Anomalies in electroweak penguins at LHCb

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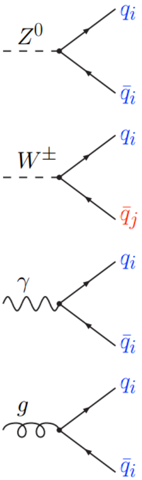
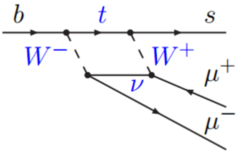
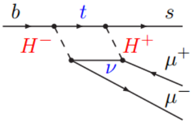
Why electroweak penguin decays?

- In SM allows only the charged interactions to change flavour.
  - Other interactions are flavour conserving.
- One can escape this constrain and produce $b \to s$ and $b \to d$ at loop level.
  - This kind of processes are suppressed in SM $\to$ Rare decays.
  - New Physics can enter in the loops.







Tools

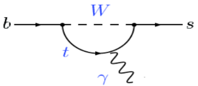
- **Operator Product Expansion and Effective Field Theory**

$$H_{eff} = -\frac{4G_f}{\sqrt{2}} VV'^{*} \sum_i \left[\underbrace{C_i(\mu)O_i(\mu)}_{\text{left-handed}} + \underbrace{C'_i(\mu)O'_i(\mu)}_{\text{right-handed}} \right],$$

i=1,2	Tree
i=3-6,8	Gluon penguin
i=7	Photon penguin
i=9,10	EW penguin
i=S	Scalar penguin
i=P	Pseudoscalar penguin

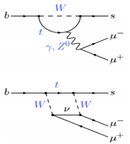
where C_i are the Wilson coefficients and O_i are the corresponding effective operators.

$b \to s\gamma$



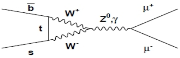
$C_7^{(\prime)}$

$b \to s\ell^+\ell^-$



$C_7^{(\prime)}, C_9^{(\prime)}, C_{10}^{(\prime)}$

$B^0_{(s)} \to \mu^+\mu^-$



$C_S^{(\prime)}, C_P^{(\prime)}, C_{10}^{(\prime)}$

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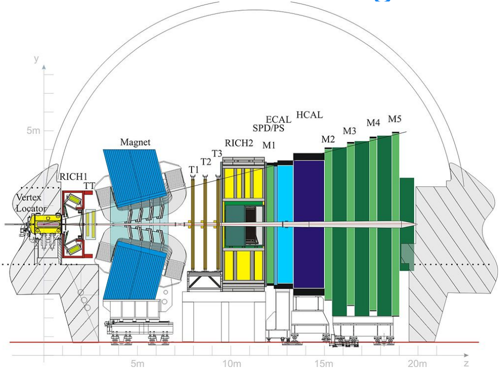

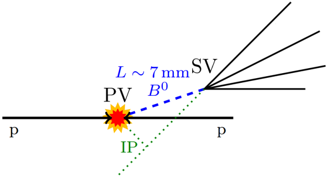

- Excellent Impact Parameter (IP) resolution ($20\ \mu$m).
  $\Rightarrow$ Identify secondary vertices from heavy flavour decays
- Proper time resolution $\sim\ 40$ fs.
  $\Rightarrow$ Good separation of primary and secondary vertices.
- Excellent momentum ($\delta p/p \sim 0.5 - 1.0\%$) and inv. mass resolution.
  $\Rightarrow$ Low combinatorial background.

LHCb detector - PID

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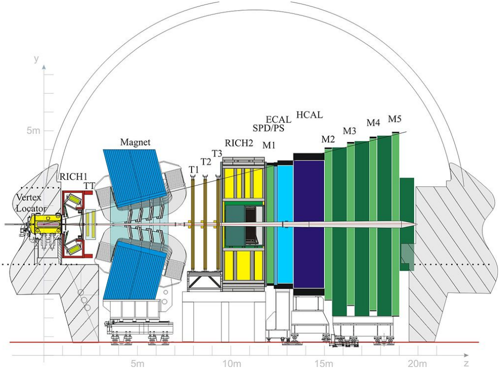

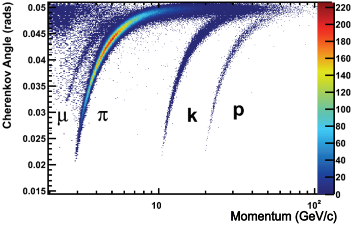

- Excellent Muon identification $\epsilon_{\mu\to\mu} \sim 97\%$, $\epsilon_{\pi\to\mu} \sim 1-3\%$
- Good $K-\pi$ separation via RICH detectors, $\epsilon_{K\to K} \sim 95\%$, $\epsilon_{\pi\to K} \sim 5\%$.
  $\Rightarrow$ Reject peaking backgrounds.
- High trigger efficiencies, low momentum thresholds. Muons: $p_T > 1.76\mathrm{GeV}$ at L0, $p_T > 1.0\mathrm{GeV}$ at HLT1,
  $B \to J/\psi X$: Trigger $\sim 90\%$.

