Introduction to Accelerator Physics

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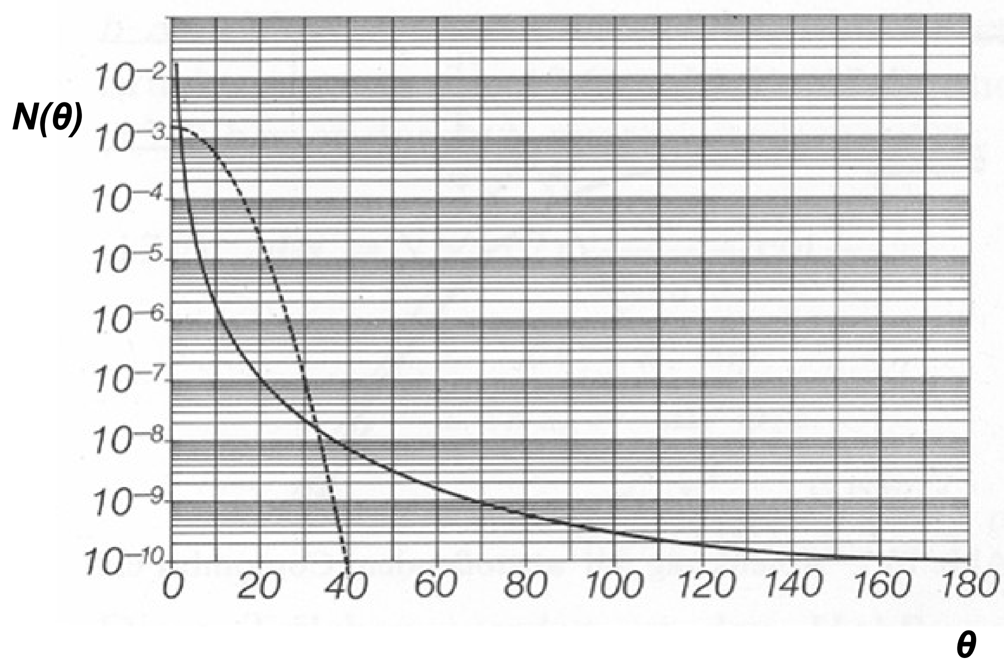
CERN-LHC

I.) The First Steps

A Bit of History

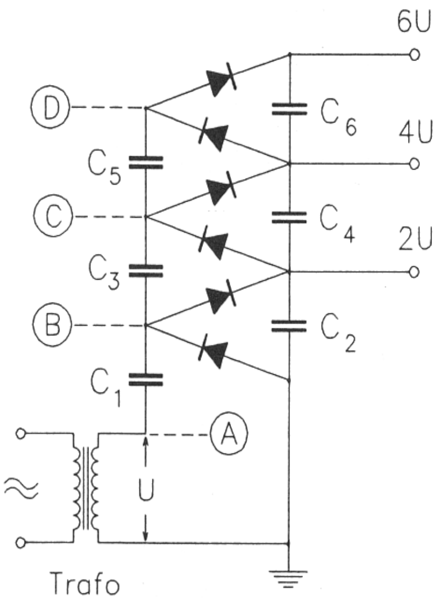
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



Rutherford Scattering, 1906

Using radioactive particle sources:
α-particles of some MeV energy

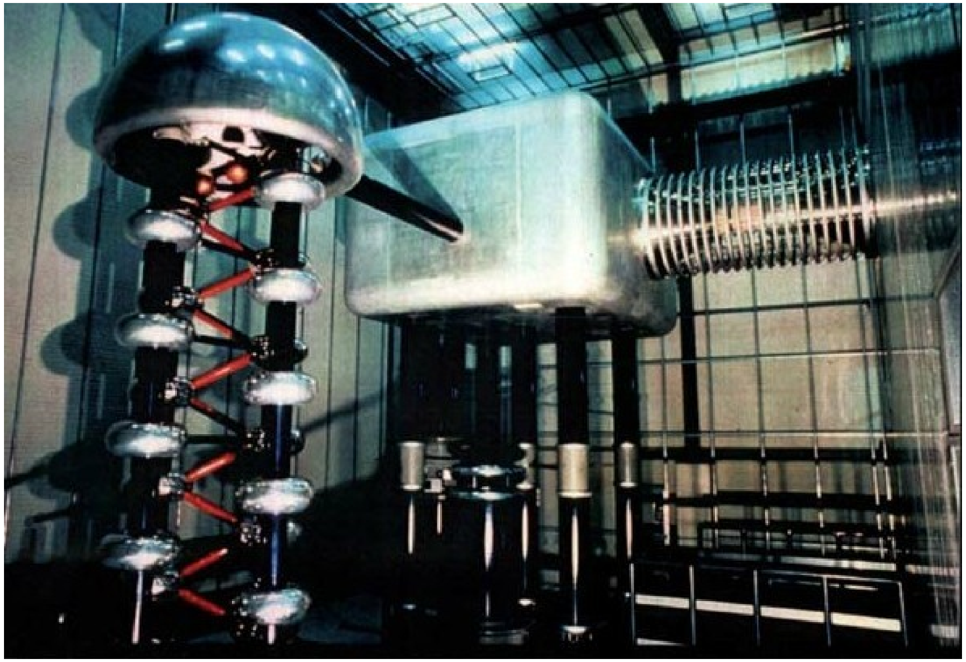
1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: ***Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam***

1932: ***First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV***

Particle source: ***Hydrogen discharge tube on 400 kV level***
Accelerator: ***evacuated glas tube***
Target: ***Li-Foil on earth potential***



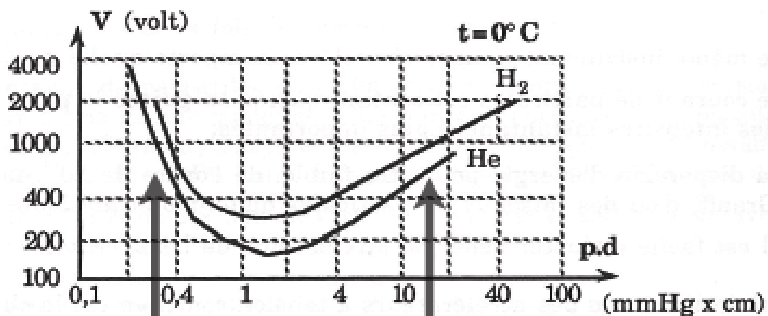
Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

robust, simple, on-knob machines
largely used in history as pre-accelerators for proton and ion beams
recently replaced by modern structures (RFQ)

Main limitation

Main limitation:
electric discharge due to too high Voltage.
Maximum limit: 1 MV

Limit set by Paschen law:
the breaking Voltage between two parallel electrodes depends only on the pressure of the gas between the electrodes and their distance

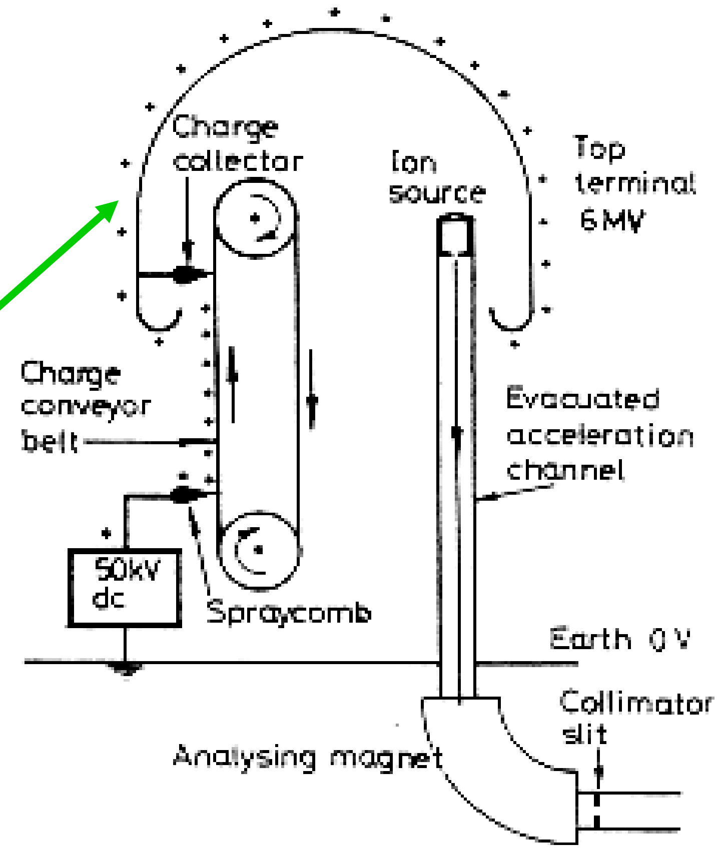


Low pressure: gas not too dense, long mean average path of electrons

High pressure: dense gas, large Voltage needed for gas ionisation

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV using high pressure gas to suppress discharge (SF_6)*

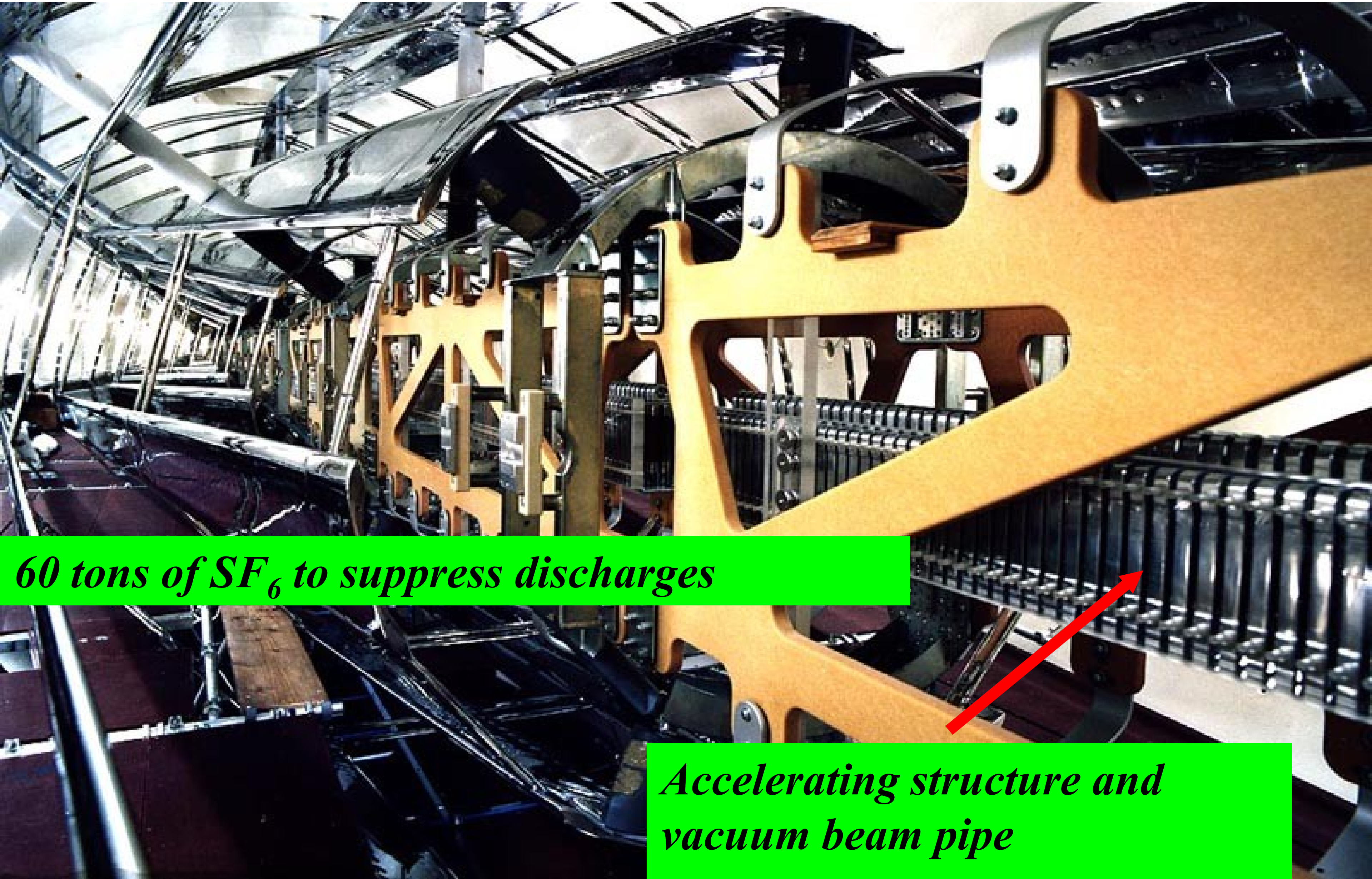
*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle": Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

