Unfolding for counting experiments

Marcin Chrząszcz[1,2], Nicola Serra[1]

[1] University of Zurich,
[2] Institute of Nuclear Physics, Krakow

June 24, 2014

Reminder 1 - Constructing Matrix unfolding

- We don't know explicate

$$\epsilon(\cos\theta_k, \cos\theta_l, \phi) \tag{1}$$

- We don't need to know it, we just need to calculate matrix elements
- Let's use PHSP MC.
- Moments for PHSP MC are:
  $v_{gen}^T = (2/3, 0, 0, 0, 0, 0, 0, 0)$
- After reconstruction we get(full q^2 range): $v_{rec}^T =$
  $(0.7069, 0.0077, -0.00236466, 0.0005, 0.0007, 0.0011, 0.0011, -0.0012)$

University of Zurich[UZH]

Reminder 2 - Constructing Matrix unfolding

- We got first column of the unfolding matrix $(\frac{3}{2} v_{gen})$.

$$\begin{pmatrix} 1.06 & \cdots & a_{1,8} \\ 0.01157 & \cdots & a_{2,8} \\ -0.003547 & \ddots & \vdots \\ 0.0007841 & \ddots & \vdots \\ 0.001126 & \ddots & \vdots \\ 0.001766 & \ddots & \vdots \\ 0.001664 & \ddots & \vdots \\ -0.001937 & \cdots & a_{8,8} \end{pmatrix}$$

- How about the others?
- We can reweight accordingly to f_x.

Reminder 3 - Constructing Matrix unfolding

- To get S_3 each event i^{th} has has weight $f_{S_3}(\cos\theta_{k_i}, \cos\theta_{l_i}, \phi_i)$
- One can calculate on MC the reweighed moments in PHPS:

$$\int PDF * f_{S_3} = \frac{32}{225} \tag{2}$$

- Our base vector now is: $v^T_{gen} = (0, \frac{32}{225}, 0, 0, 0, 0, 0, 0)$
- So lets see what do we get as reconstructed vector(after multiplying by $\frac{225}{32}$. $v^T_{rec} =$ $(0.042, 1.105, -0.005, 0.003, -0.0023, -0.005, -0.005, -0.006)$
- Please notice that weights are negative, but this is not a problem for the mean.
- Also we are avoiding the negative PDF problem :)

Reminder 4 - Constructing Matrix unfolding

- Now the matrix looks like:

$$\begin{pmatrix} 1.06 & 0.042 & \cdots & a_{1,8} \\ 0.01157 & 1.105 & \cdots & a_{2,8} \\ -0.003547 & -0.005 & \ddots & \vdots \\ 0.0007841 & -0.005 & \ddots & \vdots \\ 0.001126 & 0.003 & \ddots & \vdots \\ 0.001766 & -0.0023 & \ddots & \vdots \\ 0.001664 & -0.005 & \ddots & \vdots \\ -0.001937 & -0.006 & \cdots & a_{8,8} \end{pmatrix}$$

- The others go in the same way.
- Repenting this exercise from 1^{st} year algebra we can get the full matrix

University of Zurich[UZH]

Reminder 5

For now:

- We have proven that there has to exists unfolding matrix.
- Shown how to construct transformation matrix: $Gen \rightarrow Reco$.
- Inverting it we can have transformation matrix of $Reco \rightarrow Gen$.
- For details: LINK

What is missing?

1. ERROR!

University of Zurich[UZH]

How to?

- So lets say that transformation matrix: $Gen \rightarrow Reco$ is $\epsilon_{i,j}$.
- Each element has an error: $\delta\epsilon_{i,j}$.
- Then we can calculate the matrix: $\epsilon^{-1}_{i,j}$ (assuming it exists).
- The million dollar question is what is the error on inverted matrix?

Answer to 1M dolar quesion

- One can toy it.
- But toying is good for kids and Frequentist.