Recent measurements of $b \to s\ell\ell$

⇛ Branching fractions:

$B \to K\mu^-\mu^+$ 1606.04731

$B_s^0 \to \phi\mu^-\mu^+$ JHEP 09 (2015) 179

$B^\pm \to \pi^\pm\mu^-\mu^+$ JHEP 12 (2012) 125

$\Lambda_b \to \Lambda\mu^-\mu^+$ JHEP 06 (2015) 115

$B \to \mu^-\mu^+$ Nature 15

⇛ CP asymmetry:

$B^\pm \to \pi^\pm\mu^-\mu^+$ JHEP 10 (2015) 034

⇛ Isospin asymmetry:

$B \to K\mu^-\mu^+$ JHEP 06 (2014) 133

⇛ Lepton Universality:

$B^\pm \to K^\pm\ell\bar{\ell}$ PRL 113, (2014)

⇛ Angular:

$B^0 \to K^*\ell\bar{\ell}$ JHEP 02 (2016) 104

$B^{0,\pm} \to K^{*,\pm}\ell\bar{\ell}$ PRD 86 032012

$B_s^0 \to \phi\mu\mu$ JHEP 09 (2015) 179

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$> 2\ \sigma$ deviations from SM

Observables in $B \to K^* \mu\mu$

$\Rrightarrow$ The kinematics of $B^0 \to K^* \mu^- \mu^+$ decay is described by three angles θ_l, θ_k, ϕ and invariant mass of the dimuon system (q^2).

$\Rrightarrow$ The angular distribution can be written as:

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_l\,\mathrm{d}\cos\theta_k\,\mathrm{d}\phi}\Bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_k + F_{\mathrm{L}}\cos^2\theta_k + \tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_k\cos 2\theta_l - F_{\mathrm{L}}\cos^2\theta_k\cos 2\theta_l + S_3\sin^2\theta_k\sin^2\theta_l\cos 2\phi + S_4\sin 2\theta_k\sin 2\theta_l\cos\phi + S_5\sin 2\theta_k\sin\theta_l\cos\phi + \tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_k\cos\theta_l + S_7\sin 2\theta_k\sin\theta_l\sin\phi + S_8\sin 2\theta_k\sin 2\theta_l\sin\phi + S_9\sin^2\theta_k\sin^2\theta_l\sin 2\phi\Big].$$

Link to effective operators

$\Rrightarrow$ The observables S_i are bilinear combinations of transversity amplitudes: $A_{\perp}^{L,R}$, $A_{\parallel}^{L,R}$, $A_{0}^{L,R}$.

$\Rrightarrow$ So here is where the magic happens. At leading order the amplitudes can be written as:

$$A_{\perp}^{L,R} = \sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\text{eff}} + \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\text{eff}} + \mathcal{C}_7^{\text{eff}\prime}) \right] \xi_{\perp}(E_{K^*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\text{eff}} - \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\text{eff}} - \mathcal{C}_7^{\text{eff}\prime}) \right] \xi_{\perp}(E_{K^*})$$

$$A_{0}^{L,R} = -\frac{N m_B (1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}} \left[(\mathcal{C}_9^{\text{eff}} - \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + 2\hat{m}_b (\mathcal{C}_7^{\text{eff}} - \mathcal{C}_7^{\text{eff}\prime}) \right] \xi_{\parallel}(E_{K^*}),$$

where $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$. The $\xi_{\parallel,\perp}$ are the soft form factors.

Link to effective operators

$\Rrightarrow$ The observables S_i are bilinear combinations of transversity amplitudes: $A_{\perp}^{L,R}$, $A_{\|}^{L,R}$, $A_{0}^{L,R}$.

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$$A_{\|}^{L,R} = -\sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\rm eff} - \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\rm eff} - \mathcal{C}_7^{\rm eff\prime}) \right] \xi_{\perp}(E_{K^*})$$

$$A_{0}^{L,R} = -\frac{N m_B (1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}} \left[(\mathcal{C}_9^{\rm eff} - \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + 2\hat{m}_b (\mathcal{C}_7^{\rm eff} - \mathcal{C}_7^{\rm eff\prime}) \right] \xi_{\|}(E_{K^*}),$$

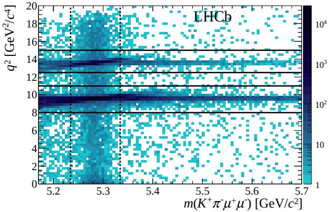
where $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$. The $\xi_{\|,\perp}$ are the soft form factors.

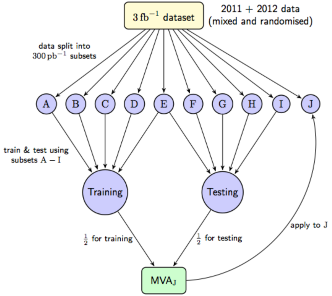
$\Rrightarrow$ Now we can construct observables that cancel the ξ soft form factors at leading order:

$$P_5^{\prime} = \frac{J_5 + \bar{J}_5}{2\sqrt{-(J_2^c + \bar{J}_2^c)(J_2^s + \bar{J}_2^s)}}$$

$B^0 \to K^*\mu^-\mu^+$, Selection

- PID, kinematics and isolation variables used in a Boosted Decision Tree (BDT) to reject background.
- Reject the regions of J/ψ and $\psi(2S)$.
- Specific vetos for backgrounds: $\Lambda_b \to pK\mu\mu$, $B_s^0 \to \phi\mu\mu$, etc.
- Using k-Fold technique and signal proxy $B \to J/\psi K^*$ for training the BDT.
- Improved selection allowed for finer binning than the 1fb^{-1} analysis.





$B^0 \to K^* \mu^- \mu^+$, Selection

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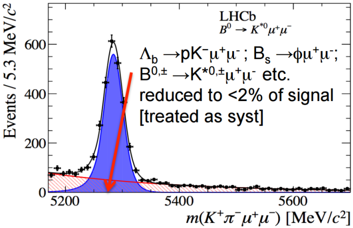
- Signal modelled by a sum of two Crystal-Ball functions.
- Shape is defined using $B \to J/\psi K^*$ and corrected for q^2 dependency.
- Combinatorial background modelled by exponent.