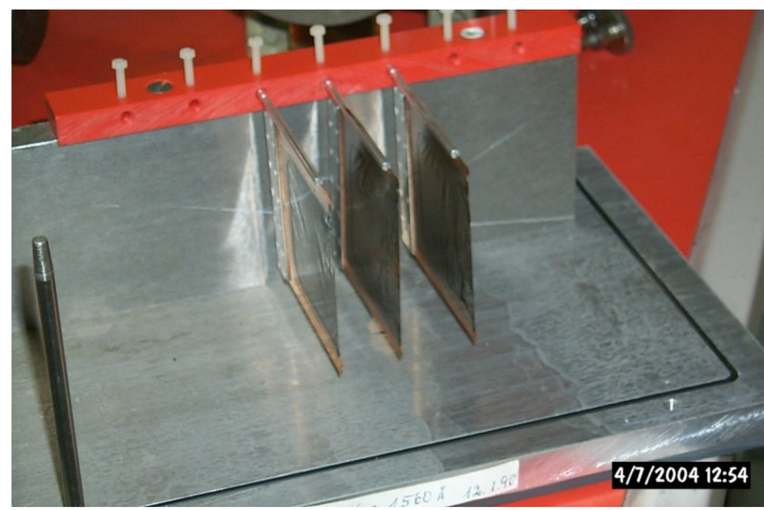
... and how it looks inside

"Vivitron" Strassbourg



The Principle of the "Steam Engine": Mechanical Transport of Charge via a rotating chain or belt

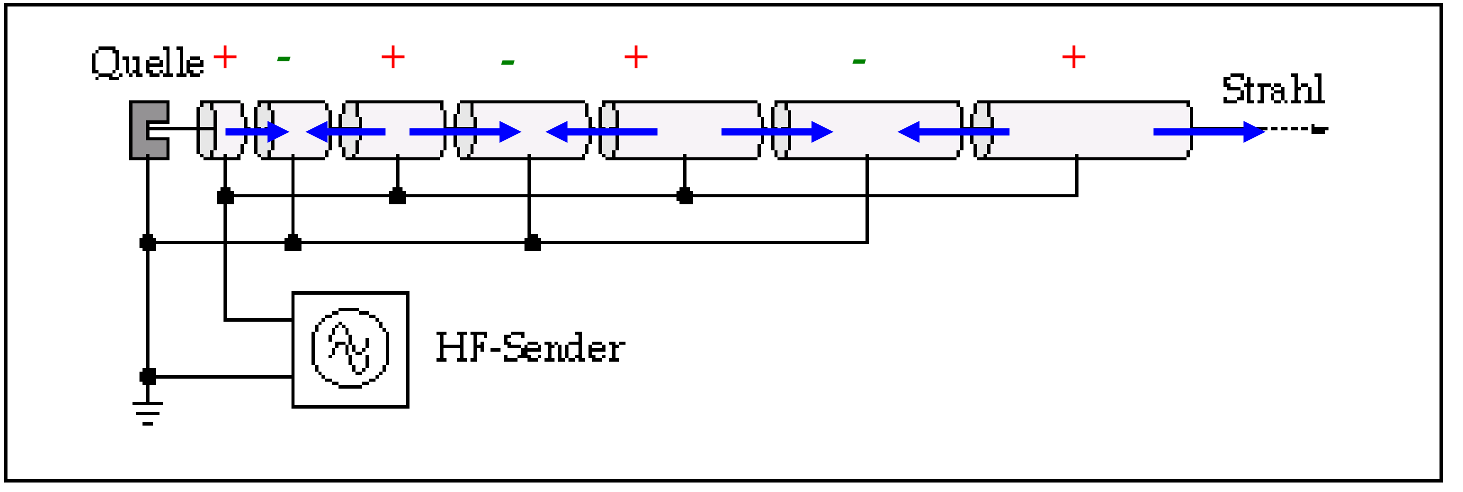
stripping foils: 1500 Å



3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

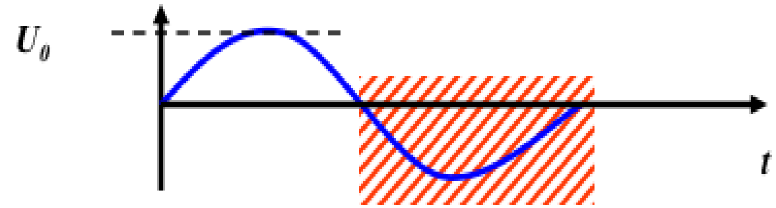
$$E_n = n * q * U_0 * \sin \psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$\rightarrow v_i = \sqrt{2E_i/m}$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0 * \sin \psi_s}}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \ldots 0.6$, Particles: Protons/Ions

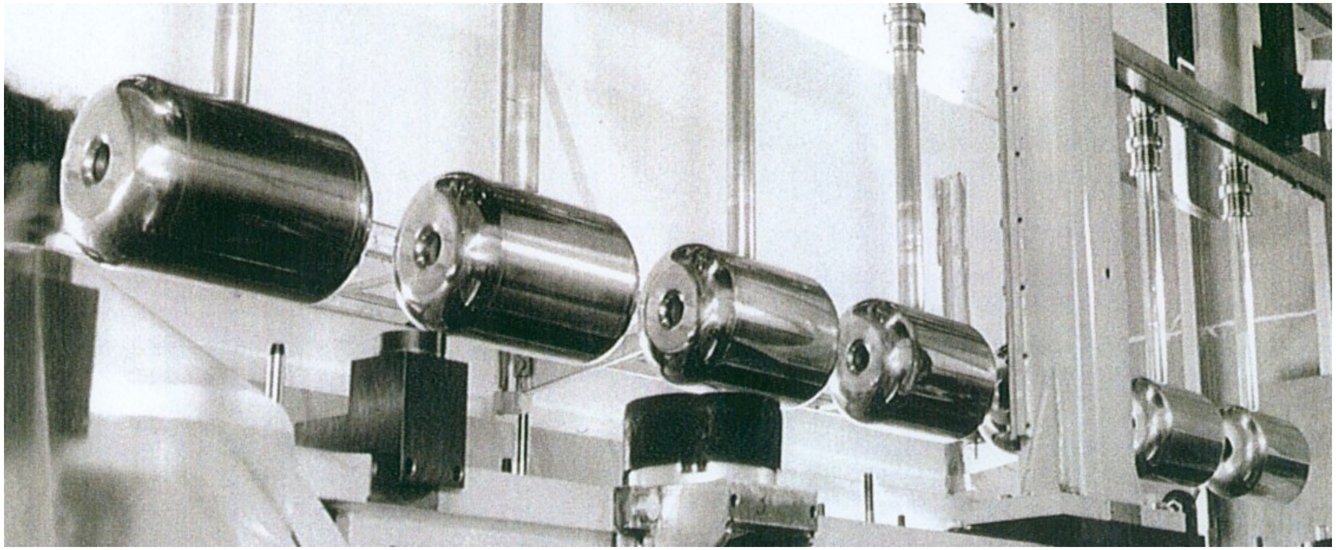
Example: DESY *Accelerating structure of the Proton Linac*