Answer to 1M dolar quesion

- One can toy it.
- But toying is good for kids and Frequentist.
- Solution comes from τ physics :) hep-ex/9909031
- One can derive(prove in the paper) the general equation:

$$\delta\epsilon^{-1}_{\alpha\ \beta} = [\epsilon^{-1}]^2_{\alpha i}[\delta\epsilon]^2_{ij}[\epsilon^{-1}]^2_{j\beta} \tag{3}$$

Matrix, $0.1 - 0.98$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9503 & -0.0003394 & -0.01202 & -0.003648 & 0.02225 & -0.04174 & -0.02515 & 0.002239 \\ -0.0009524 & 0.8219 & 0.00187 & 0.003567 & 0.002677 & 0.02974 & 0.006871 & 0.01478 \\ -0.008739 & 0.002372 & 1.031 & 0.008389 & -0.001613 & 0.00138 & 0.02255 & 9.205e-05 \\ -0.001247 & 0.003686 & 0.0106 & 0.9235 & -0.004257 & -0.001611 & 0.00106 & -0.004336 \\ 0.008308 & 0.003482 & 0.0003312 & -0.005127 & 1.185 & -0.01059 & -0.01158 & 0.002114 \\ -0.01522 & 0.03697 & 0.002679 & -0.002175 & -0.01001 & 0.9441 & 0.02969 & -0.008816 \\ -0.00842 & 0.007239 & 0.02186 & 0.001552 & -0.009282 & 0.02371 & 1.067 & -0.006338 \\ 0.0001436 & 0.01337 & 0.0004291 & -0.00291 & 0.002099 & -0.006733 & -0.006519 & 0.8188 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005272 & 0.01937 & 0.03246 & 0.02156 & 0.02205 & 0.02222 & 0.03369 & 0.01913 \\ 0.006689 & 0.04713 & 0.03204 & 0.02452 & 0.03633 & 0.02548 & 0.0338 & 0.03364 \\ 0.007514 & 0.03227 & 0.07738 & 0.04294 & 0.04653 & 0.03088 & 0.05652 & 0.03184 \\ 0.007932 & 0.0389 & 0.06793 & 0.04878 & 0.03828 & 0.03536 & 0.04956 & 0.0388 \\ 0.008985 & 0.04197 & 0.06272 & 0.0327 & 0.07225 & 0.03349 & 0.06521 & 0.04161 \\ 0.008064 & 0.03943 & 0.04825 & 0.03506 & 0.0386 & 0.0507 & 0.07108 & 0.03956 \\ 0.00767 & 0.0327 & 0.05558 & 0.03081 & 0.04743 & 0.04477 & 0.08193 & 0.03241 \\ 0.006676 & 0.0336 & 0.03765 & 0.0293 & 0.03582 & 0.03014 & 0.0391 & 0.04682 \end{pmatrix}$$

Matrix, $1.1 - 2\ GeV$

$$A_{reco \to gen} = \begin{pmatrix} 0.9519 & -0.02665 & -0.01432 & 0.002356 & 0.02539 & 0.009878 & -0.01551 & -0.01874 \\ -0.006272 & 0.8122 & -0.00351 & -0.00719 & 0.003585 & 6.784e-05 & 0.02445 & 0.008515 \\ -0.005315 & -0.003716 & 1.048 & 0.01242 & 0.01209 & -0.01478 & -0.001956 & 0.01429 \\ 0.003237 & -0.007177 & 0.01533 & 0.9184 & -0.007548 & -0.0009818 & -0.01874 & 0.009407 \\ 0.01002 & 0.004084 & 0.01391 & -0.006509 & 1.194 & -0.006516 & 0.001536 & -0.02882 \\ 0.002695 & -0.001042 & -0.01721 & -0.001842 & -0.005643 & 0.9264 & 0.02106 & 0.006755 \\ -0.004736 & 0.02346 & -0.002335 & -0.01446 & 0.001169 & 0.01697 & 1.072 & -0.003191 \\ -0.004157 & 0.007576 & 0.01377 & 0.008058 & -0.02219 & 0.005354 & -0.0008608 & 0.8304 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005202 & 0.01911 & 0.03258 & 0.02103 & 0.02252 & 0.02145 & 0.03366 & 0.01948 \\ 0.006648 & 0.04654 & 0.03227 & 0.02451 & 0.03602 & 0.02464 & 0.03298 & 0.03397 \\ 0.007557 & 0.03197 & 0.07845 & 0.04272 & 0.04744 & 0.03057 & 0.05698 & 0.03287 \\ 0.007902 & 0.03885 & 0.0678 & 0.04839 & 0.0384 & 0.03464 & 0.04925 & 0.03989 \\ 0.009015 & 0.04122 & 0.06374 & 0.03254 & 0.07349 & 0.03269 & 0.0649 & 0.04202 \\ 0.007939 & 0.0389 & 0.04793 & 0.03433 & 0.03828 & 0.04937 & 0.06985 & 0.04023 \\ 0.007651 & 0.03234 & 0.05611 & 0.03062 & 0.04776 & 0.04388 & 0.08157 & 0.03342 \\ 0.006719 & 0.03345 & 0.03868 & 0.02953 & 0.03633 & 0.03002 & 0.03989 & 0.04827 \end{pmatrix}$$