- $K\pi$ system:
  - Beside the K^* resonance there might might a tail from other higher mass states.
  - We modelled it in the analysis.
  - Reduced the systematic compared to previous analysis.



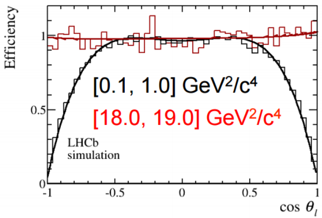
- In total we found 2398 ± 57 candidates in the $(0.1, 19)$ GeV^2 q^2 region.
- 624 ± 30 candidates in the theoretically the most interesting $(1.1 - 6.0)$ $\mathrm{GeV}^2/\mathrm{c}^4$ region.

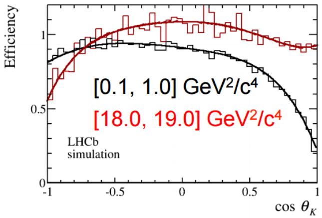
Detector acceptance

- Detector distorts our angular distribution.
- We need to model this effect.
- 4D function is used:

$$\epsilon(\cos\theta_l, \cos\theta_k, \phi, q^2) = \sum_{ijkl} c_{ijkl} P_i(\cos\theta_l) P_j(\cos\theta_k) P_k(\phi) P_l(q^2),$$

  where P_i is the Legendre polynomial of order i.
- We use up to $4^{th}, 5^{th}, 6^{th}, 5^{th}$ order for the $\cos\theta_l, \cos\theta_k, \phi, q^2$.
- 600 terms in total!





Method of moments

- Use orthogonality of spherical harmonics, $f_j(\cos\theta_l, \cos\theta_k, \phi)$:

$$\int f_i(\cos\theta_l, \cos\theta_k, \phi) \cdot f_j(\cos\theta_l, \cos\theta_k, \phi) = \delta_{ij}$$

$$M_i = \int \frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2} \frac{\mathrm{d}(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_l \, \mathrm{d}\cos\theta_k \, \mathrm{d}\phi} f_i(\cos\theta_l, \cos\theta_k, \phi)$$

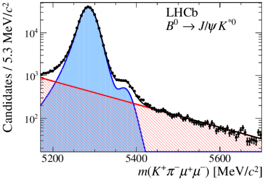
- Don't have true angular distribution but we "sample" it with our data.
- Therefore: $\int \rightarrow \sum$ and $M_i \rightarrow \widehat{M}_i$
- Acceptance corrections is included by:

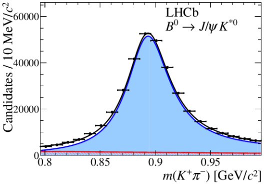
$$\widehat{M}_i = \frac{1}{\sum_e w_e} \sum w_e f_i(\cos\theta_l, \cos\theta_k, \phi)$$

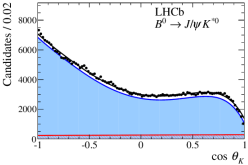
- The weight w_e accounts for the efficiency from previous slide.

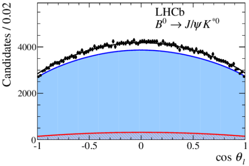
Control channel

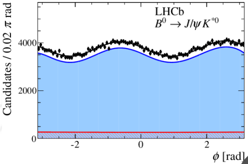
- We tested our unfolding procedure on $B \to J/\psi K^*$.
- The result is in perfect agreement with other experiments and our different analysis of this decay.