$E_{total} = 988\,MeV$

$m_0c^2 = 938\,MeV$

$p = 310\,MeV/c$

$E_{kin} = 50\,MeV$



Beam energies

1.) reminder of some relativistic formula

rest energy $E_0 = m_0c^2$

total energy $E = \gamma * E_0 = \gamma * m_0c^2$

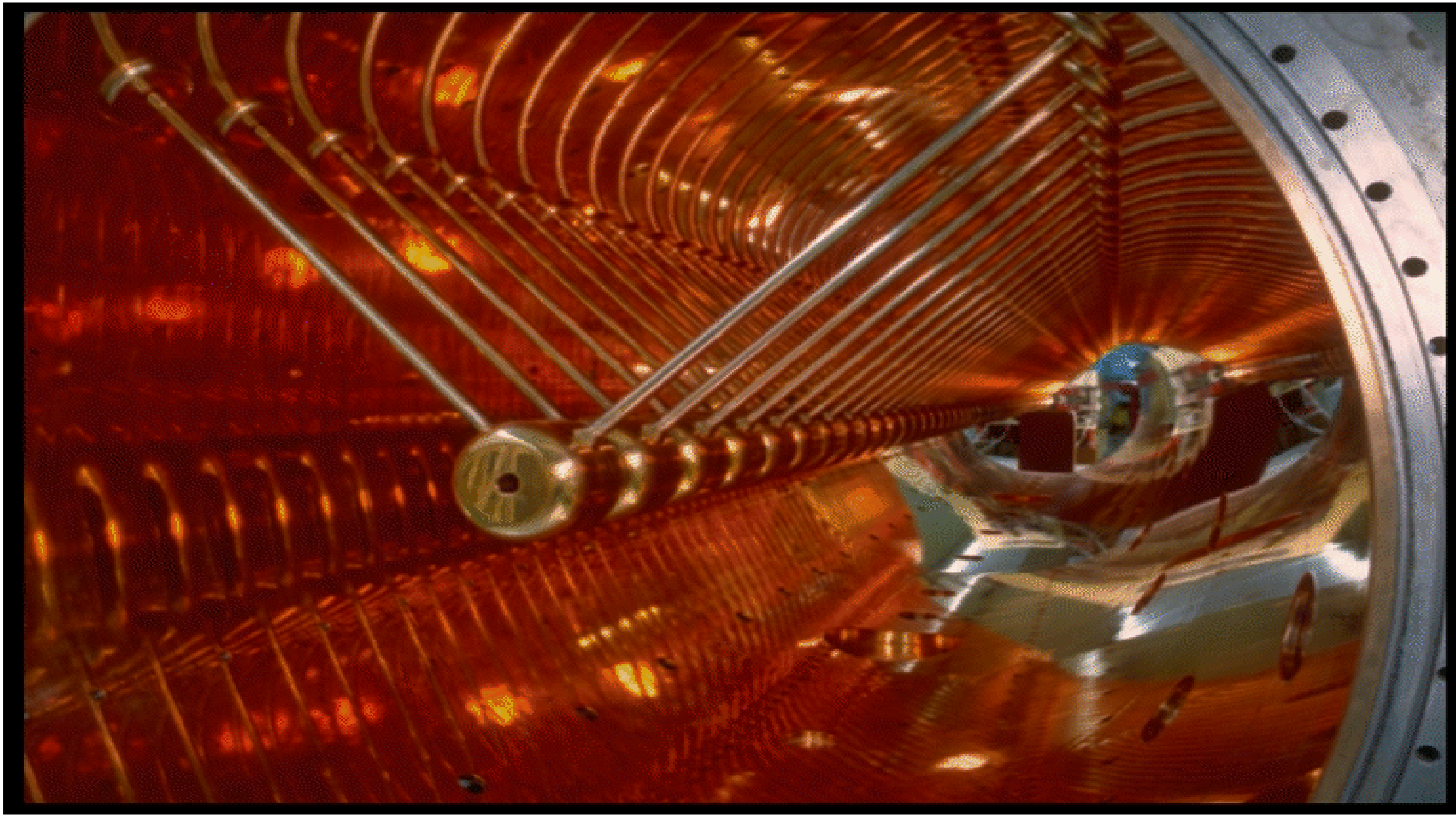
momentum $E^2 = c^2p^2 + m_0^2c^4$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

GSI: *Unilac, typical Energie ≈ 20 MeV per Nukleon, $\beta \approx 0.04 \ldots 0.6$, Protons/Ions, $\nu = 110$ MHz*

Energy Gain per „Gap“:

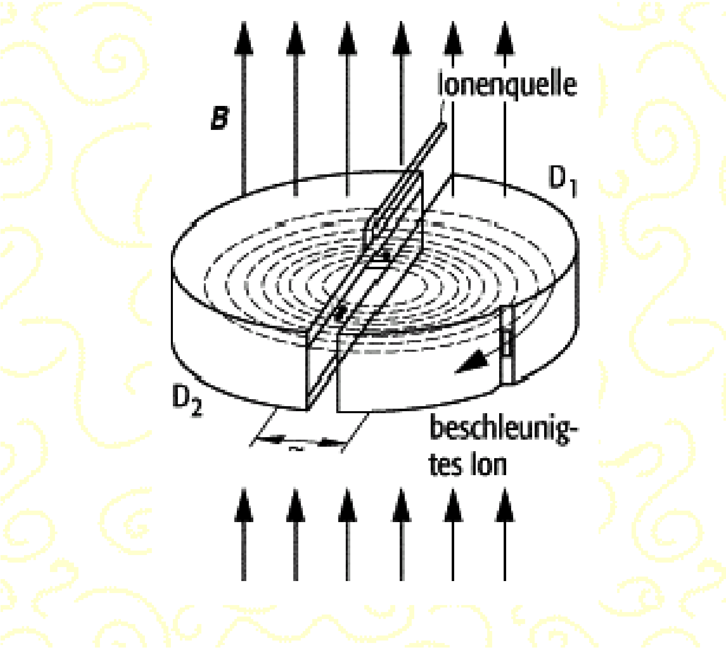
$$W = q\, U_0 \sin \omega_{RF}\, t$$



Application: ***until today THE standard proton / ion pre-accelerator***
CERN Linac 4 is being built at the moment

4.) The Cyclotron: (Livingston / Lawrence ~1930)

Idea: Bend a Linac on a Spiral
Application of a constant magnetic field
keep B = const, RF = const



➔ ***Lorentzforce***

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \rightarrow \quad B * R = p / q$$

increasing radius for
increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$\omega_z = \frac{q}{m} * B_z$$

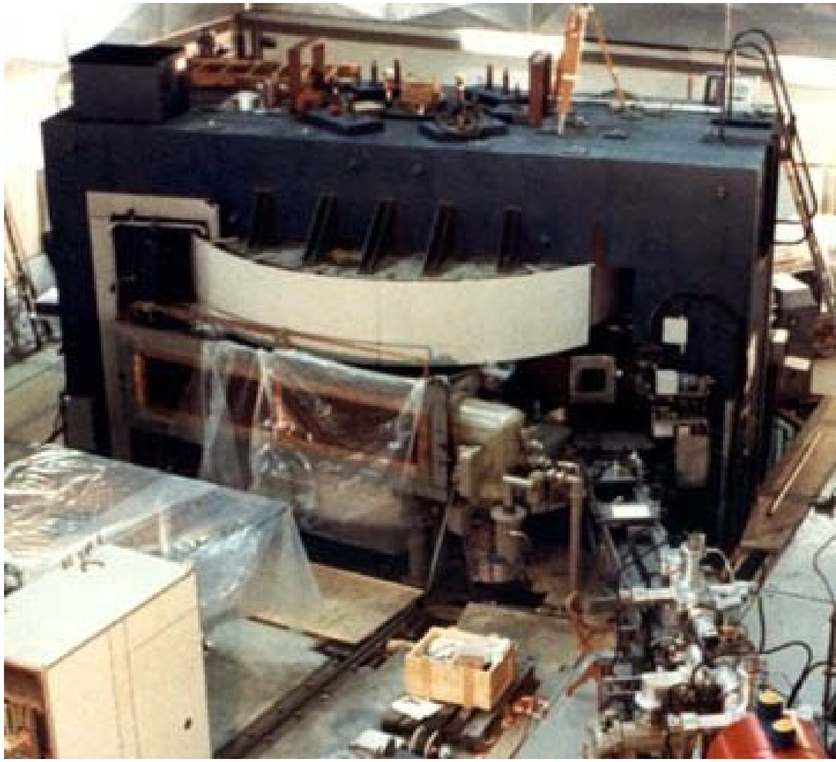
the cyclotron (rf-) frequency
is independent of the momentum

Cyclotron:

! **ω is constant** *for a given q & B*

!! *B*R = p/q*
large momentum → huge magnet

!!!! ω ~ 1/m ≠ const *works properly only for non relativistic particles*

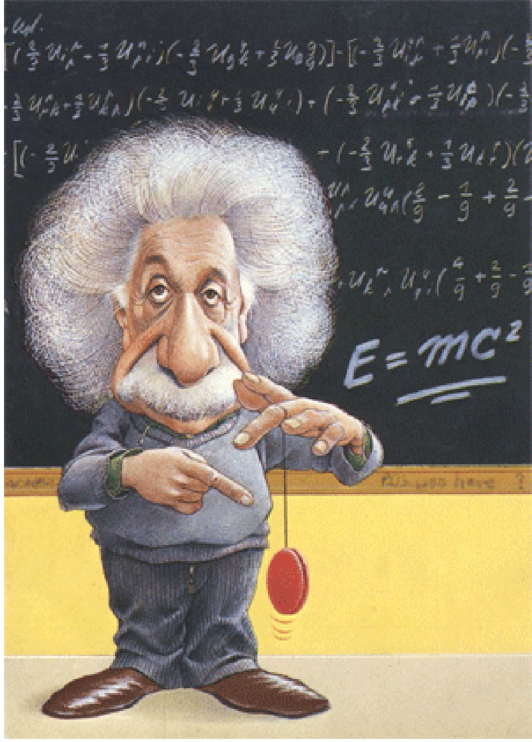


PSI Zurich

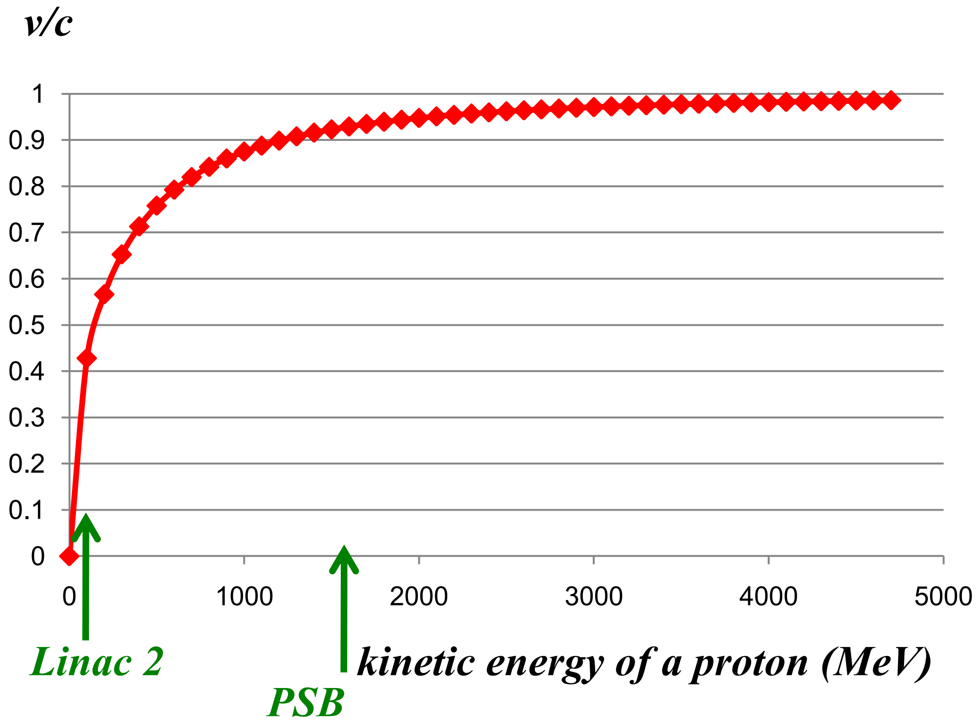
Application:
Work horses for medium energy protons
Proton / Ion Acceleration up to ≈ 60 MeV (proton energy)
nuclear physics
radio isotope production, proton / ion therapy

Beam Energy

... so sorry, here we need help from Albert:



$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}}$$



CERN Accelerators	kin. Energy	γ
Linac 2	60 MeV	1.06
PS	26 GeV	27
SPS	450 GeV	480
LHC	7 TeV	7460