Matrix, $2-3$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9453 & 0.04 & 0.00177 & -0.03031 & 0.04764 & 0.0151 & -0.01258 & -0.0410 \\ 0.01028 & 0.8527 & -5.812e-05 & -0.0136 & 0.005076 & -0.000505 & 0.01096 & 0.00640 \\ -0.0001227 & 0.001005 & 1.093 & -0.001136 & -0.005139 & 0.02279 & -0.01434 & 0.00803 \\ -0.01341 & -0.01719 & -0.001032 & 0.9839 & -0.0002435 & -0.00289 & 0.02836 & 0.01162 \\ 0.01327 & 0.007413 & -0.005841 & 0.0007396 & 1.061 & -0.00545 & 0.01544 & 0.00419 \\ 0.00528 & -0.0006148 & 0.0305 & -0.004015 & -0.00688 & 0.952 & 0.01271 & -0.0145 \\ -0.006055 & 0.01113 & -0.0148 & 0.02406 & 0.01406 & 0.01007 & 1.045 & -0.00297 \\ -0.009041 & 0.004813 & 0.00788 & 0.00857 & 0.004315 & -0.01074 & -0.003336 & 0.8538 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 004924 & 0.01926 & 0.03256 & 0.0219 & 0.01872 & 0.02106 & 0.03102 & 0.01924 \\ 006515 & 0.04928 & 0.03368 & 0.02672 & 0.03054 & 0.02542 & 0.03166 & 0.03499 \\ 007373 & 0.03367 & 0.08093 & 0.04581 & 0.04074 & 0.03073 & 0.05367 & 0.03345 \\ 007823 & 0.04156 & 0.07072 & 0.05291 & 0.03305 & 0.03571 & 0.04708 & 0.04186 \\ 008104 & 0.03815 & 0.05952 & 0.03127 & 0.05851 & 0.03006 & 0.05651 & 0.03821 \\ 007692 & 0.04103 & 0.04888 & 0.03655 & 0.03267 & 0.04975 & 0.06635 & 0.04107 \\ 007215 & 0.03316 & 0.0552 & 0.03129 & 0.03971 & 0.04303 & 0.07527 & 0.03281 \\ 006517 & 0.03465 & 0.03922 & 0.03152 & 0.03039 & 0.03023 & 0.03738 & 0.04986 \end{pmatrix}$$

