

Unfolding for counting experiments

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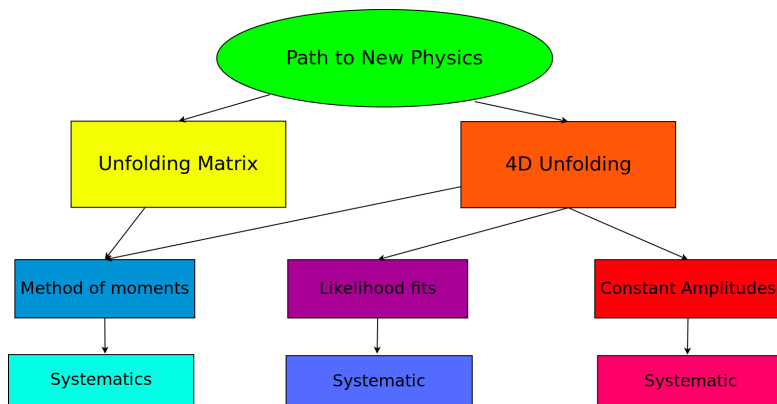
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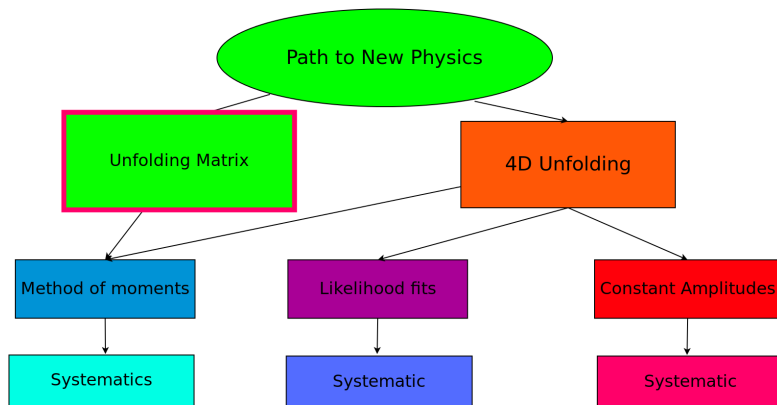
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Quo vadis $B^0 \rightarrow K^* \mu\mu$?



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- Method of moments can use 2 different unfolding techniques.
- One of them is Christophs 4D unfolding
- Easy peasy:

$$\hat{x} = \sum_i^n = \frac{f(\theta_{li}, \theta_{ki}, \phi_i)}{n} \rightarrow \frac{f(\theta_{li}, \theta_{ki}, \phi_i) \times w_i}{\sum_j^n w_j} \quad (1)$$

Very easy and has proved to work. See presentation [LINK](#)

Matrix unfolding

- Our PDF is a vector in 8 dim space, where the dimensions are: f_x^1
- Our acceptance is a function:

$$\epsilon(\cos \theta_k, \cos \theta_l, \phi) \quad (2)$$

- We assume it's a smooth function (C^∞).
- Normal moments:

$$M_x = \int PDF \times f_x \rightarrow \overline{M_x} = \int PDF \times f_x \times \epsilon(\cos \theta_k, \cos \theta_l, \phi) \quad (3)$$

- Let's play Christoph's trick:

$$\epsilon(\cos \theta_k, \cos \theta_l, \phi) = \sum A_{\alpha, \beta, \gamma} \cos^\alpha \theta_k \cos^\beta \theta_l \phi^\gamma \quad (4)$$

¹See 1st presentation on Method of moments.



Matrix unfolding

- Our Moments will become:

$$\int PDF \times f_x \times \epsilon(\cos \theta_k, \cos \theta_l, \phi) = \sum A_{\alpha, \beta, \gamma} \int PDF \times f_x \times \cos^\alpha \theta_k \cos^\beta \theta_l \phi^\gamma \quad (5)$$

- We just need to show that:

$$\int PDF \times f_x \times \cos^\alpha \theta_k \cos^\beta \theta_l \phi^\gamma = \sum_y B_y \int PDF \times f_y = \sum_y B_y M_y \quad (6)$$

- This is a bit nasty but doable, using Mathematica. You just need to check 8 base functions.
- As expected it's is linear terms.
- I also checked this using analytical calculations.