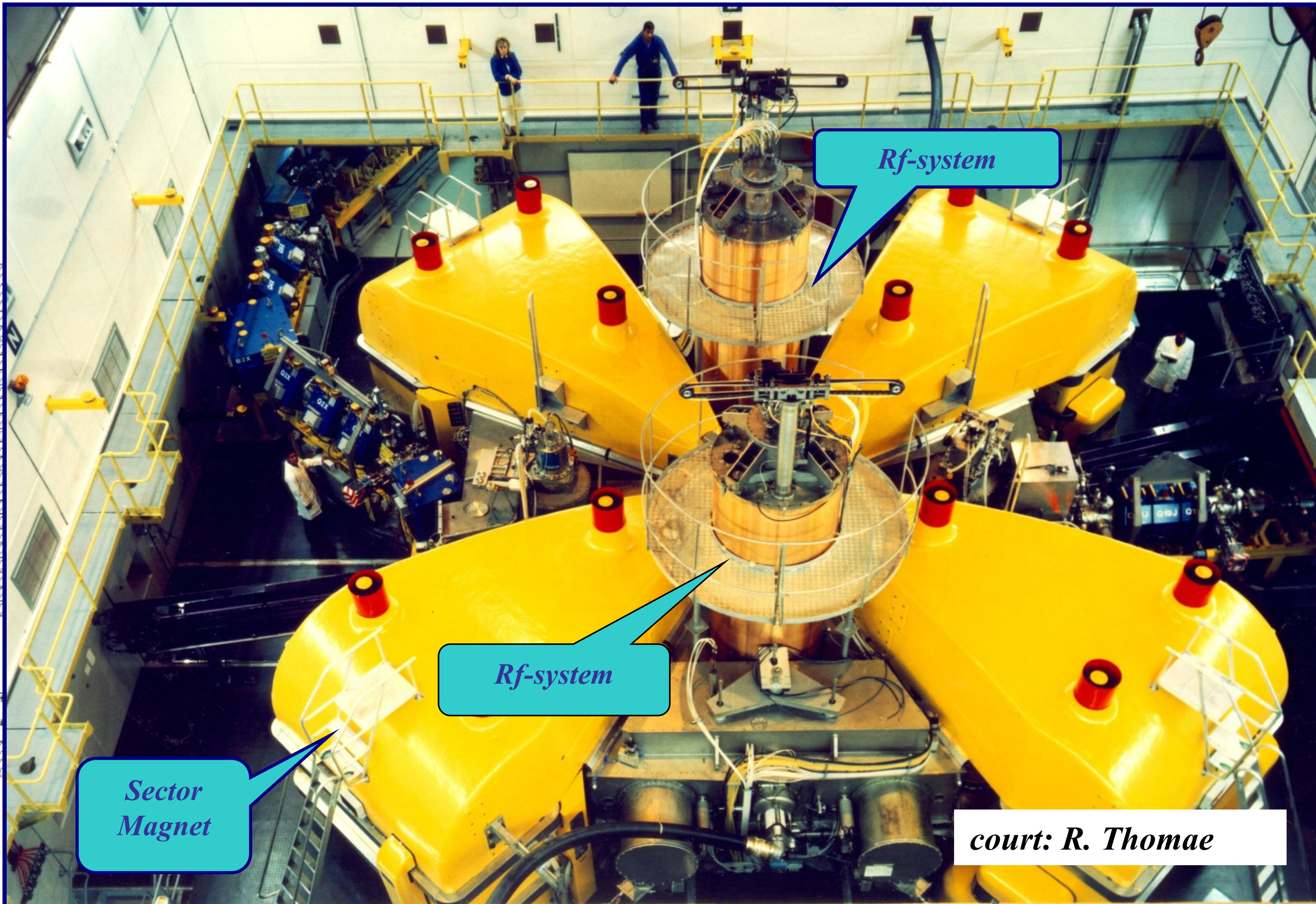
remember: proton mass = 938 MeV

Cyclotron:

modern trends: Problem: m ≠ const.

➔ non relativistic machine

$$\omega_z = \frac{e * B_z}{\gamma * m_0}$$



5.) The Betatron: Wideroe 1928/ Kerst 1940

...apply the transformer principle to an electron beam: no RF system needed, changing magnetic B field

Idea: a time varying magnetic field induces a voltage that will accelerate the particles

Farady induction law

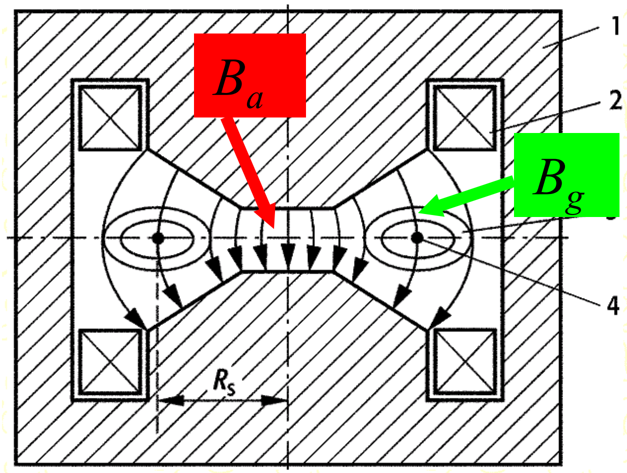
$$\oint \vec{E}\, d\vec{s} = -\int_A \dot{B}\, df = -\dot{\Phi}$$

circular orbit

$$\frac{mv^2}{r} = e * v * B$$

$$\rightarrow \quad p = e * B * r$$

schematic design



magnetic flux through this orbit area

$$\Phi = \int B df = \pi r^2 * B_a$$

induced electric field

$$\oint \vec{E}\,ds = \vec{E} * 2\pi r = -\dot{\Phi} \quad \Rightarrow \quad \vec{E} = \frac{-\pi r^2 * \dot{B}_a}{2\pi r} = -\frac{1}{2}\dot{B}_a r$$

force acting on the particle: $\dot{p} = -|\vec{E}|e = \frac{1}{2}\dot{B}_a r$

The increasing momentum of the particle has to be accompanied by a rising magnetic guide field:

$\dot{p} = e * \dot{B}_g r$ $\boldsymbol{B_g = \frac{1}{2} B_a}$

robust, compact machines,
Energy $\leq$ ***300...500 MeV,***
limit: Synchrotron radiation

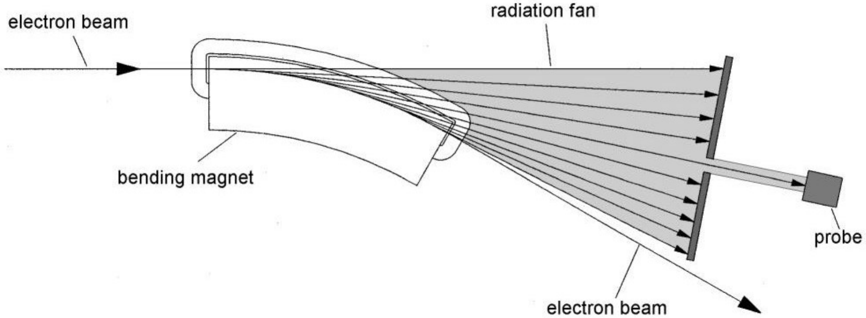
6.) Synchrotrons / Storage Rings / Colliders:

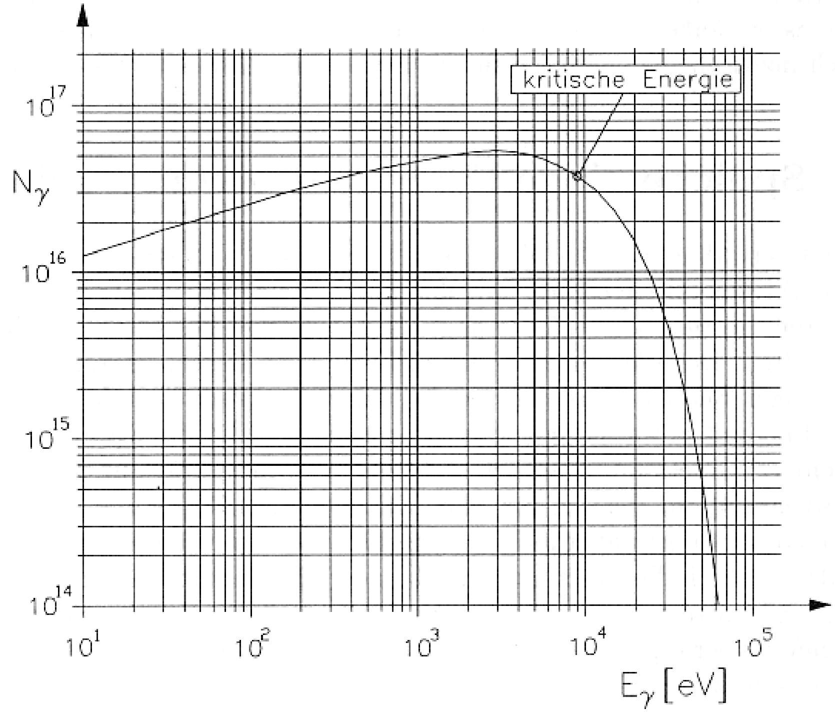
Wideroe 1943, McMillan, Veksler 1944,
Courant, Livingston, Snyder 1952

Idea: define a circular orbit of the particles,
keep the beam there during acceleration,
put magnets at this orbit to guide and focus

Advanced Photon Source,
Berkley

7.) Electron Storage Rings
Production of Synchrotron Light





$$P_s = \frac{e^2 c}{6\pi\varepsilon_0} * \frac{1}{(m_0 c^2)^4} \frac{E^4}{R^4}$$

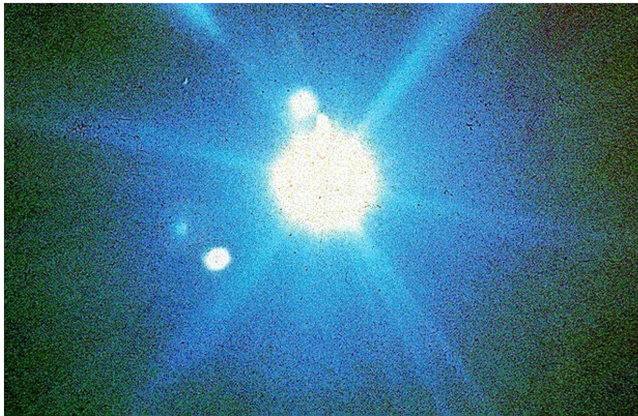
Radiation Power

$$\Delta E = \frac{e^2}{3\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{R}$$