Matrix, $3 - 4$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9485 & -0.03531 & 0.03566 & -0.01797 & -0.01638 & 0.01105 & -0.0009454 & 0.01741 \\ -0.0103 & 0.8755 & 0.01175 & -0.0118 & -0.0161 & 0.002323 & -0.0006286 & 0.002953 \\ 0.01246 & 0.01032 & 1.029 & -0.02871 & -0.001326 & -0.0006531 & 0.02134 & -0.00130 \\ -0.009267 & -0.01528 & -0.03585 & 0.983 & 0.01469 & 0.01909 & -0.0007814 & 0.01507 \\ -0.005961 & -0.02135 & -0.0002244 & 0.01444 & 0.9926 & -0.0004736 & -0.002663 & 0.003707 \\ 0.002087 & 0.002114 & -0.001774 & 0.02071 & -0.0001252 & 1.028 & -0.00786 & -0.00738 \\ 0.001439 & -0.0009891 & 0.02285 & -0.001455 & -0.001524 & -0.00625 & 1.095 & 0.008555 \\ 0.005678 & 0.002526 & -0.0023 & 0.01341 & 0.002966 & -0.004732 & 0.007783 & 0.9021 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005044 & 0.02013 & 0.03103 & 0.02225 & 0.01764 & 0.02351 & 0.03312 & 0.02092 \\ 0.006623 & 0.05256 & 0.03149 & 0.02695 & 0.02883 & 0.02852 & 0.03422 & 0.03844 \\ 0.007229 & 0.03412 & 0.07348 & 0.04506 & 0.03619 & 0.03339 & 0.05534 & 0.03524 \\ 0.007892 & 0.04374 & 0.06735 & 0.05351 & 0.03086 & 0.03969 & 0.0508 & 0.04497 \\ 0.007871 & 0.03822 & 0.05248 & 0.02984 & 0.05276 & 0.03153 & 0.05642 & 0.03947 \\ 0.008018 & 0.04425 & 0.04817 & 0.03854 & 0.03142 & 0.05783 & 0.07315 & 0.0465 \\ 0.007405 & 0.03509 & 0.05327 & 0.03269 & 0.03754 & 0.04886 & 0.08115 & 0.03668 \\ 0.006716 & 0.03761 & 0.03789 & 0.03252 & 0.02902 & 0.03459 & 0.04089 & 0.05595 \end{pmatrix}$$

Matrix, $4 - 5\ GeV$

$$A_{reco \to gen} = \begin{pmatrix} 0.9376 & -0.01202 & 0.01084 & -0.001334 & -0.01096 & -0.007861 & 0.04462 & -0.02943 \\ -0.003862 & 0.8835 & 0.005033 & -0.0009372 & 0.01946 & 0.0005903 & -0.01475 & -0.01096 \\ 0.008222 & 0.005038 & 1.028 & 0.02931 & -0.001236 & 0.002661 & -0.005131 & 0.003024 \\ 0.001399 & 0.0001528 & 0.03615 & 0.999 & 0.003816 & -0.01991 & 0.003867 & -0.004392 \\ -0.00087 & 0.02452 & -0.002084 & 0.003934 & 0.965 & 0.01721 & -0.01357 & 0.001678 \\ -0.001301 & -0.0004482 & 0.003296 & -0.02074 & 0.01785 & 1.025 & 0.01287 & 3.339e-05 \\ 0.0131 & -0.01493 & -0.005807 & 0.002846 & -0.01205 & 0.01028 & 1.067 & -0.005389 \\ -0.007891 & -0.01133 & 0.002935 & -0.002299 & 0.00155 & -0.0001701 & -0.004359 & 0.8885 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.00485 & 0.01993 & 0.03003 & 0.02205 & 0.01675 & 0.02302 & 0.03184 & 0.02001 \\ 0.006646 & 0.05384 & 0.03241 & 0.02822 & 0.02797 & 0.02913 & 0.03387 & 0.03881 \\ 0.007207 & 0.0349 & 0.07402 & 0.04631 & 0.03544 & 0.03342 & 0.05395 & 0.03486 \\ 0.00793 & 0.04514 & 0.06837 & 0.0555 & 0.0304 & 0.04028 & 0.05 & 0.04548 \\ 0.007778 & 0.038 & 0.05269 & 0.03055 & 0.05035 & 0.03124 & 0.05446 & 0.03809 \\ 0.008046 & 0.04554 & 0.04847 & 0.03966 & 0.03063 & 0.05807 & 0.07187 & 0.04624 \\ 0.007362 & 0.03556 & 0.05265 & 0.03306 & 0.03594 & 0.04858 & 0.07847 & 0.03556 \\ 0.00668 & 0.03834 & 0.03799 & 0.03361 & 0.02794 & 0.03474 & 0.03969 & 0.0549 \end{pmatrix}$$