Constructing Matrix unfolding

- Ok, so we know that the matrix exists.
- How ever we don't know explicate

$$\epsilon(\cos \theta_k, \cos \theta_l, \phi) \quad (7)$$

- We don't, we just need to calculate matrix elements
- Let's use PHSP MC.
- Moments for PHSP MC are:
 $v_{gen}^T = (2/3, 0, 0, 0, 0, 0, 0, 0)$
- After reconstruction we get: $v_{rec}^T =$
 $(0.7069, 0.0077, -0.00236466, 0.0005, 0.0007, 0.0011, 0.0011, -0.0012)$



Constructing Matrix unfolding

- We got first column of the unfolding matrix.

$$\begin{pmatrix} 1.06 & \cdots & a_{1,8} \\ 0.01157 & \cdots & a_{2,8} \\ -0.003547 & \ddots & \vdots \\ 0.0007841 & \ddots & \vdots \\ 0.001126 & \ddots & \vdots \\ 0.001766 & \ddots & \vdots \\ 0.001664 & \ddots & \vdots \\ -0.001937 & \cdots & a_{8,8} \end{pmatrix}$$

- How about the others?
- We can reweight accordingly to f_x .



Constructing Matrix unfolding

- To get S_3 each event i^{th} has has weight $f_{S_3}(\cos \theta_{k_i}, \cos \theta_{l_i}, \phi_i)$
- One can calculate on MC the reweighed moments in PHPS:

$$\int PDF * f_{S_3} = \frac{32}{225} \quad (8)$$

- Our base vector now is: $v_{gen}^T = (0, \frac{32}{225}, 0, 0, 0, 0, 0, 0)$
- So lets see what do we get as reconstructed vector(after multiplying by $\frac{225}{32} \cdot v_{rec}^T = (0.042, 1.105, -0.005, 0.003, -0.0023, -0.005, -0.005, -0.006)$)

Constructing Matrix unfolding

- Now the matrix looks like:

$$\begin{pmatrix} 1.06 & 0.042 & \cdots & a_{1,8} \\ 0.01157 & 1.105 & \cdots & a_{2,8} \\ -0.003547 & -0.005 & \ddots & \vdots \\ 0.0007841 & -0.005 & \ddots & \vdots \\ 0.001126 & 0.003 & \ddots & \vdots \\ 0.001766 & -0.0023 & \ddots & \vdots \\ 0.001664 & -0.005 & \ddots & \vdots \\ -0.001937 & -0.006 & \cdots & a_{8,8} \end{pmatrix}$$

- The others go in the same way.
- Repenting this exercise from 1st year algebra we can get the full matrix

Constructing Matrix unfolding

- The full transformation matrix from generator space to reconstructed space:

$$A_{gen \rightarrow reco} = \begin{pmatrix} 1.06 & 0.0423 & -0.0081 & 0.0022 & 0.0049 & 0.0037 & 0.0028 & -0.0065 \\ 0.0115 & 1.105 & -0.0050 & 0.0027 & -0.0018 & -0.0040 & -0.0054 & -0.0065 \\ -0.0035 & -0.0050 & 0.981 & 0.0005 & -0.0025 & 0.0002 & -0.0037 & -0.0048 \\ 0.00078 & 0.0034 & 0.0006 & 1.002 & -0.0032 & -0.0040 & 0.0003 & 0.0018 \\ 0.001126 & -0.0023 & -0.0032 & -0.0032 & 1.055 & 0.001 & -0.004 & 0.0023 \\ 0.00176 & -0.0050 & 0.00036 & -0.0040 & 0.0011 & 0.96 & -0.0057 & 0.0009 \\ 0.0016 & -0.005 & -0.003 & 0.00029 & -0.003 & -0.004 & 0.9543 & 0.0000 \\ -0.0019 & -0.0065 & -0.004 & 0.001 & 0.0018 & 0.0007 & 0.000 & 1.098 \end{pmatrix}$$

