

Comparison of angular methods III

Marcin Chrząszcz^{1,2}, Nicola Serra¹



University of
Zurich^{UZH}

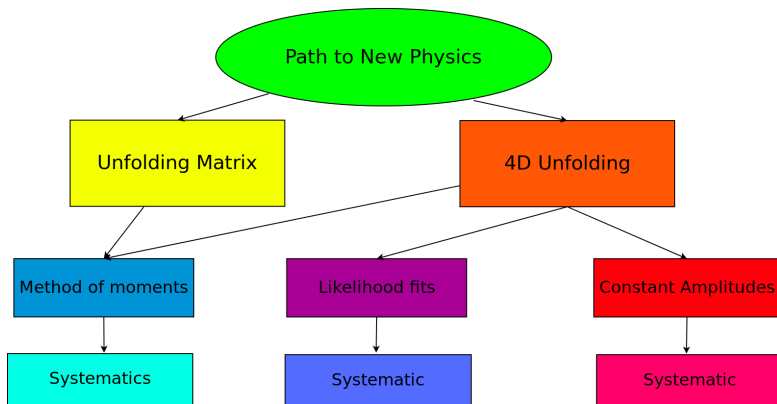


¹ University of Zurich,

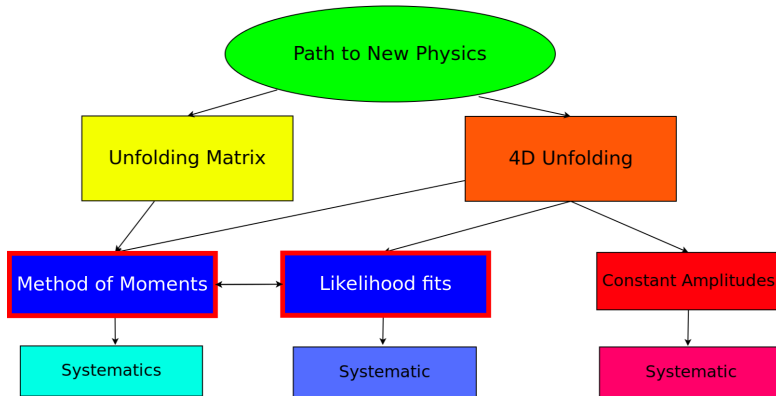
² Institute of Nuclear Physics, Krakow

May 24, 2014

Quo vadis $B^0 \rightarrow K^* \mu\mu$?



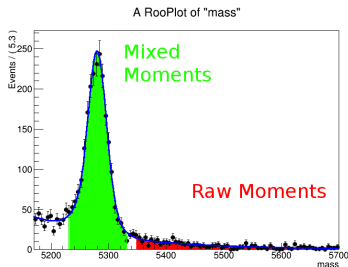
Quo vadis $B^0 \rightarrow K^* \mu\mu$?



How to use Moments

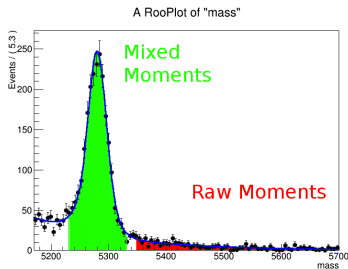
- 1 Method of moments still uses fits.
- 2 Extended likelihood fit is used to extract number of signal and background events.
- 3 Calculate raw moments in bck window $mass > 5350$ and signal window $mass \in [5230, 5330]$
- 4 Use a simple weighted average:

$$M_{mix} = \frac{n_{sig} M_{sig} + n_{bkg} M_{bkg}}{n_{sig} + n_{bkg}} \quad (1)$$



How to use Moments

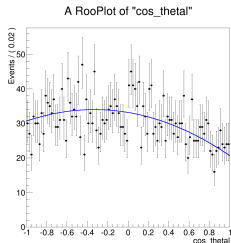
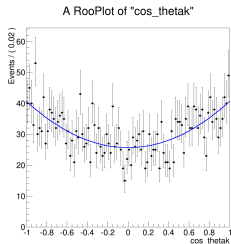
- 1 Method of moments still uses fits.
- 2 Extended likelihood fit is used to extract number of signal and background events.
- 3 Calculate raw moments in bck window $mass > 5350$ and signal window $mass \in [5230, 5330]$
- 4 Extract signal moments:



$$M_{sig} = \frac{(n_{sig} + n_{bkg})M_{mix}}{n_{sig}} - \frac{n_{bkg}m_{bkg}}{n_{sig}} \quad (1)$$