Matrix, $5 - 6$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9426 & -0.06054 & -0.01858 & -0.009577 & -0.04448 & 0.01487 & -0.05111 & 0.01211 \\ -0.01507 & 0.9261 & -0.01842 & -0.01486 & 0.007677 & -0.004233 & 0.0158 & 0.014 \\ -0.004885 & -0.01924 & 1.031 & -0.02811 & -0.01966 & -0.01106 & 0.001607 & -0.01568 \\ -0.005543 & -0.01758 & -0.03607 & 1.021 & -0.007365 & 0.003655 & -0.01436 & 0.00549 \\ -0.0161 & 0.009612 & -0.02354 & -0.007301 & 0.9042 & -0.01913 & 0.01151 & -0.005796 \\ 0.00465 & -0.004299 & -0.01486 & 0.003926 & -0.01952 & 1.059 & -0.03643 & 0.01645 \\ -0.01623 & 0.01623 & 0.002113 & -0.01218 & 0.008872 & -0.0291 & 1.07 & 0.006078 \\ 0.002753 & 0.01452 & -0.01636 & 0.004515 & -0.004271 & 0.01376 & 0.006442 & 0.9317 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005003 & 0.02191 & 0.03071 & 0.02325 & 0.01614 & 0.02449 & 0.03247 & 0.02216 \\ 0.006905 & 0.05934 & 0.03295 & 0.02971 & 0.02639 & 0.03158 & 0.0347 & 0.04249 \\ 0.007292 & 0.037 & 0.07401 & 0.04768 & 0.03295 & 0.03528 & 0.05479 & 0.03725 \\ 0.008092 & 0.04885 & 0.06895 & 0.05818 & 0.02831 & 0.04279 & 0.05104 & 0.04891 \\ 0.007625 & 0.03832 & 0.05044 & 0.02997 & 0.04563 & 0.03107 & 0.05238 & 0.03881 \\ 0.008271 & 0.04963 & 0.04961 & 0.04164 & 0.02872 & 0.06191 & 0.07339 & 0.04943 \\ 0.007462 & 0.03732 & 0.05348 & 0.03441 & 0.03342 & 0.05065 & 0.07878 & 0.03779 \\ 0.006902 & 0.0424 & 0.03877 & 0.03522 & 0.02645 & 0.03649 & 0.04033 & 0.06046 \end{pmatrix}$$

Matrix, $6 - 7$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.945 & -0.03958 & 0.001374 & 0.009617 & 0.03559 & -0.002808 & 0.02331 & -0.04465 \\ -0.01082 & 0.9286 & 0.002015 & -0.002359 & 0.01392 & -0.01108 & 0.003507 & 0.01705 \\ 0.00141 & 0.002621 & 1.031 & 0.02586 & 0.005218 & -0.01453 & -0.01824 & 0.006611 \\ 0.002335 & -0.003277 & 0.03239 & 1.016 & 0.001427 & -0.004502 & -0.01755 & -0.02446 \\ 0.009841 & 0.01654 & 0.006119 & 0.0005815 & 0.9252 & 0.008913 & 0.01408 & -0.0007158 \\ -0.00144 & -0.01259 & -0.01669 & -0.003164 & 0.007971 & 1.06 & -0.008279 & 0.01129 \\ 0.005264 & 0.003986 & -0.01699 & -0.0127 & 0.01191 & -0.006548 & 1.015 & -0.008087 \\ -0.01037 & 0.01663 & 0.00801 & -0.01934 & -0.001413 & 0.008724 & -0.007422 & 0.9235 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005032 & 0.02188 & 0.03101 & 0.02328 & 0.01671 & 0.02429 & 0.03088 & 0.022 \\ 0.006931 & 0.06018 & 0.03299 & 0.02969 & 0.02749 & 0.03099 & 0.03255 & 0.04223 \\ 0.007286 & 0.03675 & 0.0738 & 0.04685 & 0.03389 & 0.03559 & 0.05248 & 0.03688 \\ 0.008095 & 0.04878 & 0.06788 & 0.05722 & 0.02884 & 0.04292 & 0.04809 & 0.04844 \\ 0.007711 & 0.03986 & 0.05127 & 0.03021 & 0.04725 & 0.0318 & 0.05043 & 0.03921 \\ 0.008258 & 0.04926 & 0.05066 & 0.04188 & 0.03007 & 0.06099 & 0.0698 & 0.04976 \\ 0.007228 & 0.03647 & 0.05241 & 0.03325 & 0.03342 & 0.04911 & 0.07267 & 0.03646 \\ 0.006913 & 0.04264 & 0.03886 & 0.03505 & 0.02731 & 0.0368 & 0.0384 & 0.05918 \end{pmatrix}$$

University of Zurich[UZH]