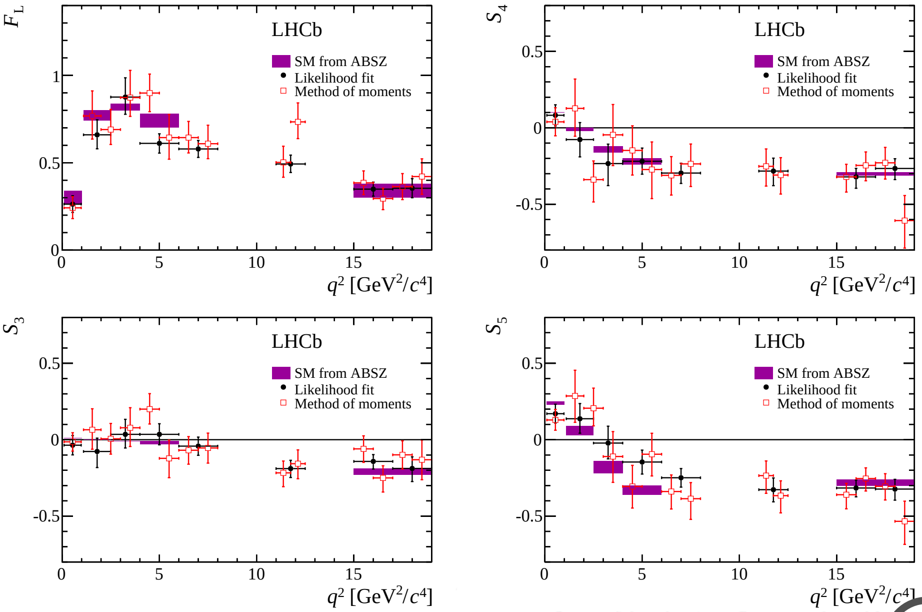








$B^0 \to K^*\mu\mu$ results



$B^0 \to K^* \mu\mu$ results

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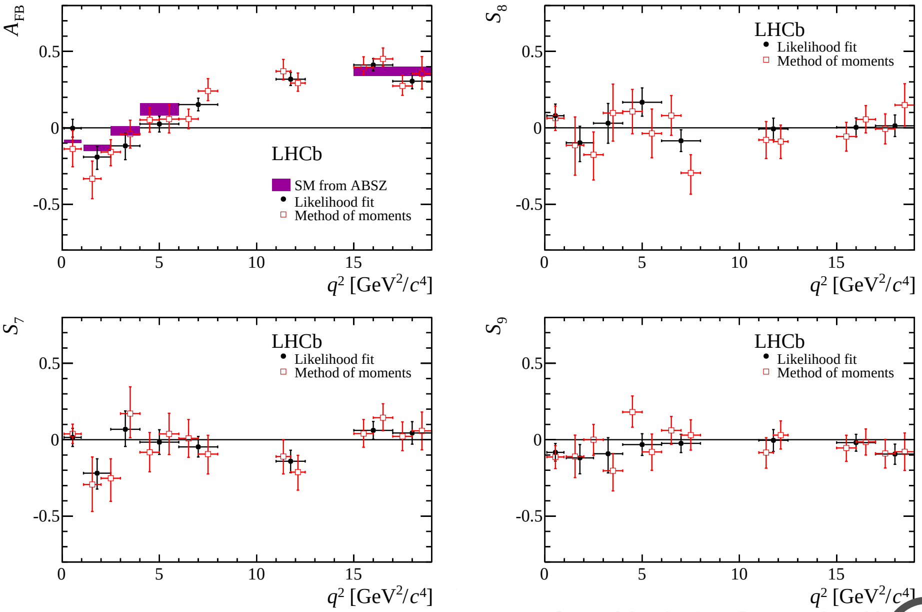

Results in $B \to K^{*}\mu\mu$

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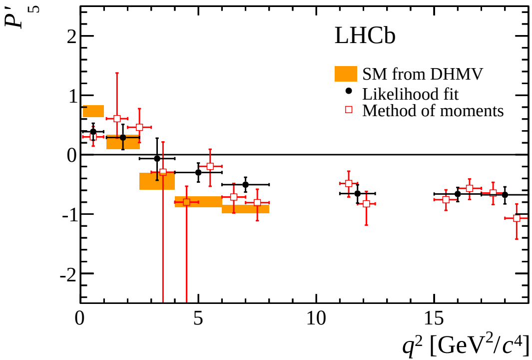
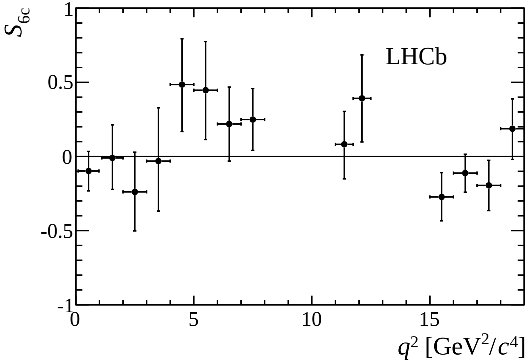


- Tension gets confirmed!
- The two bins deviate by 2.8 and $3.0\ \sigma$ from SM prediction.
- Result compatible with previous result.

- Thanks to Method of Moments there was the possibility to measure a new observable S_{6c}.



- Measurement is consistent with the SM prediction.

BF measurements of $B \to K^{*0,\pm}\mu\mu$

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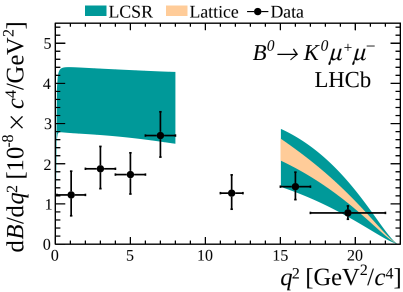
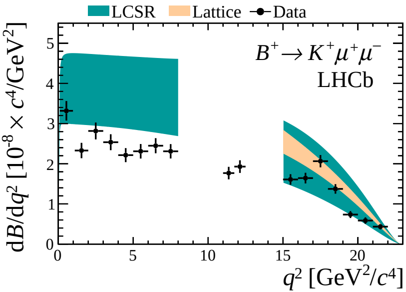
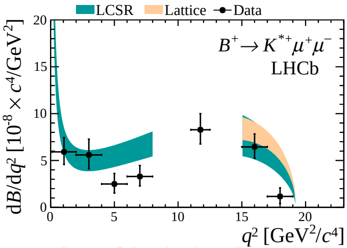
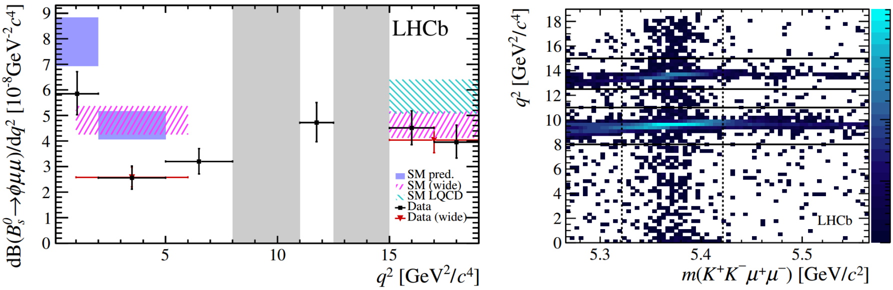




- Despite large theoretical errors the results are consistently smaller than SM prediction.