How to use fits 1/2

- Assuming we can decouple in the PDF the angular and mass part:
$$PDF(\theta_l, \theta_k, \phi, m) = PDF_1(\theta_l, \theta_k, \phi) \times PDF_2(m)$$
- PDF_1 is our PDF defined in previous presentation.
- PDF_2 is double CB as defined in Christoph toy creation.
- For background we assume 2nd order Chebyshev polynomials($PDF_{1,bck}$)
- And single exponential for bkg part($PDF_{2,bck}$)



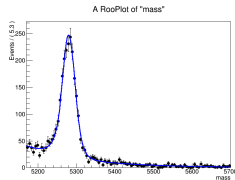
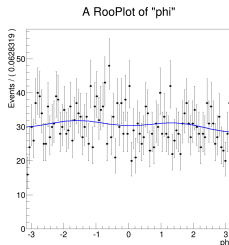
How to use fits 2/2

What is left free in the fit:

- λ for exponent.
- F_l, S_x for angular distribution.
- Relative normalization of signal and bkg.

Other remarks:

- 1 I am using good old RooFit, not custom developed fitter.
- 2 No s-wave in the fits, or systematics include in this study.
- 3 individual toys are now provided by Christoph, so one can compare 1 : 1.



General remark:

- All pull plots/fits are available: [LINK](#)
- User: lhcb, password: 2924.



MM results 1 GeV binning, Mean bias

Q^2	S_1	S_2	S_3	S_4	S_5	S_6
0	$-0.431 \pm 0.0184(-23.4)$	$0.00218 \pm 0.0186(0.118)$	$0.0389 \pm 0.0183(2.13)$	$0.0104 \pm 0.0183(0.564)$	$0.0357 \pm 0.0183(1.96)$	
1	$-0.029 \pm 0.0188(-1.56)$	$0.0247 \pm 0.0187(1.32)$	$0.016 \pm 0.0184(0.872)$	$-0.0125 \pm 0.0191(-0.657)$	$-0.00833 \pm 0.0186(-0.449)$	
2	$0.0090 \pm 0.0181(0.501)$	$-0.0226 \pm 0.0186(-1.21)$	$-0.00623 \pm 0.018(-0.347)$	$-0.0235 \pm 0.018(-1.3)$	$-0.0201 \pm 0.0183(-1.1)$	
3	$-0.046 \pm 0.0183(-2.56)$	$-0.00942 \pm 0.0183(-0.515)$	$-0.00109 \pm 0.0188(-0.0579)$	$0.0108 \pm 0.0188(0.577)$	$-0.0519 \pm 0.0184(-2.83)$	
4	$-0.031 \pm 0.0183(-1.71)$	$0.0268 \pm 0.0181(1.48)$	$-0.00687 \pm 0.0182(-0.378)$	$-0.00106 \pm 0.0187(-0.0568)$	$0.00908 \pm 0.0189(0.481)$	
5	$-0.034 \pm 0.0178(-1.96)$	$0.0231 \pm 0.0189(1.22)$	$-0.0169 \pm 0.0183(-0.92)$	$-0.0138 \pm 0.0183(-0.75)$	$9.56e - 05 \pm 0.0184(0.00521)$	
6	$-0.031 \pm 0.0186(-1.69)$	$0.0184 \pm 0.0183(1)$	$-0.000708 \pm 0.0182(-0.039)$	$-0.00728 \pm 0.0189(-0.386)$	$0.0223 \pm 0.0182(1.22)$	
7	$0.0042 \pm 0.0188(0.22)$	$0.022 \pm 0.0181(1.22)$	$-0.00412 \pm 0.0188(-0.22)$	$0.0044 \pm 0.0187(0.236)$	$0.0291 \pm 0.0188(1.55)$	
8	$0.0061 \pm 0.018(0.34)$	$0.0118 \pm 0.0184(0.643)$	$-0.0169 \pm 0.0185(-0.915)$	$0.0244 \pm 0.0182(1.34)$	$0.00532 \pm 0.0188(0.283)$	
9	$0.0056 \pm 0.0184(0.30)$	$-0.00938 \pm 0.0182(-0.514)$	$-0.0405 \pm 0.0182(-2.22)$	$0.00885 \pm 0.0182(0.487)$	$0.0361 \pm 0.0185(1.96)$	
10	$-0.0156 \pm 0.0185(-0.84)$	$-0.00611 \pm 0.018(-0.34)$	$-0.0241 \pm 0.0184(-1.31)$	$0.0204 \pm 0.0186(1.09)$	$-0.00378 \pm 0.0189(-0.201)$	
11	$-0.0079 \pm 0.018(-0.44)$	$0.0539 \pm 0.0182(2.96)$	$-0.0063 \pm 0.0189(-0.334)$	$-0.0297 \pm 0.019(-1.57)$	$-0.00305 \pm 0.0183(-0.166)$	