Matrix, $7 - 8$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9439 & -0.008577 & 0.02194 & -0.03527 & 0.02117 & 0.002818 & -0.03045 & 0.03674 \\ -0.00281 & 0.9327 & 0.01528 & -0.01142 & 0.003977 & -0.003849 & 0.005332 & -0.001996 \\ 0.01215 & 0.01497 & 1.049 & 0.02072 & -0.01013 & 0.00439 & 0.0268 & -0.01252 \\ -0.01552 & -0.0141 & 0.0269 & 1.038 & 0.008406 & 0.01559 & 0.006097 & -0.006599 \\ 0.00792 & 0.004397 & -0.01175 & 0.008599 & 0.913 & -0.01238 & 0.01077 & 0.004526 \\ -0.003936 & -0.002701 & 0.006165 & 0.01572 & -0.01315 & 1.056 & 0.005698 & -0.02622 \\ -0.01108 & 0.005669 & 0.02694 & 0.004684 & 0.00747 & 0.004389 & 1.031 & -0.0009495 \\ 0.009494 & -0.001074 & -0.01283 & -0.00596 & 0.00317 & -0.0205 & -0.0007477 & 0.9244 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.005117 & 0.02247 & 0.03197 & 0.02424 & 0.01667 & 0.02467 & 0.03167 & 0.0222 \\ 0.007024 & 0.0611 & 0.03411 & 0.03076 & 0.02738 & 0.03196 & 0.03395 & 0.04353 \\ 0.007528 & 0.03781 & 0.07762 & 0.05016 & 0.03419 & 0.03637 & 0.05471 & 0.03789 \\ 0.00836 & 0.05008 & 0.07234 & 0.0615 & 0.02922 & 0.04396 & 0.05051 & 0.04956 \\ 0.007784 & 0.0398 & 0.05285 & 0.03146 & 0.0471 & 0.03192 & 0.05186 & 0.03956 \\ 0.008409 & 0.05064 & 0.0521 & 0.04352 & 0.02981 & 0.06265 & 0.07273 & 0.04945 \\ 0.007446 & 0.03773 & 0.05481 & 0.03522 & 0.03376 & 0.05073 & 0.07631 & 0.03711 \\ 0.007025 & 0.04375 & 0.04004 & 0.03654 & 0.02726 & 0.03676 & 0.03919 & 0.06031 \end{pmatrix}$$

University of Zurich[UZH]

Matrix, $15 - 16\ GeV$

$$A_{reco \to gen} = \begin{pmatrix} 0.9339 & 0.00108 & 0.01361 & -0.003327 & 0.02336 & 0.01977 & 0.01204 & 0.06036 \\ -0.002786 & 0.9655 & 0.002191 & -0.01751 & 0.003735 & -0.003235 & -0.01139 & 0.003084 \\ 0.002358 & 0.001539 & 1.044 & 0.0005145 & 0.02315 & 0.01266 & -0.01177 & 0.01238 \\ -0.006399 & -0.02075 & 0.0008438 & 1.065 & 0.003937 & 0.001169 & 0.01186 & 0.007426 \\ 0.005634 & 0.007771 & 0.03177 & 0.005357 & 0.8611 & 0.006598 & 0.02392 & 0.01592 \\ 0.008184 & -0.002466 & 0.01451 & -0.0001025 & 0.005234 & 1.106 & -0.02518 & -0.001139 \\ 0.005487 & -0.01252 & -0.0114 & 0.01094 & 0.01915 & -0.02023 & 1.003 & 0.002107 \\ 0.01222 & -0.0006629 & 0.008504 & 0.003367 & 0.01214 & 0.0004481 & 0.001254 & 0.9418 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.007227 & 0.03343 & 0.04627 & 0.03662 & 0.02185 & 0.03722 & 0.04363 & 0.03258 \\ 0.01039 & 0.09654 & 0.05184 & 0.04878 & 0.03787 & 0.05022 & 0.04962 & 0.06603 \\ 0.01081 & 0.05899 & 0.1128 & 0.07597 & 0.04595 & 0.05597 & 0.07601 & 0.05719 \\ 0.0122 & 0.07955 & 0.107 & 0.09418 & 0.04019 & 0.06834 & 0.07171 & 0.07577 \\ 0.01096 & 0.05911 & 0.07402 & 0.04603 & 0.06126 & 0.04891 & 0.07197 & 0.05855 \\ 0.01244 & 0.08032 & 0.07744 & 0.06709 & 0.04246 & 0.09896 & 0.1062 & 0.07742 \\ 0.0106 & 0.05862 & 0.07647 & 0.05126 & 0.04564 & 0.07754 & 0.1063 & 0.05511 \\ 0.0103 & 0.06658 & 0.05991 & 0.05607 & 0.03788 & 0.05824 & 0.05712 & 0.09277 \end{pmatrix}$$