- Inverting the matrix is simple, and doable

$$A_{reco \rightarrow gen} = \begin{pmatrix} 0.9434 & -0.036 & 0.007 & -0.0020 & -0.0044 & -0.0038 & -0.0030 & 0.0054 \\ -0.009 & 0.90 & 0.0045 & -0.0024 & 0.0016 & 0.003873 & 0.00527 & 0.005 \\ 0.003 & 0.00454 & 1.019 & -0.00058 & 0.0025 & -0.000291 & 0.004 & 0.004 \\ -0.00071 & -0.0030 & -0.0007 & 0.9977 & 0.0030 & 0.004206 & -0.0003 & -0.0017 \\ -0.001 & 0.0020 & 0.0031 & 0.0030 & 0.9483 & -0.0010 & 0.004626 & -0.0019 \\ -0.001 & 0.004 & -0.0003 & 0.0042 & -0.001087 & 1.037 & 0.0063 & -0.0009 \\ -0.0017 & 0.0053 & 0.0042 & -0.0002 & 0.00370 & 0.0050 & 1.048 & 0.0000 \\ 0.0016 & 0.0053 & 0.00452 & -0.001 & -0.001582 & -0.0007213 & 0.000 & 0.9105 \end{pmatrix}$$



Sensitivity to unknowns

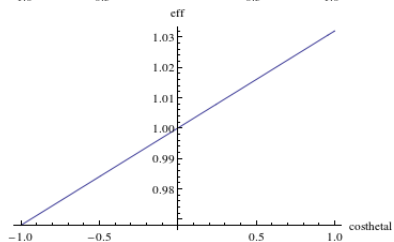
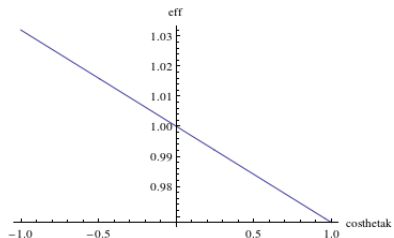
- We are unfolding based on MC.
- There are MC/Data differences, which can have impact on the unfolding.

Let's put small modification:

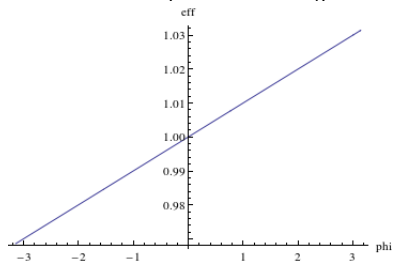
$$w_j \rightarrow \overline{w}_j = \frac{1}{\text{eff}(\cos \theta_{l_j}, \cos \theta_{k_j}, \phi_j)} \times \text{corr}(\cos \theta_{l_j}, \cos \theta_{k_j}, \phi_j) \quad (9)$$

Unfortunately God didn't allowed me sneak peak into his cards so I don't know $\text{corr}(\cos \theta_{l_j}, \cos \theta_{k_j}, \phi_j)$, but let's try out some functions and see what happens :)

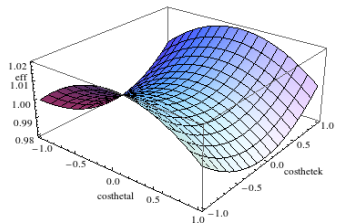
Corr1 functions



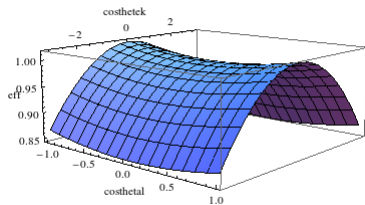
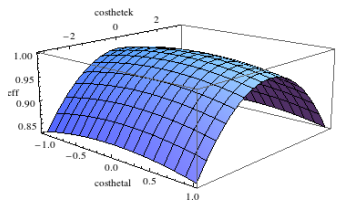
$$\text{corr1}(\text{cos}_l, \text{cos}_k, \phi) = 1 + 0.032 \text{cos}_l - 0.032 \text{cos}_k + 0.01\phi$$



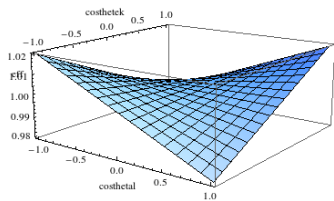
Corr2 functions



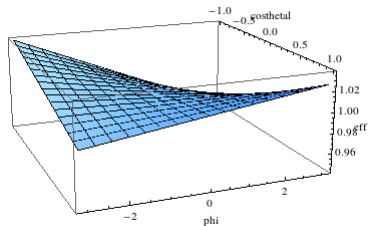
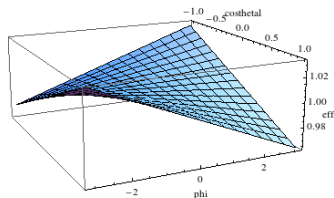
$$corr2(\cos_l, \cos_k, \phi) = -0.02 \cos_l^2 + 0.02 \cos_k^2 - 0.015 \phi^2 + 1$$