Q^2	S_7	S_8	S_9
0	$-0.0179 \pm 0.018(-0.997)$	$-0.0311 \pm 0.0187(-1.66)$	$0.0274 \pm 0.0184(1.49)$
1	$-0.0221 \pm 0.0183(-1.21)$	$-0.0592 \pm 0.0185(-3.2)$	$0.0151 \pm 0.0181(0.836)$
2	$-0.00449 \pm 0.0185(-0.243)$	$-0.00996 \pm 0.0188(-0.529)$	$0.00609 \pm 0.0186(0.328)$
3	$0.0274 \pm 0.0177(1.55)$	$-0.00217 \pm 0.0186(-0.117)$	$-0.00915 \pm 0.0187(-0.49)$
4	$0.0156 \pm 0.0181(0.861)$	$-0.0141 \pm 0.0186(-0.759)$	$0.00498 \pm 0.0178(0.279)$
5	$0.0158 \pm 0.0186(0.848)$	$0.00727 \pm 0.0191(0.381)$	$0.029 \pm 0.0185(1.57)$
6	$-0.00232 \pm 0.0186(-0.125)$	$0.00974 \pm 0.0177(0.551)$	$-0.00613 \pm 0.018(-0.341)$
7	$-0.00853 \pm 0.0184(-0.463)$	$-0.00649 \pm 0.0182(-0.356)$	$0.00402 \pm 0.0183(0.22)$
8	$-0.0137 \pm 0.0185(-0.739)$	$-0.0167 \pm 0.0185(-0.903)$	$-0.0278 \pm 0.0183(-1.52)$
9	$-0.0229 \pm 0.019(-1.2)$	$-0.0506 \pm 0.0183(-2.77)$	$-0.0269 \pm 0.0183(-1.46)$
10	$-0.0155 \pm 0.0185(-0.84)$	$0.0276 \pm 0.0184(1.5)$	$0.0137 \pm 0.0185(0.742)$
11	$-0.0503 \pm 0.0183(-2.75)$	$-0.0137 \pm 0.0181(-0.758)$	$0.0169 \pm 0.0187(0.902)$



MM results 1 GeV binning, Sigma bias

Q^2	F_1	S_3	S_4	S_5	S_6
0	$0.969 \pm 0.014(2.24)$	$0.979 \pm 0.0142(1.45)$	$0.958 \pm 0.0136(3.11)$	$0.969 \pm 0.0139(2.21)$	$0.975 \pm 0.014(1.8)$
1	$0.998 \pm 0.0154(0.128)$	$0.99 \pm 0.0141(0.684)$	$0.982 \pm 0.014(1.31)$	$1 \pm 0.0142(-0.327)$	$0.988 \pm 0.0152(0.812)$
2	$0.957 \pm 0.0134(3.23)$	$0.989 \pm 0.0142(0.807)$	$0.96 \pm 0.0142(2.79)$	$0.965 \pm 0.0135(2.6)$	$0.973 \pm 0.0143(1.9)$
3	$0.971 \pm 0.0139(2.11)$	$0.971 \pm 0.0137(2.1)$	$0.996 \pm 0.0152(0.277)$	$0.995 \pm 0.0142(0.374)$	$0.967 \pm 0.014(2.39)$
4	$0.971 \pm 0.014(2.09)$	$0.965 \pm 0.0138(2.51)$	$0.966 \pm 0.0137(2.46)$	$0.994 \pm 0.0139(0.44)$	$1 \pm 0.0142(0.00431)$
5	$0.947 \pm 0.0136(3.88)$	$1 \pm 0.0138(-0.165)$	$0.967 \pm 0.0138(2.4)$	$0.978 \pm 0.0137(1.63)$	$0.981 \pm 0.0132(1.46)$
6	$0.983 \pm 0.0142(1.19)$	$0.981 \pm 0.0139(1.35)$	$0.967 \pm 0.0134(2.46)$	$1 \pm 0.0145(-0.126)$	$0.969 \pm 0.0134(2.29)$
7	$1 \pm 0.0144(-0.01)$	$0.968 \pm 0.0142(2.26)$	$0.993 \pm 0.0143(0.474)$	$0.989 \pm 0.0136(0.775)$	$0.991 \pm 0.0138(0.671)$
8	$0.956 \pm 0.0133(3.33)$	$0.98 \pm 0.0138(1.43)$	$0.983 \pm 0.0145(1.18)$	$0.966 \pm 0.0136(2.46)$	$0.996 \pm 0.0145(0.309)$
9	$0.959 \pm 0.013(3.14)$	$0.968 \pm 0.0145(2.21)$	$0.954 \pm 0.0128(3.64)$	$0.963 \pm 0.013(2.8)$	$0.985 \pm 0.0142(1.03)$
10	$0.972 \pm 0.0131(2.11)$	$0.955 \pm 0.0131(3.46)$	$0.978 \pm 0.0141(1.59)$	$0.991 \pm 0.014(0.635)$	$1.01 \pm 0.0137(-0.502)$
11	$0.952 \pm 0.014(3.43)$	$0.969 \pm 0.0141(2.22)$	$0.999 \pm 0.0146(0.0445)$	$1 \pm 0.0144(-0.22)$	$0.979 \pm 0.0143(1.45)$