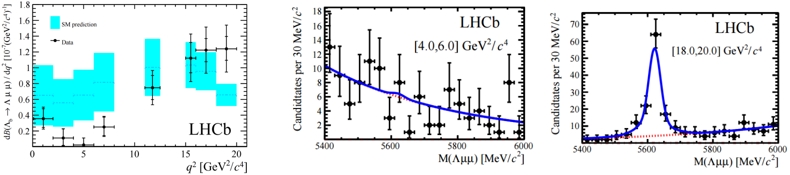


BF measurements of $B_s^0 \to \phi\mu\mu$

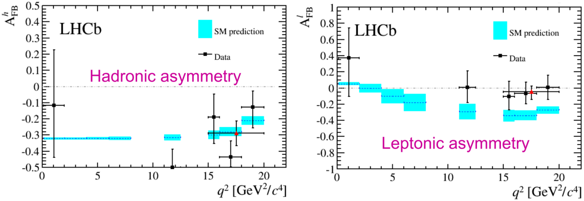


- Suppressed by $\frac{f_s}{f_d}$.
- Cleaner because of narrow ϕ resonance.
- $3.3\ \sigma$ deviation in SM in the $1 - 6\mathrm{GeV}^2/\mathrm{c}^4$ bin.
- Angular part in agreement with SM (S_5 is not accessible).

Measurements of $\Lambda_b \to \Lambda\mu\mu$



- In total ~ 300 candidates in data set.
- Decay not present in the low q^2.
- For the bins in which we have $> 3\ \sigma$ significance the forward backward asymmetry is measured for the hadronic and leptonic system.

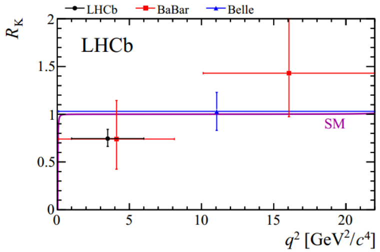


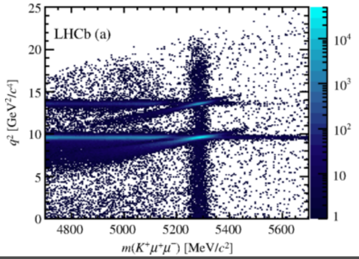
Lepton universality test

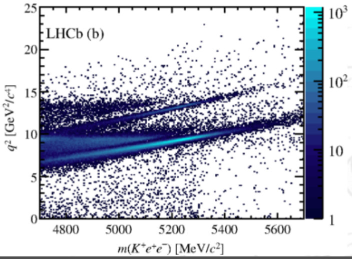
Phys. Rev. Lett. 113, 151601 (2014)

- Challenging analysis due to bremsstrahlung.
- Migration of events modeled by MC.
- Correct for bremsstrahlung.
- Take double ratio with $B^+ \to J/\psi K^+$ to cancel systematics.
- In 3fb^{-1}, LHCb measures $R_K = 0.745^{+0.090}_{-0.074}(stat.)^{+0.036}_{-0.036}(syst.)$
- Consistent with SM at 2.6σ.

$$R_K = \frac{\int_{q^2=1\,\mathrm{GeV}^2/c^4}^{q^2=6\,\mathrm{GeV}^2/c^4} (\mathrm{d}\mathcal{B}[B^+ \to K^+\mu^+\mu^-]/\mathrm{d}q^2)\mathrm{d}q^2}{\int_{q^2=1\,\mathrm{GeV}^2/c^4}^{q^2=6\,\mathrm{GeV}^2/c^4} (\mathrm{d}\mathcal{B}[B^+ \to K^+e^+e^-]/\mathrm{d}q^2)\mathrm{d}q^2} = 1 \pm \mathcal{O}(10^{-3})$$







Angular analysis of $B^0 \to K^*ee$

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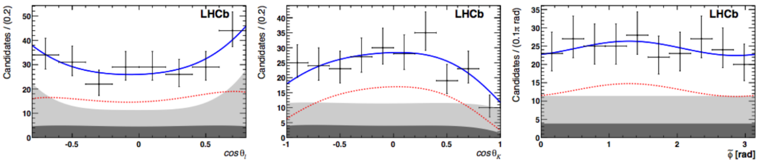
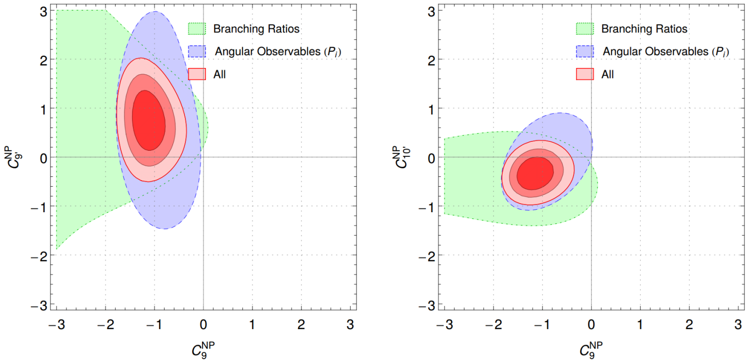
- With the full data set ($3\mathrm{fb}^{-1}$) we performed angular analysis in $0.0004 < q^2 < 1\ \mathrm{GeV}^2/\mathrm{c}^4$.
- Electrons channels are extremely challenging experimentally:
  - Bremsstrahlung.
  - Trigger efficiencies.
- Determine the angular observables: F_{L}, $A_{\mathrm{T}}^{(2)}$, $A_{\mathrm{T}}^{\mathrm{Re}}$, $A_{\mathrm{T}}^{\mathrm{Im}}$:
- Results in full agreement with the SM.
- Similar strength on C_7 Wilson coefficient as from $b \to s\gamma$ decays.