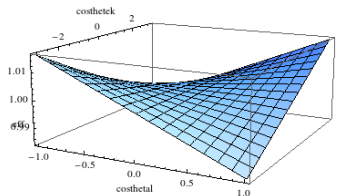
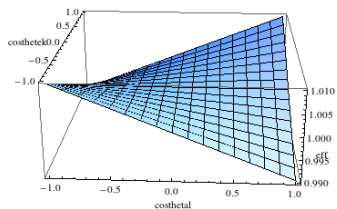
Corr3 functions



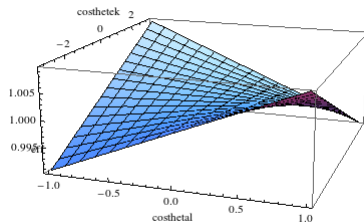
$$\text{corr3}(\cos_l, \cos_k, \phi) = 0.02 \cos_l \cos_k + 0.01 \cos_k \phi - 0.01 \phi \cos_l + 1$$



Corr4 functions



$$\text{corr3}(\cos_l, \cos_k, \phi) = 0.01 \cos_k \cos_l \phi + 1$$



| q^2 | Mean of the pull | | | | | | |
|-------|----------------------------|----------------------------|---------------------------|-----------------------|---------------------------|---------------------------|------------------------|
| | S_3 | S_4 | S_5 | S_{6s} | S_{6c} | S_7 | S_8 |
| 0 | $0.0085 \pm 0.026(0.3)$ | $0.13 \pm 0.027(4.6)$ | $-0.025 \pm 0.027(-0.92)$ | $0.46 \pm 0.027(17)$ | $-0.13 \pm 0.028(-4.7)$ | $0.029 \pm 0.028(1)$ | $-0.66 \pm 0.027(-25)$ |
| 1 | $0.0094 \pm 0.028(0.33)$ | $0.078 \pm 0.028(2.8)$ | $-0.02 \pm 0.028(-0.73)$ | $0.24 \pm 0.028(8.5)$ | $-0.075 \pm 0.028(-2.7)$ | $-0.016 \pm 0.026(-0.63)$ | $-0.39 \pm 0.028(-14)$ |
| 2 | $-0.02 \pm 0.027(-0.72)$ | $0.027 \pm 0.026(1)$ | $-0.024 \pm 0.027(-0.91)$ | $0.18 \pm 0.027(6.8)$ | $-0.024 \pm 0.027(-0.89)$ | $-0.067 \pm 0.027(-2.5)$ | $-0.39 \pm 0.028(-14)$ |
| 3 | $0.013 \pm 0.028(0.46)$ | $-0.0089 \pm 0.027(-0.32)$ | $0.055 \pm 0.026(2.1)$ | $0.12 \pm 0.027(4.7)$ | $0.11 \pm 0.027(3.9)$ | $-0.018 \pm 0.028(-0.63)$ | $-0.43 \pm 0.027(-16)$ |
| 4 | $-0.0054 \pm 0.029(-0.18)$ | $-0.066 \pm 0.027(-2.4)$ | $0.037 \pm 0.028(1.3)$ | $0.22 \pm 0.027(8.1)$ | $0.099 \pm 0.027(3.7)$ | $-0.1 \pm 0.026(-3.8)$ | $-0.41 \pm 0.026(-15)$ |
| 5 | $0.06 \pm 0.027(2.2)$ | $-0.069 \pm 0.026(-2.6)$ | $-0.013 \pm 0.028(-0.49)$ | $0.21 \pm 0.027(7.8)$ | $0.093 \pm 0.027(3.5)$ | $-0.083 \pm 0.028(-3)$ | $-0.41 \pm 0.028(-15)$ |
| 6 | $0.0064 \pm 0.026(0.25)$ | $-0.051 \pm 0.027(-1.9)$ | $-0.029 \pm 0.028(-1)$ | $0.26 \pm 0.028(9.2)$ | $0.14 \pm 0.027(5.1)$ | $-0.081 \pm 0.027(-3)$ | $-0.45 \pm 0.028(-16)$ |
| 8 | $0.023 \pm 0.027(0.85)$ | $-0.031 \pm 0.028(-1.1)$ | $0.0042 \pm 0.028(0.15)$ | $0.21 \pm 0.026(7.8)$ | $0.12 \pm 0.028(4.2)$ | $-0.13 \pm 0.027(-4.8)$ | $-0.48 \pm 0.026(-18)$ |
| 9 | $-0.017 \pm 0.027(-0.63)$ | $-0.015 \pm 0.026(-0.56)$ | $-0.0052 \pm 0.026(-0.2)$ | $0.27 \pm 0.027(10)$ | $0.046 \pm 0.026(1.7)$ | $-0.12 \pm 0.026(-4.4)$ | $-0.5 \pm 0.026(-19)$ |
| 10 | $-0.054 \pm 0.027(-2)$ | $-0.056 \pm 0.026(-2.2)$ | $0.036 \pm 0.026(1.4)$ | $0.16 \pm 0.028(5.7)$ | $0.077 \pm 0.028(2.8)$ | $-0.034 \pm 0.027(-1.3)$ | $-0.39 \pm 0.028(-14)$ |
| 11 | $0.023 \pm 0.027(0.88)$ | $0.021 \pm 0.027(0.8)$ | $-0.011 \pm 0.027(-0.41)$ | $0.14 \pm 0.027(5)$ | $0.042 \pm 0.027(1.6)$ | $-0.098 \pm 0.028(-3.5)$ | $-0.3 \pm 0.027(-11)$ |