Energy Loss per turn

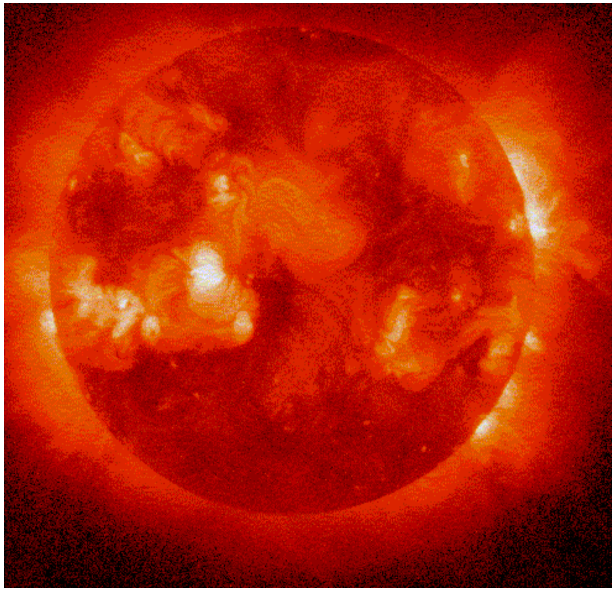
$$\omega_c = \frac{3c\gamma^3}{2R}$$

„typical Frequency“ of emitted light

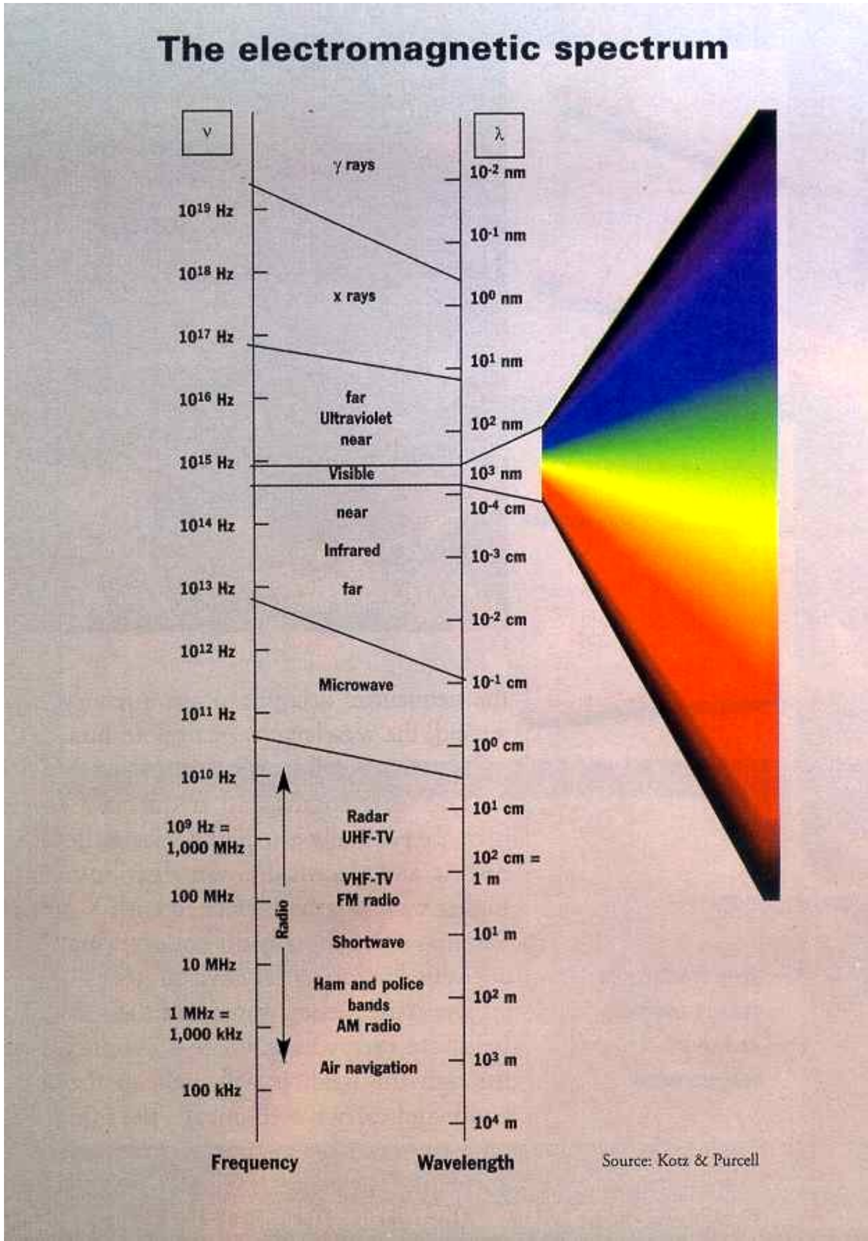


Application of Synchrotron Light
Analysis at Atoms & Molecules

The electromagnetic Spectrum:



having a closer look at the sun ...



Light:

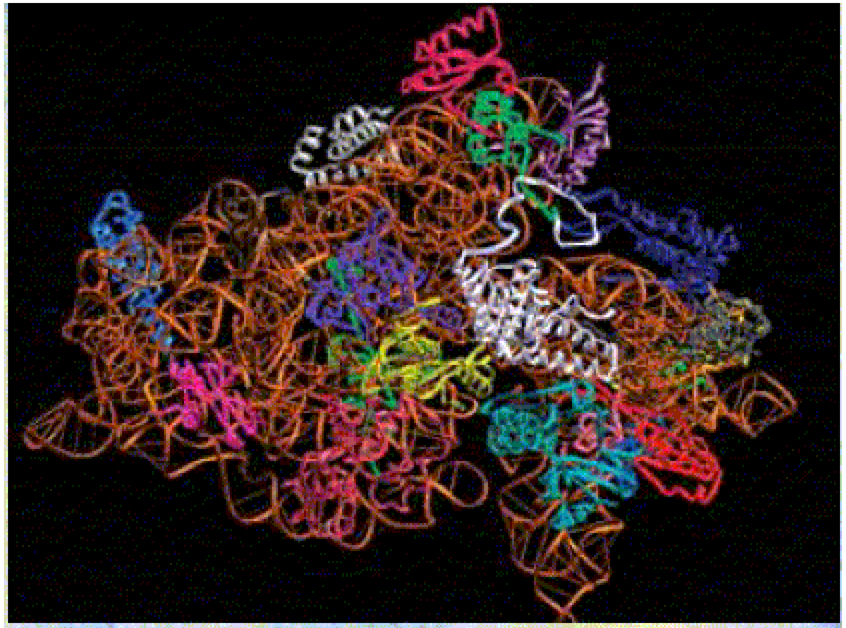
$\lambda \approx 400$ nm … 800 nm

1 Oktave

Analysis of Cell structures

Structure of a Ribosom

Ribosomen are responsible for the protein production in living cells.
The structure of these Ribosom molecules can be analysed using brilliant synchrotron light from electron storage rings
(Quelle: Max-Planck-Arbeitsgruppen für Strukturelle Molekularbiologie)



Structure of the ribosome, the "protein factory" in living cells

Angiographie

x-ray method applicable for the imaging of coronar heart arteria

8.) Synchrotrons as Collider Rings (1960 ...):

Beam energies

1.) reminder of some relativistic formula

total energy $$E^2 = p^2c^2 + m_0^2c^4$$

$$\longrightarrow \quad cp = \sqrt{E^2 - m_0^2c^4} = \sqrt{(\gamma m_0 c^2)^2 - (m_0 c^2)^2} = \sqrt{\gamma^2 - 1}\, m_0 c^2$$

$$\longrightarrow \quad cp = \gamma\beta * m_0 c^2$$

2.) energy balance of colliding particles

rest energy of a particle $$E_0^2 = (m_0 c^2)^2 = E^2 - p^2c^2$$

in exactly the same way we define a center of mass energy of a system of particles:

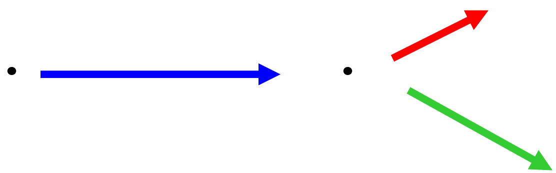
$$E_{cm}^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i cp_i\right)^2$$

two colliding particles

$$E_{cm}^{2} = \left(\gamma_1 m_1 + \gamma_2 m_2\right)^2 c^4 - \left(cp_1 + cp_2\right)^2$$

$$E_{cm}^{2} = \left(\gamma_1 m_1 + \gamma_2 m_2\right)^2 c^4 - \left(\gamma_1 \beta_1 m_1 + \gamma_2 \beta_2 m_2\right)^2 c^4$$

Example 1): proton beam on fixed proton



$$\gamma_2 = 1$$

$$m_1 = m_1 = m_p$$

$$\beta_2 = 0$$

$$E_{cm}^{2} = \left(\gamma_1 + 1\right)^2 m_p^2 c^4 - \left(\gamma_1 \beta_1 m_1\right)^2 c^4$$

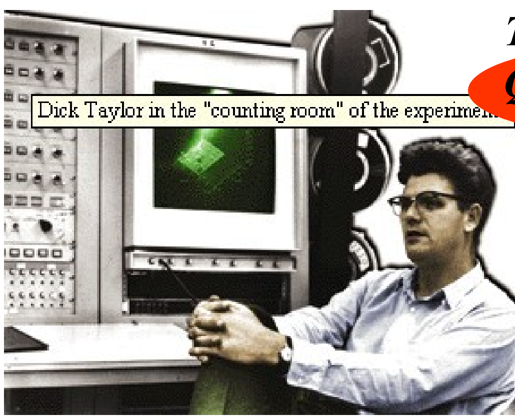
remember: $\beta\gamma = \sqrt{\gamma^2 - 1}$

$$E_{cm}^{2} = (\gamma_1 + 1)^2 m_p^2 c^4 - (\gamma_1^2 - 1) * m_p^2 c^4$$

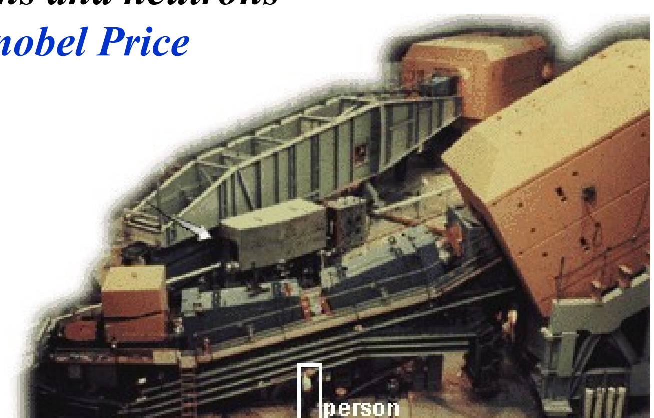
$$E_{cm}^{2} = 2(\gamma_1 - 1) * m_p^2 c^4$$

$$E_{cm} = \sqrt{2(\gamma_1 - 1)} * m_p c^2$$

Descovery of the Quarks: electron beam on fixed proton / neutron target



Taylor/Kendall/Friedman: Discovery of the Quark structure of protons and neutrons 1966-1978 ..... 1990 Knobel Price