University of Zurich[UZH]

Matrix, $16 - 17\ GeV$

$A_{reco \to gen} =$

$$\begin{pmatrix}
0.9496 & -0.0144 & -0.04947 & -0.01837 & -0.01716 & 0.03543 & 0.02941 & 0.03469 \\
-0.00478 & 0.9532 & -0.002416 & -0.01565 & -0.002176 & -0.04158 & -0.008659 & 0.008457 \\
-0.02091 & -0.003799 & 1.017 & 0.005475 & -0.002693 & -0.002398 & -0.004536 & 0.004765 \\
-0.01131 & -0.01774 & 0.005349 & 1.036 & -0.01962 & 0.01559 & -0.002843 & -0.0163 \\
-0.003551 & -0.000527 & -0.003863 & -0.01833 & 0.9273 & 0.0113 & 0.0199 & -0.01963 \\
0.01641 & -0.05085 & -0.003484 & 0.0132 & 0.01329 & 1.057 & 0.005759 & -0.002536 \\
0.006702 & -0.007821 & -0.007193 & -0.001249 & 0.01617 & 0.004364 & 1.02 & -0.01442 \\
0.008137 & 0.006787 & 0.007275 & -0.01416 & -0.01677 & -0.003623 & -0.01468 & 0.9239
\end{pmatrix}$$

$deltaA_{reco \to gen} =$

$$\begin{pmatrix}
0.008642 & 0.03749 & 0.05212 & 0.03973 & 0.02823 & 0.04084 & 0.05259 & 0.03702 \\
0.01184 & 0.1057 & 0.05526 & 0.05126 & 0.04746 & 0.05261 & 0.05504 & 0.07191 \\
0.01242 & 0.0635 & 0.1195 & 0.07948 & 0.05588 & 0.05834 & 0.08514 & 0.06119 \\
0.01395 & 0.08487 & 0.113 & 0.09802 & 0.04932 & 0.07284 & 0.08126 & 0.08416 \\
0.0131 & 0.07069 & 0.08419 & 0.05207 & 0.07888 & 0.05367 & 0.08458 & 0.06597 \\
0.01405 & 0.08461 & 0.08303 & 0.07213 & 0.05018 & 0.102 & 0.1159 & 0.08509 \\
0.01235 & 0.0632 & 0.08559 & 0.05716 & 0.05601 & 0.08193 & 0.1196 & 0.06223 \\
0.01171 & 0.07316 & 0.06348 & 0.06076 & 0.04535 & 0.06206 & 0.06367 & 0.101
\end{pmatrix}$$