| q^2 | Mean of the pull | | | | | | |
|-------|--------------------------|----------------------------|----------------------------|---------------------------|----------------------------|-----------------------------|--------------------------|
| | S_3 | S_4 | S_5 | S_{6s} | S_{6c} | S_7 | S_8 |
| 0 | $-0.21 \pm 0.026(-8.1)$ | $0.061 \pm 0.026(2.3)$ | $-0.016 \pm 0.026(-0.63)$ | $0.048 \pm 0.026(1.8)$ | $-0.0062 \pm 0.027(-0.23)$ | $-0.012 \pm 0.028(-0.44)$ | $0.0091 \pm 0.026(0.36)$ |
| 1 | $-0.12 \pm 0.028(-4.5)$ | $0.032 \pm 0.027(1.2)$ | $-0.0071 \pm 0.026(-0.27)$ | $0.02 \pm 0.027(0.75)$ | $-0.086 \pm 0.027(-3.2)$ | $-0.03 \pm 0.025(-1.2)$ | $0.021 \pm 0.027(0.79)$ |
| 2 | $-0.15 \pm 0.027(-5.8)$ | $-0.013 \pm 0.026(-0.49)$ | $0.011 \pm 0.026(0.44)$ | $-0.039 \pm 0.026(-1.5)$ | $-0.032 \pm 0.027(-1.2)$ | $-0.066 \pm 0.027(-2.5)$ | $0.018 \pm 0.028(0.64)$ |
| 3 | $-0.15 \pm 0.027(-5.6)$ | $0.025 \pm 0.026(0.96)$ | $0.016 \pm 0.027(0.58)$ | $-0.062 \pm 0.027(-2.3)$ | $0.037 \pm 0.026(1.4)$ | $0.026 \pm 0.027(0.96)$ | $-0.016 \pm 0.027(-0.5)$ |
| 4 | $-0.12 \pm 0.028(-4.5)$ | $-0.024 \pm 0.026(-0.92)$ | $0.045 \pm 0.027(1.6)$ | $0.0075 \pm 0.026(0.29)$ | $0.015 \pm 0.027(0.53)$ | $-0.036 \pm 0.026(-1.4)$ | $0.037 \pm 0.026(1.4)$ |
| 5 | $-0.095 \pm 0.027(-3.6)$ | $-0.032 \pm 0.026(-1.2)$ | $0.014 \pm 0.026(0.52)$ | $-0.013 \pm 0.026(-0.48)$ | $-0.0093 \pm 0.027(-0.35)$ | $-0.013 \pm 0.026(-0.51)$ | $0.042 \pm 0.027(1.6)$ |
| 6 | $-0.17 \pm 0.025(-6.5)$ | $0.008 \pm 0.027(0.3)$ | $0.012 \pm 0.027(0.45)$ | $0.029 \pm 0.027(1.1)$ | $0.0072 \pm 0.027(0.27)$ | $-0.0012 \pm 0.026(-0.046)$ | $0.011 \pm 0.027(0.42)$ |
| 8 | $-0.13 \pm 0.026(-5.1)$ | $-0.0077 \pm 0.027(-0.28)$ | $0.05 \pm 0.027(1.9)$ | $-0.03 \pm 0.026(-1.2)$ | $-0.012 \pm 0.028(-0.44)$ | $-0.046 \pm 0.026(-1.7)$ | $0.031 \pm 0.026(1.2)$ |
| 9 | $-0.15 \pm 0.026(-5.7)$ | $-0.0083 \pm 0.026(-0.32)$ | $0.03 \pm 0.026(1.2)$ | $0.044 \pm 0.027(1.6)$ | $-0.07 \pm 0.026(-2.7)$ | $-0.022 \pm 0.026(-0.84)$ | $-0.045 \pm 0.026(-1.7)$ |
| 10 | $-0.15 \pm 0.025(-5.8)$ | $-0.032 \pm 0.025(-1.3)$ | $0.059 \pm 0.026(2.2)$ | $-0.072 \pm 0.028(-2.6)$ | $-0.029 \pm 0.027(-1.1)$ | $0.064 \pm 0.027(2.4)$ | $0.014 \pm 0.027(0.51)$ |
| 11 | $-0.067 \pm 0.026(-2.6)$ | $0.017 \pm 0.026(0.65)$ | $-0.015 \pm 0.026(-0.56)$ | $-0.051 \pm 0.026(-1.9)$ | $-0.0086 \pm 0.026(-0.33)$ | $-0.018 \pm 0.028(-0.67)$ | $0.017 \pm 0.027(0.62)$ |