Q^2	S_7	S_8	S_9
0	$0.958 \pm 0.0139(2.98)$	$0.994 \pm 0.0151(0.42)$	$0.972 \pm 0.0145(1.91)$
1	$0.969 \pm 0.0138(2.24)$	$0.98 \pm 0.0145(1.4)$	$0.963 \pm 0.0137(2.74)$
2	$0.983 \pm 0.0145(1.17)$	$0.998 \pm 0.0146(0.147)$	$0.986 \pm 0.0143(0.98)$
3	$0.942 \pm 0.013(4.43)$	$0.987 \pm 0.0132(0.957)$	$0.996 \pm 0.0145(0.295)$
4	$0.965 \pm 0.0137(2.58)$	$0.986 \pm 0.0138(1)$	$0.965 \pm 0.0131(2.24)$
5	$0.979 \pm 0.0135(1.55)$	$0.978 \pm 0.0149(1.46)$	$0.981 \pm 0.0143(1.33)$
6	$0.983 \pm 0.0139(1.22)$	$0.942 \pm 0.013(4.48)$	$0.955 \pm 0.0136(3.33)$
7	$0.979 \pm 0.0129(1.63)$	$0.97 \pm 0.0145(2.06)$	$0.972 \pm 0.0135(2.05)$
8	$0.979 \pm 0.014(1.5)$	$0.985 \pm 0.0139(1.08)$	$0.971 \pm 0.0136(2.13)$
9	$0.995 \pm 0.0139(0.368)$	$0.975 \pm 0.0142(1.73)$	$0.975 \pm 0.0137(1.82)$
10	$0.988 \pm 0.0132(0.907)$	$0.977 \pm 0.0142(1.59)$	$0.978 \pm 0.0142(1.56)$
11	$0.971 \pm 0.0149(1.97)$	$0.968 \pm 0.0142(2.25)$	$0.988 \pm 0.0137(0.886)$

Conclusions:

- MM does not bias the mean!
- There is a trend that it overestimates the errors.
- From all possible things that could go wrong this is the best.