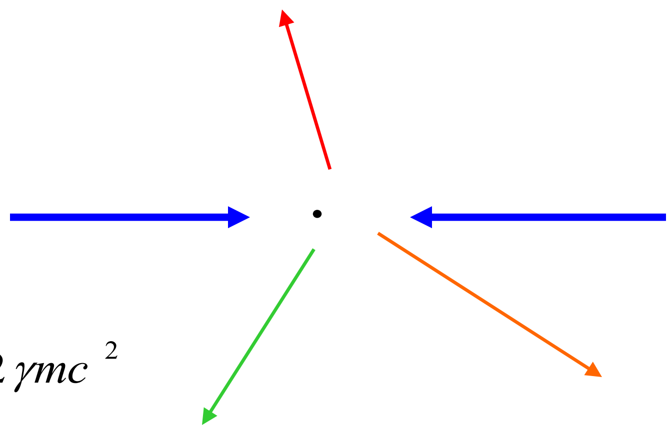


Example 2 : particle anti-particle collider

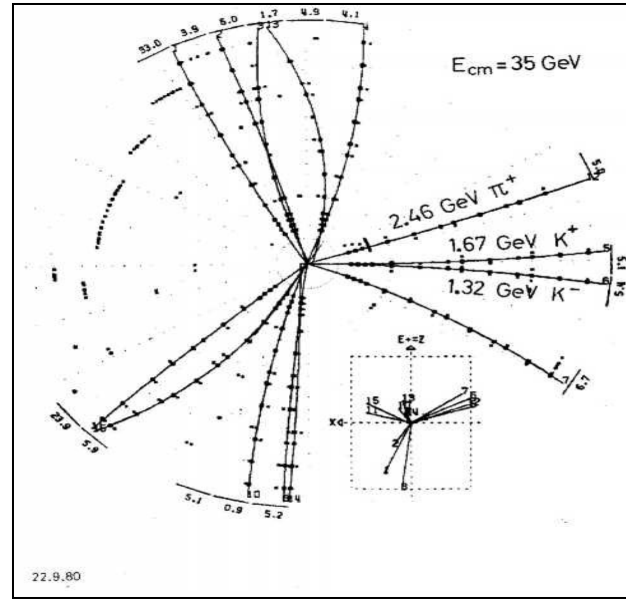
$e+ / e-$, $p / \bar{p}$, $m+ / m-$

* store both counter rotating particle beams in the same magnet lattice
* no conservation of quantum numbers required

$$E_{cm}^2 = (\gamma_1 m_1 + \gamma_2 m_2)^2 c^4 - (cp_1 + cp_2)^2$$



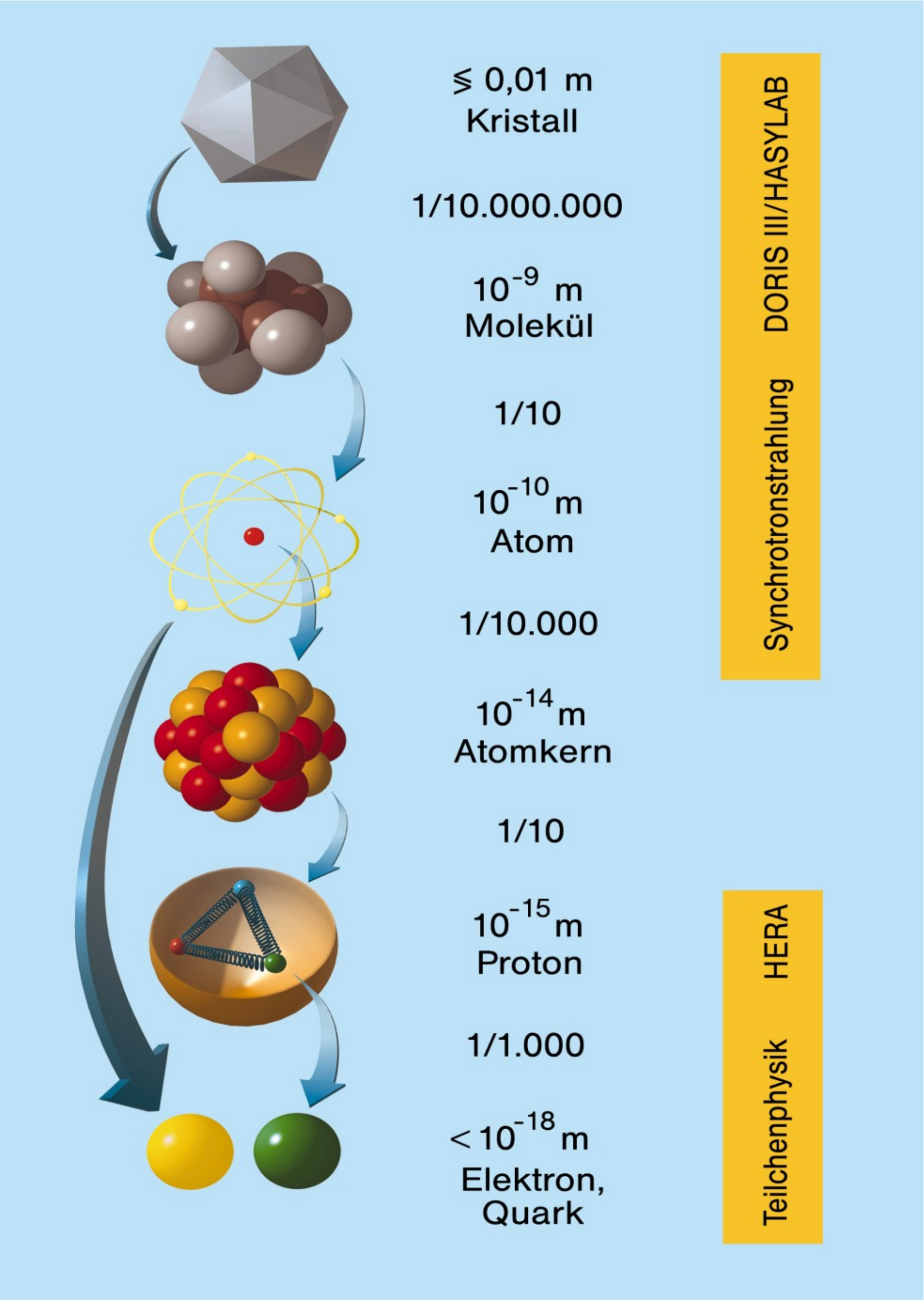
$$E_{cm} = 2\gamma mc^2$$



1979 PETRA Collider at DESY
discovery of the gluon

Colliders: * working at highest energies ("cm")
* store the particles for long time in an accelerator
* bring two beams into collision
* particle density !!
* preparation / technical design / field qualities are extreme

Structure of Matter



9.) Storage Rings for Structure Analysis

synchrotron light: nm

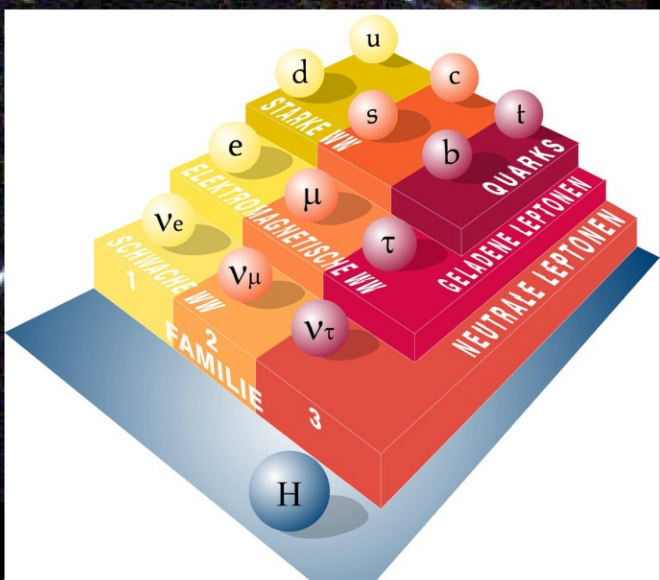
electron scattering: Å ... 10 $^{-18}$ ***m***

de Broglie:

$$\lambda = \frac{h}{p} = \frac{ch}{E} \qquad E \approx pc$$

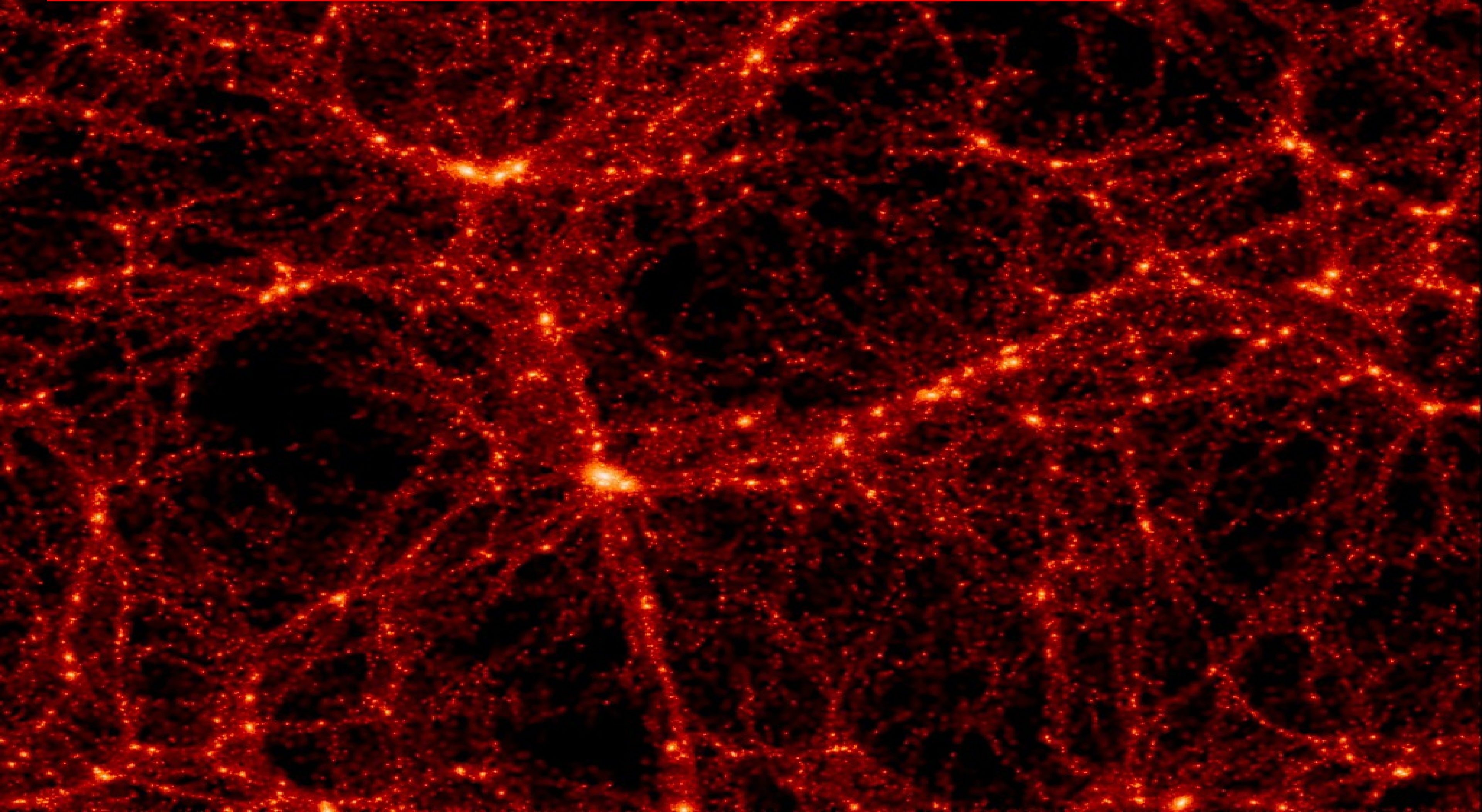
10.) Storage Rings to Explain the Universe