| q^2 | Mean of the pull | | | | | | |
|-------|------------------------------|-----------------------------|---------------------------|-----------------------------|------------------------|----------------------------|----------------------------|
| | S_3 | S_4 | S_5 | S_{6S} | S_{6c} | S_7 | S_8 |
| 0 | $-0.021 \pm 0.026(-0.81)$ | $0.041 \pm 0.026(1.5)$ | $0.009 \pm 0.027(0.34)$ | $0.043 \pm 0.026(1.6)$ | $0.13 \pm 0.028(4.8)$ | $-0.0072 \pm 0.028(-0.26)$ | $0.044 \pm 0.026(1.7)$ |
| 1 | $-0.014 \pm 0.028(-0.51)$ | $0.03 \pm 0.027(1.1)$ | $0.022 \pm 0.027(0.82)$ | $0.015 \pm 0.028(0.53)$ | $0.078 \pm 0.028(2.8)$ | $-0.037 \pm 0.025(-1.4)$ | $0.057 \pm 0.027(2.1)$ |
| 2 | $-0.037 \pm 0.027(-1.4)$ | $-0.0013 \pm 0.027(-0.048)$ | $-0.015 \pm 0.027(-0.54)$ | $-0.052 \pm 0.026(-2)$ | $0.1 \pm 0.027(3.8)$ | $-0.051 \pm 0.026(-1.9)$ | $0.022 \pm 0.028(0.78)$ |
| 3 | $-0.015 \pm 0.027(-0.55)$ | $0.036 \pm 0.028(1.3)$ | $-0.039 \pm 0.027(-1.4)$ | $-0.072 \pm 0.027(-2.7)$ | $0.17 \pm 0.027(6.2)$ | $-0.0044 \pm 0.028(-0.15)$ | $-0.024 \pm 0.027(-0.9)$ |
| 4 | $-0.00047 \pm 0.029(-0.017)$ | $-0.012 \pm 0.027(-0.43)$ | $0.0099 \pm 0.028(0.35)$ | $-0.002 \pm 0.026(-0.076)$ | $0.17 \pm 0.028(6.2)$ | $-0.062 \pm 0.027(-2.3)$ | $0.0086 \pm 0.027(0.32)$ |
| 5 | $0.046 \pm 0.027(1.7)$ | $-0.02 \pm 0.026(-0.76)$ | $-0.04 \pm 0.027(-1.5)$ | $-0.012 \pm 0.027(-0.44)$ | $0.13 \pm 0.027(4.8)$ | $-0.04 \pm 0.027(-1.5)$ | $-0.0056 \pm 0.027(-0.2)$ |
| 6 | $-0.013 \pm 0.026(-0.52)$ | $0.041 \pm 0.027(1.5)$ | $-0.033 \pm 0.028(-1.2)$ | $-0.0019 \pm 0.027(-0.068)$ | $0.15 \pm 0.027(5.4)$ | $-0.034 \pm 0.027(-1.3)$ | $-0.021 \pm 0.028(-0.7)$ |
| 8 | $0.039 \pm 0.026(1.5)$ | $0.01 \pm 0.028(0.36)$ | $-0.027 \pm 0.028(-0.96)$ | $-0.024 \pm 0.026(-0.9)$ | $0.12 \pm 0.027(4.3)$ | $-0.092 \pm 0.026(-3.5)$ | $-0.036 \pm 0.027(-1.3)$ |
| 9 | $-0.01 \pm 0.027(-0.38)$ | $0.0024 \pm 0.027(0.09)$ | $-0.018 \pm 0.026(-0.68)$ | $0.022 \pm 0.028(0.79)$ | $0.068 \pm 0.027(2.6)$ | $-0.069 \pm 0.026(-2.6)$ | $-0.078 \pm 0.026(-3)$ |
| 10 | $-0.017 \pm 0.027(-0.62)$ | $-0.015 \pm 0.026(-0.57)$ | $0.012 \pm 0.027(0.46)$ | $-0.074 \pm 0.028(-2.6)$ | $0.1 \pm 0.027(3.8)$ | $-0.003 \pm 0.027(-0.11)$ | $-0.043 \pm 0.029(-1.5)$ |
| 11 | $0.032 \pm 0.026(1.2)$ | $0.024 \pm 0.026(0.91)$ | $-0.04 \pm 0.026(-1.5)$ | $-0.066 \pm 0.027(-2.5)$ | $0.098 \pm 0.027(3.7)$ | $-0.043 \pm 0.028(-1.6)$ | $-0.0018 \pm 0.028(-0.06)$ |