Fit 1GeV binning, Mean bias

Q^2	F_1	S_3	S_4	S_5	S_6
0	$-0.54 \pm 0.02091(-25.82)$	$0.01246 \pm 0.0213(0.585)$	$0.01828 \pm 0.02126(0.8598)$	$0.02513 \pm 0.02078(1.209)$	$0.0646 \pm 0.02081(3.104)$
1	$-0.27 \pm 0.02232(-12.1)$	$0.03261 \pm 0.02401(1.358)$	$0.08175 \pm 0.02376(3.441)$	$-0.1473 \pm 0.0246(-5.987)$	$-0.244 \pm 0.02276(-10.72)$
2	$-0.7574 \pm 0.02208(-34.3)$	$-0.0307 \pm 0.02267(-1.354)$	$-0.07512 \pm 0.02344(-3.204)$	$0.03054 \pm 0.02523(1.21)$	$-0.2317 \pm 0.02327(-9.957)$
3	$-0.7548 \pm 0.02333(-32.35)$	$-0.01416 \pm 0.02193(-0.6458)$	$-0.1786 \pm 0.02354(-7.588)$	$0.2604 \pm 0.02232(11.67)$	$-0.05868 \pm 0.02325(-2.524)$
4	$-0.5667 \pm 0.02163(-26.2)$	$0.0153 \pm 0.02265(0.6756)$	$-0.2313 \pm 0.02105(-10.99)$	$0.4372 \pm 0.02207(19.81)$	$0.03735 \pm 0.02385(1.566)$
5	$-0.4589 \pm 0.02156(-21.29)$	$0.0305 \pm 0.02265(1.347)$	$-0.2186 \pm 0.02141(-10.21)$	$0.5118 \pm 0.02161(23.69)$	$0.1218 \pm 0.02321(5.249)$
6	$-0.3408 \pm 0.02351(-14.5)$	$-0.005133 \pm 0.0235(-0.2184)$	$-0.3205 \pm 0.02116(-15.15)$	$0.628 \pm 0.02207(28.45)$	$0.2972 \pm 0.02256(13.17)$
7	$-0.2309 \pm 0.02362(-9.778)$	$-0.003081 \pm 0.02186(-0.1409)$	$-0.4115 \pm 0.02268(-18.14)$	$0.6794 \pm 0.02274(29.88)$	$0.4305 \pm 0.02204(19.53)$
8	$0.3543 \pm 0.02837(12.49)$	$0.157 \pm 0.02254(6.969)$	$-0.3984 \pm 0.02223(-17.93)$	$0.3877 \pm 0.02171(17.86)$	$0.8208 \pm 0.02474(33.18)$
9	$0.2385 \pm 0.02778(8.586)$	$0.1185 \pm 0.02156(5.499)$	$-0.3564 \pm 0.02171(-16.42)$	$0.3164 \pm 0.02264(13.98)$	$0.7744 \pm 0.02437(31.78)$
10	$0.2395 \pm 0.0273(8.774)$	$0.2025 \pm 0.02106(9.617)$	$-0.3328 \pm 0.02126(-15.66)$	$0.2583 \pm 0.02247(11.5)$	$0.5419 \pm 0.02453(22.09)$
11	$0.139 \pm 0.02676(5.194)$	$0.3433 \pm 0.02149(15.98)$	$-0.3043 \pm 0.02151(-14.15)$	$0.1217 \pm 0.02339(5.205)$	$0.3523 \pm 0.02402(14.67)$

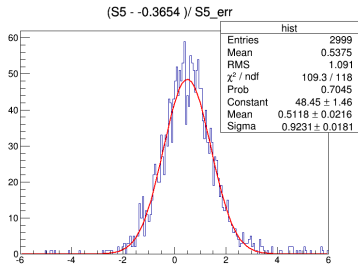
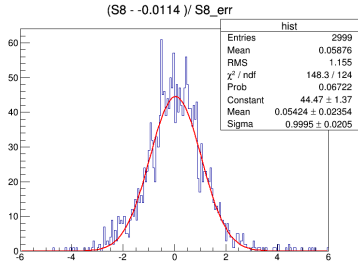
Q^2	S_7	S_8	S_9
0	$-0.03252 \pm 0.02039(-1.595)$	$-0.02852 \pm 0.02148(-1.328)$	$0.03152 \pm 0.0207(1.523)$
1	$-0.01799 \pm 0.02456(-0.7325)$	$-0.03414 \pm 0.02435(-1.402)$	$0.04347 \pm 0.02277(1.909)$
2	$-0.1108 \pm 0.02502(-4.429)$	$-0.007344 \pm 0.02559(-0.287)$	$0.04804 \pm 0.02257(2.129)$
3	$-0.009689 \pm 0.02367(-0.4093)$	$0.03343 \pm 0.0234(1.429)$	$-0.01668 \pm 0.0209(-0.7982)$
4	$-0.03027 \pm 0.02361(-1.282)$	$0.05424 \pm 0.02354(2.304)$	$-0.02207 \pm 0.02167(-1.018)$
5	$-0.07159 \pm 0.02324(-3.08)$	$0.02229 \pm 0.02396(0.9304)$	$-0.03396 \pm 0.02147(-1.582)$
6	$-0.06034 \pm 0.02395(-2.52)$	$0.04234 \pm 0.02284(1.854)$	$-0.01762 \pm 0.02174(-0.8102)$
7	$-0.05002 \pm 0.02386(-2.097)$	$0.03382 \pm 0.02289(1.477)$	$0.009742 \pm 0.02147(0.4538)$
8	$-0.05097 \pm 0.0245(-2.08)$	$-0.02328 \pm 0.02359(-0.9872)$	$-0.03285 \pm 0.023(-1.428)$
9	$-0.03206 \pm 0.02543(-1.261)$	$-0.009805 \pm 0.02334(-0.4201)$	$-0.04927 \pm 0.02272(-2.169)$
10	$-0.02589 \pm 0.02487(-1.041)$	$-0.005148 \pm 0.02376(-0.2166)$	$-0.02395 \pm 0.02422(-0.9889)$
11	$-0.04134 \pm 0.02664(-1.552)$	$-0.03522 \pm 0.02517(-1.399)$	$-0.02359 \pm 0.0246(-0.9592)$