Precision Measurements of the Standard Model,
Search for Higgs, Supersymmetry, Dark Matter
Physics beyond the Standard Model



Reconstruction of Dark Matter distribution based on observations

Budget: ***Dark Matter: 33 %***
Dark Energy: 66 %
Anything else (including us) 1%

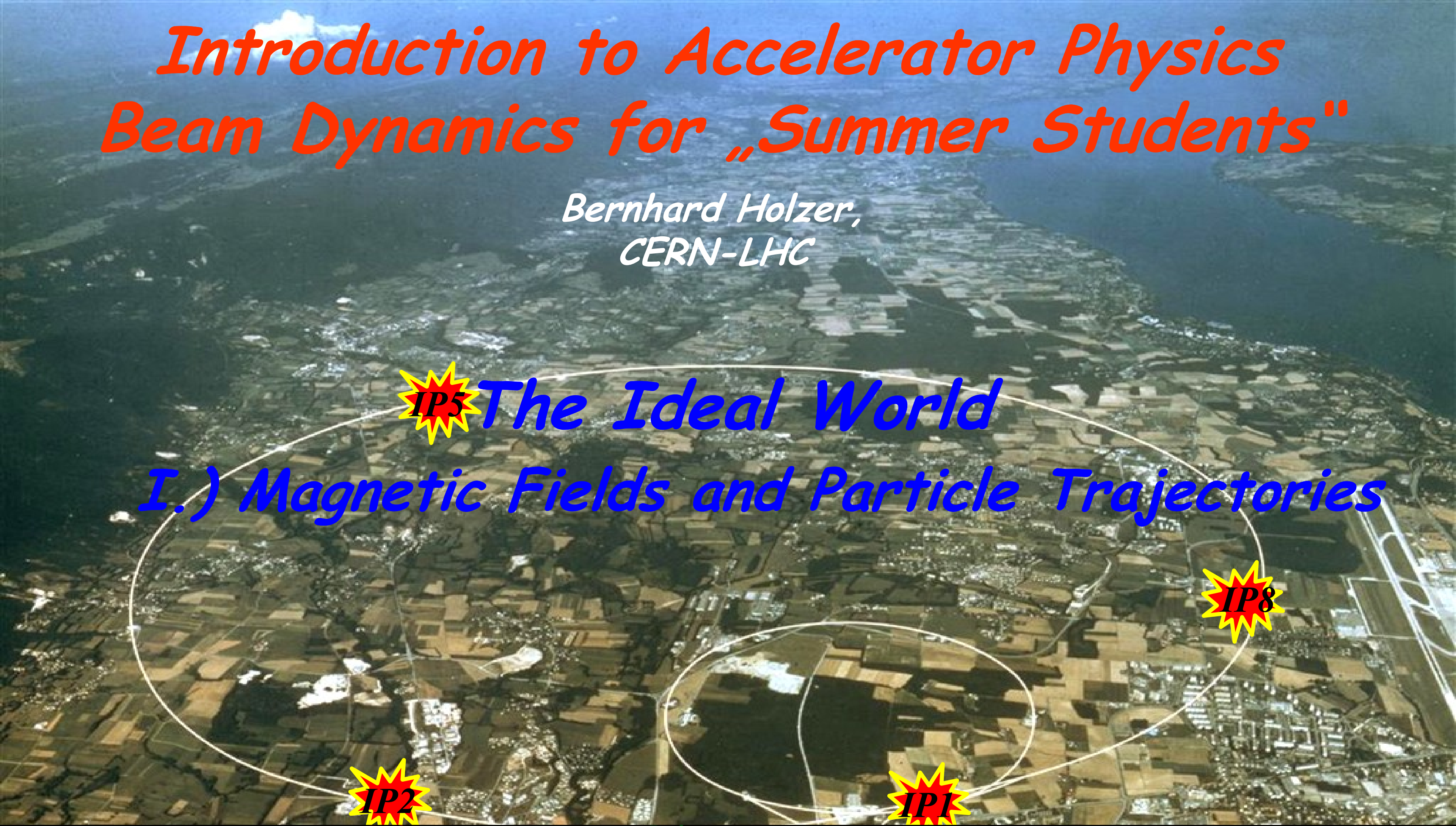


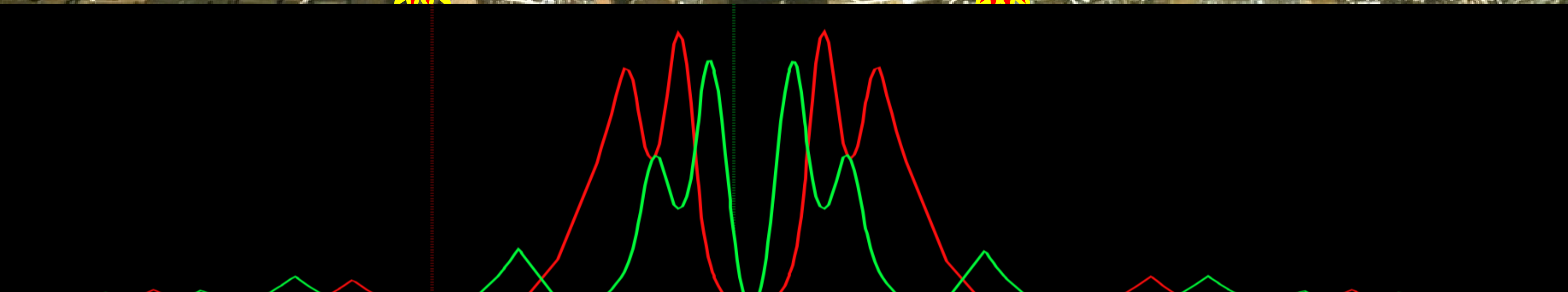
II

Introduction to Accelerator Physics
Beam Dynamics for „Summer Students"

Bernhard Holzer,
CERN-LHC

The Ideal World
I.) Magnetic Fields and Particle Trajectories





TEST:

$$D_n = \beta_C \sin n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^{n} \cos\left(i\phi_C - \frac{1}{2}\phi_C \pm \varphi_m\right) * \sqrt{\frac{\beta_m}{\beta_C}} - \\ - \cos n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^{n} \sqrt{\beta_m \beta_C} * \sin\left(i\phi_C - \frac{1}{2}\phi_C \pm \varphi_m\right)$$

$$D_n = \sqrt{\beta_m \beta_C} * \sin n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^{n} \cos\left((2i-1)\frac{\phi_C}{2} \pm \varphi_m\right) - \\ - \sqrt{\beta_m \beta_C} * \delta_{\text{supr}} * \cos n\phi_C \sum_{i=1}^{n} \sin\left((2i-1)\frac{\phi_C}{2} \pm \varphi_m\right)$$

Remembering the trigonometric gymnastics shown above we get

$$D_n = \delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \sin n\phi_C * \sum_{i=1}^{n} \cos\left((2i-1)\frac{\phi_C}{2}\right) * 2\cos\varphi_m - \\ - \delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos n\phi_C \sum_{i=1}^{n} \sin\left((2i-1)\frac{\phi_C}{2}\right) * 2\cos\varphi_m$$

$$D_n = 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos\varphi_m \left\{ \sum_{i=1}^{n} \cos\left((2i-1)\frac{\phi_C}{2}\right) * \sin(n\phi_C) - \\ - \sum_{i=1}^{n} \sin\left((2i-1)\frac{\phi_C}{2}\right) * \cos(n\phi_C) \right\}$$

$$D_n = 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos\varphi_m \sin(n\phi_C) \frac{\sin\frac{n\phi_C}{2} * \cos\frac{n\phi_C}{2}}{\sin\frac{\phi_C}{2}} - \\ - 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos\varphi_m * \cos(n\Phi_C) * \frac{\sin\frac{n\Phi_C}{2} * \sin\frac{n\Phi_C}{2}}{\sin\frac{\Phi_C}{2}}$$