Matrix, $17 - 18$ *GeV*

$$A_{reco \to gen} = \begin{pmatrix} 0.9722 & -0.2207 & 0.04866 & 0.03008 & 0.06913 & -0.04369 & 0.006754 & -0.007125 \\ -0.05524 & 1.005 & 0.006893 & 0.001597 & -0.02202 & 0.0168 & 0.01972 & -0.03585 \\ 0.003177 & 0.007009 & 0.9282 & 0.0363 & 0.006305 & -0.01634 & -0.001624 & 0.02907 \\ 0.01302 & 0.001604 & 0.05221 & 0.9617 & 0.01874 & -0.01516 & -0.01685 & -0.00448 \\ 0.02335 & -0.02845 & 0.01286 & 0.02353 & 0.9206 & 0.005032 & -0.02313 & 0.002084 \\ -0.01782 & 0.02651 & -0.02096 & -0.01789 & 0.002636 & 1.073 & 0.05697 & 0.02173 \\ 0.001236 & 0.02227 & 0.001961 & -0.01673 & -0.02081 & 0.04564 & 1.002 & 0.02384 \\ -0.00329 & -0.04029 & 0.03169 & -0.004072 & 0.003657 & 0.02022 & 0.02588 & 0.993 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.01158 & 0.05859 & 0.05715 & 0.04579 & 0.03679 & 0.05273 & 0.06339 & 0.05361 \\ 0.01675 & 0.1375 & 0.06136 & 0.0578 & 0.05652 & 0.06728 & 0.06803 & 0.09741 \\ 0.01504 & 0.07891 & 0.1195 & 0.08187 & 0.06421 & 0.06836 & 0.0968 & 0.07722 \\ 0.01743 & 0.107 & 0.1147 & 0.1026 & 0.05713 & 0.08438 & 0.09246 & 0.104 \\ 0.01671 & 0.08622 & 0.09098 & 0.05799 & 0.09323 & 0.06518 & 0.09928 & 0.08555 \\ 0.01823 & 0.1077 & 0.08822 & 0.07697 & 0.06018 & 0.1254 & 0.1384 & 0.108 \\ 0.01557 & 0.07927 & 0.08985 & 0.06133 & 0.06639 & 0.09943 & 0.1417 & 0.08085 \\ 0.01581 & 0.09842 & 0.07226 & 0.06849 & 0.0572 & 0.07807 & 0.08073 & 0.134 \end{pmatrix}$$

University of Zurich[UZH]

Matrix, $17 - 18\ GeV$

$$A_{reco \to gen} = \begin{pmatrix} 0.9668 & -0.1503 & -0.06883 & -0.03643 & -0.1029 & -0.001732 & 0.02973 & 0.01421 \\ -0.04222 & 1.008 & -0.02683 & -0.00634 & 0.0238 & -0.01704 & -0.02964 & -0.001006 \\ -0.01775 & -0.02415 & 0.9854 & 0.01759 & -0.02388 & -0.02996 & 0.05574 & 0.02075 \\ -0.01405 & -0.001046 & 0.02347 & 0.9961 & -0.03245 & 0.05951 & -0.03554 & -0.03533 \\ -0.02836 & 0.02182 & -0.03202 & -0.02734 & 0.9501 & 0.003661 & -0.005823 & 0.001794 \\ -0.004889 & -0.02396 & -0.03633 & 0.05887 & 0.01071 & 1.14 & 0.02636 & -0.02449 \\ 0.01562 & -0.02848 & 0.05775 & -0.02848 & -0.01448 & 0.0121 & 1.131 & -0.007396 \\ 0.01017 & -0.002444 & 0.02482 & -0.02862 & -0.001555 & -0.02052 & -0.001701 & 0.8984 \end{pmatrix}$$

$$\delta A_{reco \to gen} = \begin{pmatrix} 0.01571 & 0.07502 & 0.08616 & 0.06693 & 0.05243 & 0.07755 & 0.1012 & 0.06491 \\ 0.0219 & 0.1898 & 0.08893 & 0.08128 & 0.08134 & 0.1012 & 0.1096 & 0.1178 \\ 0.02111 & 0.1138 & 0.1845 & 0.1207 & 0.09673 & 0.1026 & 0.155 & 0.09628 \\ 0.02383 & 0.1479 & 0.1734 & 0.1504 & 0.08372 & 0.1266 & 0.1453 & 0.1269 \\ 0.02303 & 0.1239 & 0.1389 & 0.0845 & 0.1362 & 0.09892 & 0.1606 & 0.1086 \\ 0.02532 & 0.1566 & 0.1338 & 0.116 & 0.08793 & 0.1951 & 0.2209 & 0.139 \\ 0.02247 & 0.1179 & 0.1425 & 0.0934 & 0.1013 & 0.1557 & 0.2343 & 0.1029 \\ 0.02003 & 0.1234 & 0.09631 & 0.08929 & 0.07676 & 0.1063 & 0.112 & 0.1622 \end{pmatrix}$$

University of Zurich[UZH]

Summary and plans

- WE have all the matrix
- Error evaluated.
- In principle, everything is ready for method of moments to be written down and reviewed.
- I want to play one more trick, using symmetry of $\cos\theta_k$ and ϕ we can probably set lot of matrix elements to 0.