Theory implications

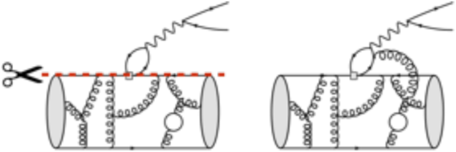
- A fit prepared by `S. Descotes-Genon, L. Hofer, J. Matias, J. Virto`.
- Took into the fit:
  - $\mathcal{B}(B \to X_s \gamma) = (3.36 \pm 0.23) \times 10^{-4}$, Misiak et. al. PRL 114, 221801 (2015)
  - $\mathcal{B}(B \to \mu\mu)$, theory: Bobeth PRD 89, (2014), experiment: LHCb+CMS average (2015)
  - $\mathcal{B}(B \to X_s \mu\mu)$, Huber et al Nucl Phys B802, 2008
  - $\mathcal{B}(B \to K \mu\mu)$, Bouchard et al JHEP11 (2011) 122
  - $B_{(s)} \to K^*(\phi)\mu\mu$, Horgan et al PRL 112, (2014)
  - $B \to Kee$, $B \to K^*ee$ and R_k.

- A fit prepared by `S. Descotes-Genon, L. Hofer, J. Matias, J. Virto`.
- The data can be explained by modifying the C_9 Wilson coefficient.
- Overall there is $> 4\,\sigma$ discrepancy wrt. SM prediction.



If not NP?

- We are not there yet!
- There might be something not taken into account in the theory.
- Resonances (J/ψ, $\psi(2S)$) tails can mimic NP effects.
- There might be some non factorizable QCD corrections.
  " However, the central value of this effect would have to be significantly larger than expected on the basis of existing estimates" D.Straub, arXiv:1503.06199 .



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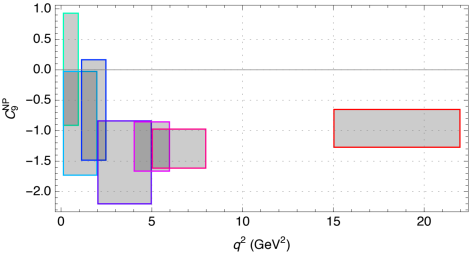
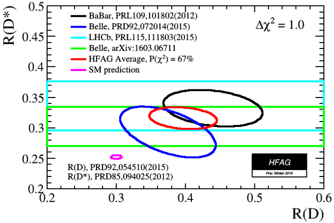


There is more!

- There is one other Lepton Universality Violation decay recently measured by LHCb.
- $R(D^*) = \dfrac{\mathcal{B}(B \to D^* \tau \nu)}{\mathcal{B}(B \to D^* \mu \nu)}$
- Clean SM prediction: $R(D^*) = 0.252(3)$, PRD 85 094025 (2012)
- LHCb result: $R(D^*) = 0.336 \pm 0.027 \pm 0.030$
- HFAG average: $R(D^*) = 0.322 \pm 0.022$
- $4.0\ \sigma$ discrepancy wrt. SM.



Conclusions

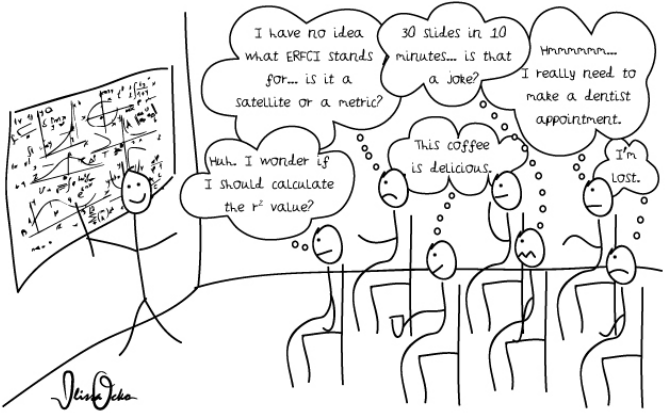
- Clear tensions wrt. SM predictions!
- Measurements cluster in the same direction.
- We are not opening the champagne yet!
- Still need improvement both on theory and experimental side.
- Time will tell if this is QCD+fluctuations or new Physics:

Conclusions

- Clear tensions wrt. SM predictions!
- Measurements cluster in the same direction.
- We are not opening the champagne yet!
- Still need improvement both on theory and experimental side.
- Time will tell if this is QCD+fluctuations or new Physics:

"... when you have eliminated all the
Standard Model explanations, whatever remains,
however improbable, must be New Physics."
Prof. Joaquim Matias

Thank you for the attention!