| q^2 | Mean of the pull | | | | | | |
|-------|----------------------------|-----------------------------|----------------------------|---------------------------|----------------------------|-------------------------|----------------------------|
| | S_3 | S_4 | S_5 | S_{6s} | S_{6c} | S_7 | S_8 |
| 0 | $-0.019 \pm 0.026(-0.71)$ | $0.048 \pm 0.027(1.8)$ | $0.018 \pm 0.027(0.67)$ | $0.059 \pm 0.027(2.2)$ | $-0.015 \pm 0.028(-0.55)$ | $0.061 \pm 0.027(2.2)$ | $0.012 \pm 0.027(0.44)$ |
| 1 | $-0.014 \pm 0.028(-0.51)$ | $0.043 \pm 0.027(1.6)$ | $0.013 \pm 0.027(0.49)$ | $0.024 \pm 0.028(0.86)$ | $-0.038 \pm 0.028(-1.4)$ | $0.024 \pm 0.026(0.94)$ | $0.037 \pm 0.027(1.3)$ |
| 2 | $-0.03 \pm 0.027(-1.1)$ | $-0.0021 \pm 0.027(-0.076)$ | $-0.01 \pm 0.027(-0.39)$ | $-0.017 \pm 0.027(-0.61)$ | $-0.016 \pm 0.027(-0.58)$ | $0.013 \pm 0.027(0.49)$ | $0.027 \pm 0.028(0.98)$ |
| 3 | $-0.007 \pm 0.027(-0.26)$ | $0.03 \pm 0.028(1.1)$ | $-0.036 \pm 0.027(-1.3)$ | $-0.074 \pm 0.027(-2.8)$ | $0.041 \pm 0.027(1.5)$ | $0.08 \pm 0.028(2.9)$ | $-0.0083 \pm 0.027(-0.31)$ |
| 4 | $0.00089 \pm 0.029(0.031)$ | $-0.021 \pm 0.027(-0.77)$ | $0.0032 \pm 0.028(0.11)$ | $0.0031 \pm 0.026(0.12)$ | $0.012 \pm 0.027(0.44)$ | $0.019 \pm 0.026(0.71)$ | $0.034 \pm 0.027(1.3)$ |
| 5 | $0.044 \pm 0.028(1.6)$ | $-0.022 \pm 0.026(-0.82)$ | $-0.041 \pm 0.027(-1.5)$ | $-0.014 \pm 0.027(-0.53)$ | $-0.029 \pm 0.027(-1.1)$ | $0.041 \pm 0.027(1.5)$ | $0.042 \pm 0.028(1.5)$ |
| 6 | $-0.011 \pm 0.026(-0.42)$ | $0.041 \pm 0.027(1.5)$ | $-0.045 \pm 0.028(-1.6)$ | $0.011 \pm 0.027(0.41)$ | $-0.0089 \pm 0.027(-0.33)$ | $0.067 \pm 0.027(2.5)$ | $0.036 \pm 0.027(1.3)$ |
| 8 | $0.05 \pm 0.026(1.9)$ | $0.0021 \pm 0.028(0.074)$ | $-0.025 \pm 0.028(-0.91)$ | $-0.023 \pm 0.026(-0.87)$ | $-0.024 \pm 0.028(-0.85)$ | $0.022 \pm 0.026(0.82)$ | $0.026 \pm 0.026(0.98)$ |
| 9 | $-0.0064 \pm 0.027(-0.23)$ | $0.012 \pm 0.027(0.44)$ | $-0.0075 \pm 0.026(-0.28)$ | $0.029 \pm 0.027(1.1)$ | $-0.087 \pm 0.027(-3.2)$ | $0.036 \pm 0.027(1.3)$ | $-0.02 \pm 0.026(-0.76)$ |
| 10 | $-0.019 \pm 0.027(-0.71)$ | $-0.0051 \pm 0.025(-0.2)$ | $0.0081 \pm 0.026(0.31)$ | $-0.077 \pm 0.028(-2.7)$ | $-0.03 \pm 0.027(-1.1)$ | $0.11 \pm 0.027(4.1)$ | $0.013 \pm 0.028(0.46)$ |
| 11 | $0.027 \pm 0.026(1)$ | $0.032 \pm 0.027(1.2)$ | $-0.038 \pm 0.027(-1.4)$ | $-0.048 \pm 0.026(-1.8)$ | $-0.021 \pm 0.027(-0.77)$ | $0.019 \pm 0.028(0.69)$ | $0.035 \pm 0.027(1.3)$ |



