▶ Try to capture most features of the expansion (better convergence)
- ▶ Parametrize the ratios  $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$  instead
- ▶ The poles should not modify the asymptotic behaviour at  $|q^2| \rightarrow \infty$

$$\mathcal{H}_\lambda(z) = \frac{1 - z z_{J/\psi}^*}{z - z_{J/\psi}} \frac{1 - z z_{\psi(2S)}^*}{z - z_{\psi(2S)}} \hat{\mathcal{H}}_\lambda(z)$$

$$\hat{\mathcal{H}}_\lambda(z) = \left[ \sum_{k=0}^K \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$$

where  $\alpha_k^{(\lambda)}$  are complex coefficients, and the expansion is truncated after the term  $z^K$ . We will take  $K = 2$  (16 real parameters).

# Experimental constraints on $z$ parametrisation

[Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305]

## Experimental constraints :

- ▶ The residues of the poles are given by  $B \rightarrow K^* \psi_n$  :

$$\mathcal{H}_\lambda(q^2 \rightarrow M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \dots$$

- ▶ Angular analyses [Belle, Babar, LHCb] determine :

$$|r_\perp^{\psi_n}|, |r_\parallel^{\psi_n}|, |r_0^{\psi_n}|, \arg\{r_\perp^{\psi_n} r_0^{\psi_n*}\}, \arg\{r_\parallel^{\psi_n} r_0^{\psi_n*}\},$$

where  $r_\lambda^{\psi_n} \equiv \text{Res}_{q^2 \rightarrow M_{\psi_n}^2} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 \mathcal{F}_\lambda(M_{\psi_n}^2)}$

- ▶ We produce correlated pseudo-observables from a fit (5+5).



# Prior Fit to $\simeq$ parametrisation

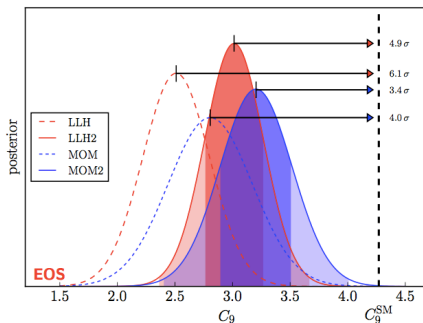
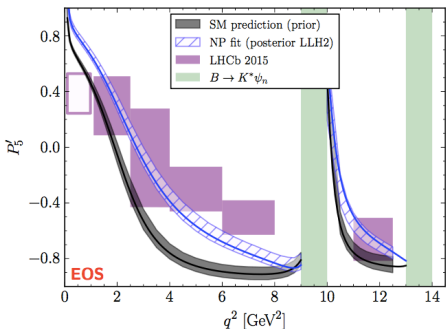
(Prior) Fit to Experimental and theoretical pseudo-observables :

$k$	0	1	2
$\text{Re}[\alpha_k^{(\perp)}]$	$-0.06 \pm 0.21$	$-6.77 \pm 0.27$	$18.96 \pm 0.59$
$\text{Re}[\alpha_k^{(\parallel)}]$	$-0.35 \pm 0.62$	$-3.13 \pm 0.41$	$12.20 \pm 1.34$
$\text{Re}[\alpha_k^{(0)}]$	$0.05 \pm 1.52$	$17.26 \pm 1.64$	–
$\text{Im}[\alpha_k^{(\perp)}]$	$-0.21 \pm 2.25$	$1.17 \pm 3.58$	$-0.08 \pm 2.24$
$\text{Im}[\alpha_k^{(\parallel)}]$	$-0.04 \pm 3.67$	$-2.14 \pm 2.46$	$6.03 \pm 2.50$
$\text{Im}[\alpha_k^{(0)}]$	$-0.05 \pm 4.99$	$4.29 \pm 3.14$	–

Table: Mean values and standard deviations (in units of  $10^{-4}$ ) of the prior PDF for the parameters  $\alpha_k^{(\lambda)}$ .

# New Physics Analysis

SM predictions and Fit including  $B \rightarrow K^* \mu^+ \mu^-$  data and  $C_9^{\text{NP}}$  :

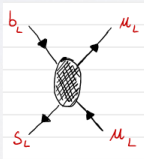
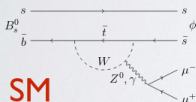


The NP hypothesis with  $C_9^{\text{NP}} \sim -1$  is favored strongly in the global fit

# Scale of NP?

$$b \rightarrow s \mu \mu \quad (\text{LHCb from 2013})$$

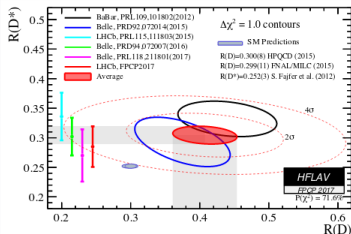
- 1) Angular observables in  $B \rightarrow K^* \mu^+ \mu^- \sim 4\sigma$  (!)
  - 2) Branching ratios  $\gtrsim 3.5\sigma$  (!)
  - 3) LFU violation in  $R_K$   $2.6\sigma$
  - 4) LFU violation in  $R_{K^*}$  (2 bins)  $2.3\sigma, 2.6\sigma$
- “clean” only  $\approx 4\sigma$