Backup

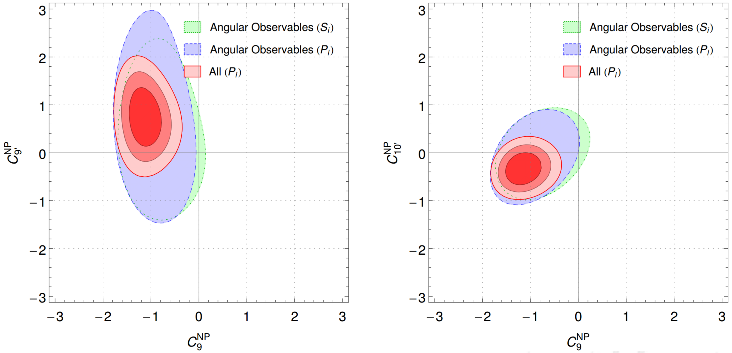
Theory implications

Coefficient	Best fit	1σ	3σ	Pull$_{SM}$	p-value (%)
$\mathcal{C}_7^{NP}$	−0.02	[−0.04, −0.00]	[−0.07, 0.04]	1.1	16.0
$\mathcal{C}_9^{NP}$	−1.11	[−1.32, −0.89]	[−1.71, −0.40]	**4.5**	62.0
$\mathcal{C}_{10}^{NP}$	0.58	[0.34, 0.84]	[−0.11, 1.41]	2.5	25.0
$\mathcal{C}_{7'}^{NP}$	0.02	[−0.01, 0.04]	[−0.05, 0.09]	0.7	15.0
$\mathcal{C}_{9'}^{NP}$	0.49	[0.21, 0.77]	[−0.33, 1.35]	1.8	19.0
$\mathcal{C}_{10'}^{NP}$	−0.27	[−0.46, −0.08]	[−0.84, 0.28]	1.4	17.0
$\mathcal{C}_9^{NP} = \mathcal{C}_{10}^{NP}$	−0.21	[−0.40, 0.00]	[−0.74, 0.55]	1.0	16.0
$\mathcal{C}_9^{NP} = -\mathcal{C}_{10}^{NP}$	−0.69	[−0.88, −0.51]	[−1.27, −0.18]	**4.1**	55.0
$\mathcal{C}_{9'}^{NP} = \mathcal{C}_{10'}^{NP}$	−0.09	[−0.35, 0.17]	[−0.88, 0.66]	0.3	14.0
$\mathcal{C}_{9'}^{NP} = -\mathcal{C}_{10'}^{NP}$	0.20	[0.08, 0.32]	[−0.15, 0.56]	1.7	19.0
$\mathcal{C}_9^{NP} = -\mathcal{C}_{9'}^{NP}$	−1.09	[−1.28, −0.88]	[−1.62, −0.42]	**4.8**	72.0
$\mathcal{C}_9^{NP} = -\mathcal{C}_{10}^{NP} = -\mathcal{C}_{9'}^{NP} = -\mathcal{C}_{10'}^{NP}$	−0.68	[−0.49, −0.49]	[−1.36, −0.15]	3.9	50.0
$\mathcal{C}_9^{NP} = -\mathcal{C}_{10}^{NP} = \mathcal{C}_{9'}^{NP} = -\mathcal{C}_{10'}^{NP}$	−0.17	[−0.29, −0.06]	[−0.54, 0.18]	1.5	18.0

Table 2: *Best-fit points, confidence intervals, pulls for the SM hypothesis and p-values for different one-dimensional NP scenarios.*

If not NP?

- How about our clean P_i observables?
- The QCD cancel as mentioned only at leading order.
- Comparison to normal observables with the optimised ones.



Transversity amplitudes

⇛ One can link the angular observables to transversity amplitudes

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2+|A_\parallel^L|^2+|A_\perp^R|^2+|A_\parallel^R|^2\right]+\frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R\,*}+A_\parallel^L A_\parallel^{R\,*}\right),$$

$$J_{1c} = |A_0^L|^2+|A_0^R|^2+\frac{4m_\ell^2}{q^2}\left[|A_t|^2+2\mathrm{Re}(A_0^L A_0^{R\,*})\right]+\beta_\ell^2\,|A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2+|A_\parallel^L|^2+|A_\perp^R|^2+|A_\parallel^R|^2\right], \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2+|A_0^R|^2\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2-|A_\parallel^L|^2+|A_\perp^R|^2-|A_\parallel^R|^2\right], \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L\,*}+A_0^R A_\parallel^{R\,*})\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L\,*}-A_0^R A_\perp^{R\,*})-\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^*+A_\parallel^{R\,*}A_S)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L\,*}-A_\parallel^R A_\perp^{R\,*})\right], \qquad J_{6c} = 4\beta_\ell\,\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^*+A_0^{R\,*}A_S),$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(\mathrm{A_0^L A_\parallel^{L\,*}}-\mathrm{A_0^R A_\parallel^{R\,*}})+\frac{\mathrm{m}_\ell}{\sqrt{\mathrm{q}^2}}\mathrm{Im}(\mathrm{A_\perp^L A_S^*}-\mathrm{A_\perp^{R\,*}A_S}))\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(\mathrm{A_0^L A_\perp^{L\,*}}+\mathrm{A_0^R A_\perp^{R\,*}})\right], \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(\mathrm{A_\parallel^{L\,*}A_\perp^L}+\mathrm{A_\parallel^{R\,*}A_\perp^R})\right],$$

Link to effective operators

⇛ So here is where the magic happens. At leading order the amplitudes can be written as (soft form factors):

$$A_{\perp}^{L,R} = \sqrt{2}Nm_B(1-\hat{s})\left[(\mathcal{C}_9^{\text{eff}} + \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}}(\mathcal{C}_7^{\text{eff}} + \mathcal{C}_7^{\text{eff}\prime})\right]\xi_{\perp}(E_{K^*})$$

$$A_{\parallel}^{L,R} = -\sqrt{2}Nm_B(1-\hat{s})\left[(\mathcal{C}_9^{\text{eff}} - \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}}(\mathcal{C}_7^{\text{eff}} - \mathcal{C}_7^{\text{eff}\prime})\right]\xi_{\perp}(E_{K^*})$$

$$A_{0}^{L,R} = -\frac{Nm_B(1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}}\left[(\mathcal{C}_9^{\text{eff}} - \mathcal{C}_9^{\text{eff}\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + 2\hat{m}_b(\mathcal{C}_7^{\text{eff}} - \mathcal{C}_7^{\text{eff}\prime})\right]\xi_{\parallel}(E_{K^*}),$$

where $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$. The $\xi_{\parallel,\perp}$ are the form factors.