- Let's try to understand if we can understand why this happens:
- Let's calculate what should I expect with the unfolding
- This is up to normalization!
 - $M_5 = 0.4S_5 \rightarrow M_5 = 0.00512S_3 + 0.4S_5 - 0.002S_7$
 - $M_8 = 0.32S_8 \rightarrow M_8 = 0.0016S_4 + 0.00512S_7 + 0.32S_8$
 - $M_7 = 0.4S_7 \rightarrow M_7 = 0.002S_5 + 0.4S_7 + 0.00512S_8$
 - $M_3 = 0.32S_3 \rightarrow M_3 = 0.32S_3 - 0.0008S_9$
- The way you can look at this is that i just shown you how our unfolding matrix works like.



- Let's try to understand if we can understand why this happens:
- Let's calculate what should I expect with the unfolding
- This is up to normalization!
 - $M_5 = 0.4S_5 \rightarrow M_5 = 0.4S_5$
 - $M_8 = 0.32S_8 \rightarrow M_8 = 0.32S_5$
 - $M_7 = 0.4S_7 \rightarrow M_7 = 0.4S_7$
 - $M_3 = 0.32S_3 \rightarrow M_3 = -0.0036 + 0.0012FI + 0.32S_3$
- The way you can look at this is that i just shown you how our unfolding matrix works like.



Summary

- Developed a systematic way how to get Unfolding matrix
- Moments are resistant against variety of unfolding discrepancies.
- This might lead to reduced systematics in the future.