- With agreement with Michel previous studies LINK



Conclusion on 1GeV binning

- Fits Highly bias and not usable.
- MM work fine.
- Christian developed his own fitter
- Let's try comparing those(not very fair comparison)!



MM vs Fit¹

Q^2	F_l^{MM}	F_l^{Fit}
0	0.0397	0.025
1	0.0802	0.051
2	0.0869	0.057
3	0.0893	0.050
4	0.0878	0.051
5	0.0848	0.048
6	0.0812	0.047
7	0.0775	0.046
8	0.0657	0.041
9	0.0687	0.044
10	0.0755	0.045
11	0.0967	0.061

Q^2	S_6^{MM}	S_6^{Fit}
0	0.0632	0.058
1	0.0936	0.085
2	0.097	0.093
3	0.101	0.085
4	0.101	0.083
5	0.0999	0.085
6	0.0975	0.080
7	0.0941	0.079
8	0.0873	0.073
9	0.0925	0.080
10	0.104	0.100
11	0.137	0.121

Q^2	S_3^{MM}	S_3^{Fit}
0	0.0656	0.058
1	0.109	0.100
2	0.116	0.102
3	0.12	0.102
4	0.118	0.101
5	0.115	0.098
6	0.112	0.092
7	0.108	0.087
8	0.0989	0.090
9	0.104	0.098
10	0.114	0.108
11	0.146	0.155

Q^2	S_7^{MM}	S_7^{Fit}
0	0.0574	0.051
1	0.118	0.100
2	0.131	0.105
3	0.134	0.111
4	0.131	0.111
5	0.125	0.104
6	0.119	0.102
7	0.113	0.097
8	0.0958	0.085
9	0.101	0.091
10	0.112	0.101
11	0.144	0.136

Q^2	S_4^{MM}	S_4^{Fit}
0	0.0639	0.054
1	0.123	0.115
2	0.134	0.114
3	0.137	0.123
4	0.134	0.116
5	0.129	0.112
6	0.123	0.104
7	0.117	0.100
8	0.099	0.086
9	0.103	0.098
10	0.112	0.109
11	0.144	0.160

Q^2	S_8^{MM}	S_8^{Fit}
0	0.0641	0.058
1	0.123	0.113
2	0.134	0.120
3	0.138	0.119
4	0.135	0.112
5	0.13	0.108
6	0.124	0.103
7	0.118	0.101
8	0.104	0.092
9	0.109	0.095
10	0.121	0.115
11	0.156	0.149

Q^2	S_5^{MM}	S_5^{Fit}
0	0.0563	0.050
1	0.118	0.114
2	0.131	0.108
3	0.133	0.117
4	0.129	0.105
5	0.122	0.106
6	0.116	0.097
7	0.109	0.094
8	0.0884	0.080
9	0.0925	0.085
10	0.101	0.096
11	0.13	0.144

Q^2	S_9^{MM}	S_9^{Fit}
0	0.0656	0.057
1	0.109	0.099
2	0.116	0.109
3	0.12	0.104
4	0.119	0.102
5	0.116	0.097
6	0.112	0.094
7	0.108	0.091
8	0.0993	0.079
9	0.105	0.088
10	0.115	0.103
11	0.149	0.143

¹Taken from Christoph's presentation



Conclusion

- Method of moments a bit worse ($\sim 15\%$) than optimistic fit.
- MM overestimates the error by few %, fits underestimates the error by few %.
- Fit needs more study to understand why roofit is biased and Christoph fit is not.