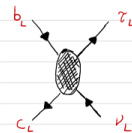
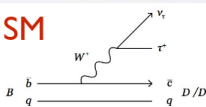
$$d_{\text{eff}} = \frac{1}{\Lambda_{R_K}^2} \bar{s}_L \gamma^\mu b_L \bar{\mu}_L \gamma_\mu \mu_L + h.c.$$

$$|C_\mu^{\text{NP}}| \gg |C_e^{\text{NP}}| \quad \Lambda_{R_K} = 31 \text{ TeV}$$

$$b \rightarrow c \tau \nu \quad \text{Babar+Belle+LHCb from 2012}$$



SM



$$d_{\text{eff}} = -\frac{2}{\Lambda_{R_D}^2} \bar{c}_L \gamma^\mu b_L \bar{\tau}_L \gamma_\mu \nu_L + h.c.$$

$$|C_\tau^{\text{NP}}| \gg |C_\mu^{\text{NP}}|, |C_e^{\text{NP}}| \quad \Lambda_{R_D} = 3.4 \text{ TeV}$$

⇒ Stolen from M. Nardecchia

# Conclusions

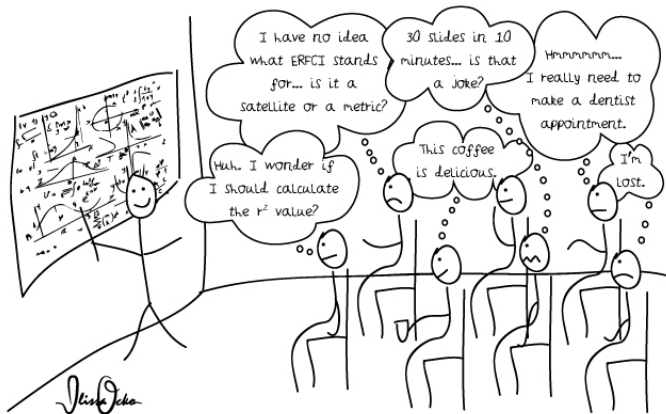
- Clear tensions wrt. SM predictions!
- Measurements cluster in the same direction.
- We are not opening the champagne yet!
- Still need improvement both on theory and experimental side.
- Time will tell if this is QCD+fluctuations or new Physics:

# Conclusions

- Clear tensions wrt. SM predictions!
- Measurements cluster in the same direction.
- We are not opening the champagne yet!
- Still need improvement both on theory and experimental side.
- Time will tell if this is QCD+fluctuations or new Physics:

“... when you have eliminated all the Standard Model explanations, whatever remains, however improbable, must be New Physics.”  
prof. Joaquim Matias

# Thank you for the attention!



# Backup

# Amplitudes method

⇒ Fit for amplitudes as (continuous) functions of  $q^2$  in the region:  $q^2 \in [1.1.6.0] \text{ GeV}^2/c^4$ .

⇒ Needs some Ansatz:

$$A(q^2) = \alpha + \beta q^2 + \frac{\gamma}{q^2}$$

⇒ The assumption is tested extensively with toys.

⇒ Set of 3 complex parameters  $\alpha, \beta, \gamma$  per vector amplitude:

- $L, R, 0, \parallel, \perp, \Re, \Im \mapsto 3 \times 2 \times 3 \times 2 = 36$  DoF.
- Scalar amplitudes: +4 DoF.
- Symmetries of the amplitudes reduces the total budget to: 28.

⇒ The technique is described in [JHEP06\(2015\)084](#).

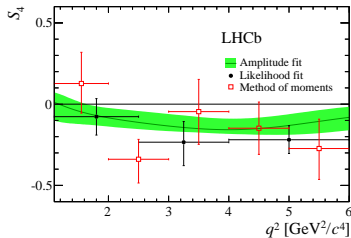
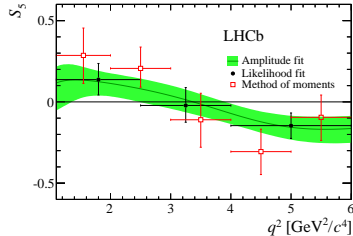
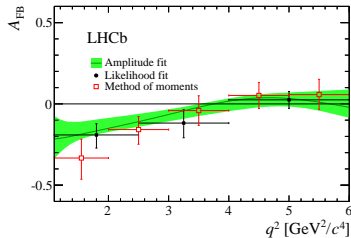
⇒ Allows to build the observables as continuous functions of  $q^2$ :

- At current point the method is limited by statistics.
- In the future the power of this method will increase.

⇒ Allows to measure the zero-crossing points for free and with smaller errors than previous methods.



# Amplitudes - results



Zero crossing points:

$$q_0(S_4) < 2.65 \quad \text{at } 95\% \text{ CL}$$

$$q_0(S_5) \in [2.49, 3.95] \quad \text{at } 68\% \text{ CL}$$

$$q_0(A_{FB}) \in [3.40, 4.87] \quad \text{at } 68\% \text{ CL}$$