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$\Rrightarrow$ Now we can construct observables that cancel the ξ form factors at leading order:

$$P_5' = \frac{J_5 + \bar{J}_5}{2\sqrt{-(J_2^c + \bar{J}_2^c)(J_2^s + \bar{J}_2^s)}}$$

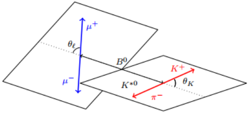
$B^0 \to K^*\mu^-\mu^+$ kinematics

⇛ The kinematics of $B^0 \to K^*\mu^-\mu^+$ decay is described by three angles θ_l, θ_k, ϕ and invariant mass of the dimuon system (q^2).

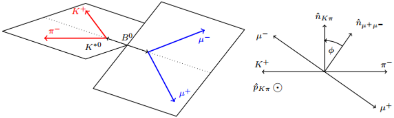
⇛ $\cos\theta_k$: the angle between the direction of the kaon in the K^* ($\overline{K}^*$) rest frame and the direction of the K^* ($\overline{K}^*$) in the B^0 ($\overline{B}^0$) rest frame.

⇛ $\cos\theta_l$: the angle between the direction of the μ^- (μ^+) in the dimuon rest frame and the direction of the dimuon in the B^0 ($\overline{B}^0$) rest frame.

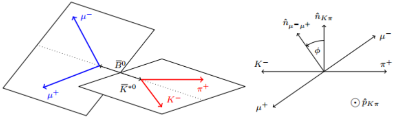
⇛ ϕ: the angle between the plane containing the μ^- and μ^+ and the plane containing the kaon and pion from the K^*.



(a) θ_K and θ_ℓ definitions for the B^0 decay



(b) ϕ definition for the B^0 decay



(c) ϕ definition for the $\overline{B}^0$ decay

$B^0 \to K^*\mu^-\mu^+$ kinematics

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$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$
$$+ J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_K \sin\theta_l \cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7 \sin 2\theta_K \sin\theta_l \sin\phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin\phi$$
$$+ J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi \Big],$$

⇛ This is the most general expression of this kind of decay.

⇛ The CP averaged angular observables are defined:

$$S_i = \frac{J_i + \bar{J}_i}{(d\Gamma + d\bar{\Gamma})/dq^2}$$

Link to effective operators

$\Rrightarrow$ The observables J_i are bilinear combinations of transversity amplitudes: $A_\perp^{L,R}$, $A_\parallel^{L,R}$, $A_0^{L,R}$.
$\Rrightarrow$ So here is where the magic happens. At leading order the amplitudes can be written as:

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\begin{aligned}
A_\perp^{L,R} &= \sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\rm eff} + \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\rm eff} + \mathcal{C}_7^{\rm eff\prime}) \right] \xi_\perp(E_{K^*}) \\
A_\parallel^{L,R} &= -\sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\rm eff} - \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10}^{\prime}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\rm eff} - \mathcal{C}_7^{\rm eff\prime}) \right] \xi_\perp(E_{K^*}) \\
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Symmetries in $B \to K^* \mu\mu$

$\Rrightarrow$ We have 12 angular coefficients (S_i).
$\Rrightarrow$ There exist 4 symmetry transformations that leave the angular distributions unchanged:

$$n_{\|} = \begin{pmatrix} A_{\|}^{L} \\ A_{\|}^{R*} \end{pmatrix}, \quad n_{\perp} = \begin{pmatrix} A_{\perp}^{L} \\ -A_{\perp}^{R*} \end{pmatrix}, \quad n_{0} = \begin{pmatrix} A_{0}^{L} \\ A_{0}^{R*} \end{pmatrix}.$$

$$n_i^{'} = U n_i = \begin{bmatrix} e^{i\phi_L} & 0 \\ 0 & e^{-i\phi_R} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i .$$

$\Rrightarrow$ Using this symmetries one can show that there are 8 independent observables. The pdf can be written as:

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2} \frac{\mathrm{d}(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_l\,\mathrm{d}\cos\theta_k\,\mathrm{d}\phi}\Bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_k \\ + F_{\mathrm{L}}\cos^2\theta_k + \tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_k\cos 2\theta_l \\ - F_{\mathrm{L}}\cos^2\theta_k\cos 2\theta_l + S_3\sin^2\theta_k\sin^2\theta_l\cos 2\phi \\ + S_4\sin 2\theta_k\sin 2\theta_l\cos\phi + S_5\sin 2\theta_k\sin\theta_l\cos\phi \\ + \tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_k\cos\theta_l + S_7\sin 2\theta_k\sin\theta_l\sin\phi \\ + S_8\sin 2\theta_k\sin 2\theta_l\sin\phi + S_9\sin^2\theta_k\sin^2\theta_l\sin 2\phi\Big].$$