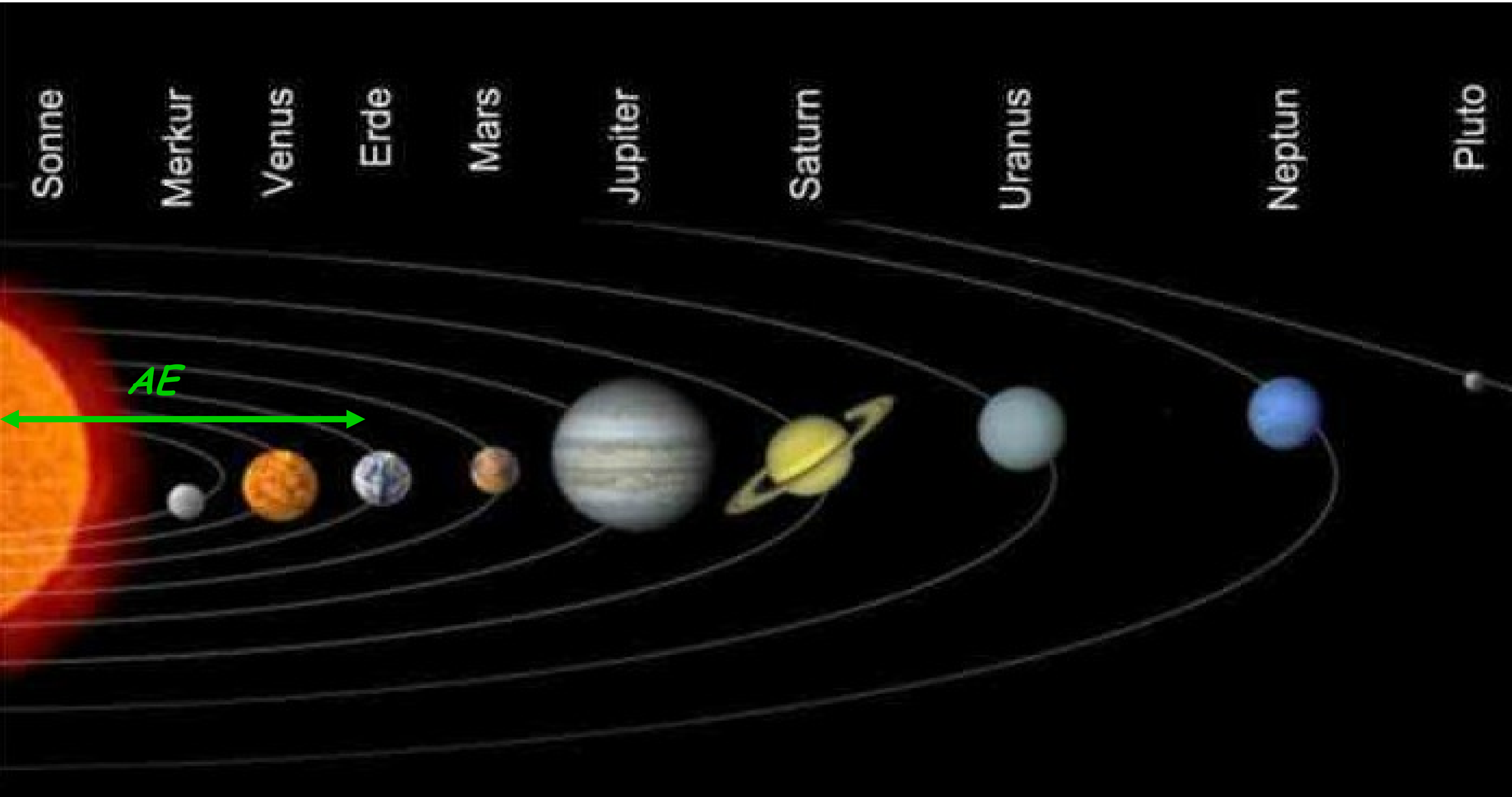
$$D_n = \frac{2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\phi_C}{2}} \left\{ 2\sin\frac{n\phi_C}{2}\cos\frac{n\phi_C}{2} * \cos\frac{n\phi_C}{2}\sin\frac{n\phi_C}{2} - \\ - (\cos^2\frac{n\phi_C}{2} - \sin^2\frac{n\phi_C}{2})\sin^2\frac{n\phi_C}{2} \right\}$$

replace by ...

"after some TLC transformations" ***... or ...*** ***" after some beer"***

Largest storage ring: The Solar System

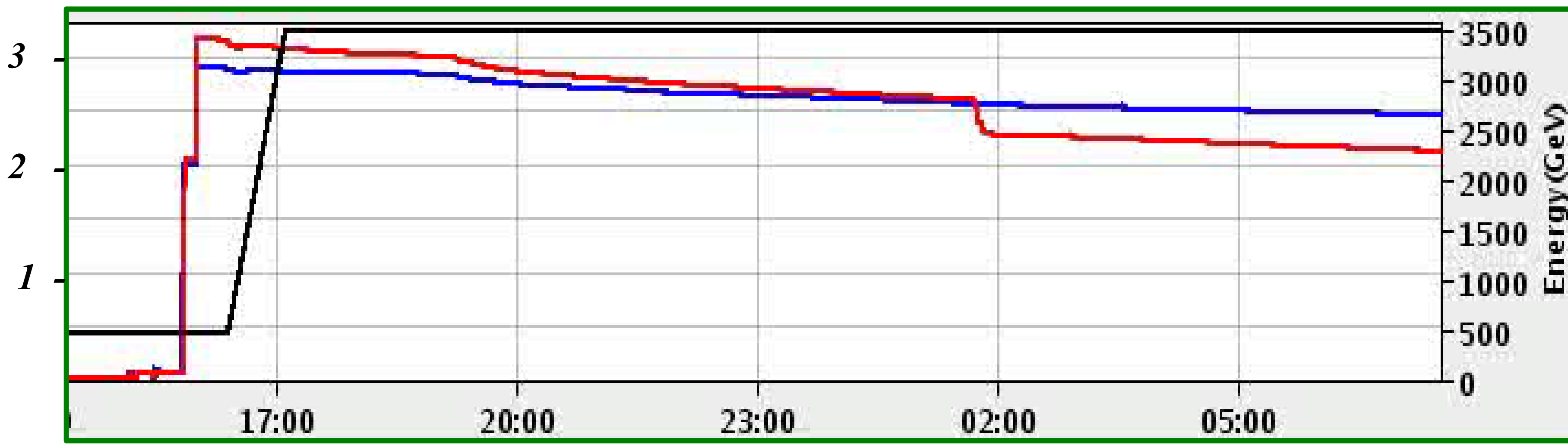
astronomical unit: average distance earth-sun
$1AE \approx 150 \cdot 10^6$ km
Distance Pluto-Sun ≈ 40 AE



Luminosity Run of a typical storage ring:

LHC Storage Ring: Protons accelerated and stored for 12 hours
distance of particles travelling at about $v \approx c$
$L = 10^{10}$-10^{11} km
... several times Sun - Pluto and back

intensity (10^{11})



→ *guide the particles on a well defined orbit („design orbit")*
→ *focus the particles to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.*

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times B)$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \, m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * 300 \frac{MV}{m}$$

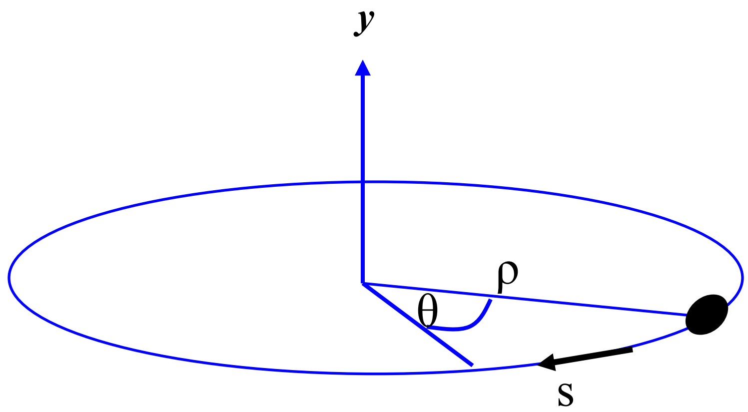
equivalent el. field … E

technical limit for el. field:

$$E \leq 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\, v\, B$

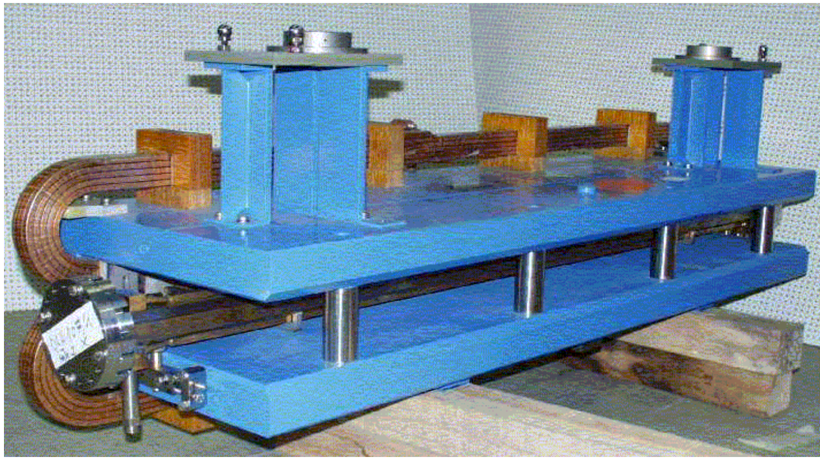
centrifugal force $F_{centr} = \dfrac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^2}{\rho} = e\, v\, B$$

$$\frac{p}{e} = B\,\rho$$

$B\rho$ = "beam rigidity"

2.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

Normalise magnetic field to momentum:

$$\frac{p}{e} = B\rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{eB}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

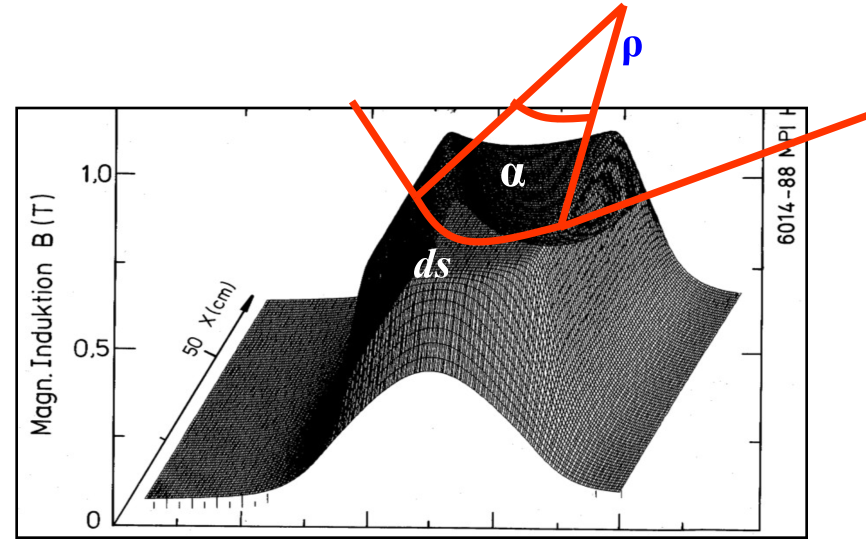
Example LHC:

$$\left.\begin{aligned} B &= 8.3\,T \\ p &= 7000\,\frac{GeV}{c} \end{aligned}\right\}$$

$$\frac{1}{\rho} = e\,\frac{8.3\,Vs/m^2}{7000 * 10^9\,eV/c} = \frac{8.3\,s * 3 * 10^8\,m/s}{7000 * 10^9\,m^2}$$

$$\frac{1}{\rho} = 0.333\,\frac{8.3}{7000}\,1/m$$

The Magnetic Guide Field



field map of a storage ring dipole magnet

$\rho = 2.53\ km$ → $2\pi\rho = 17.6\ km$

$\approx 66\%$

$B \approx 1 \ldots 8\ T$

rule of thumb:

$$\frac{1}{\rho} \approx 0.3\,\frac{B\,[T]}{p\,[GeV/c]}$$

„normalised bending strength"

The Problem:

LHC Design Magnet current: I=11850 A

and the machine is 27 km long !!!

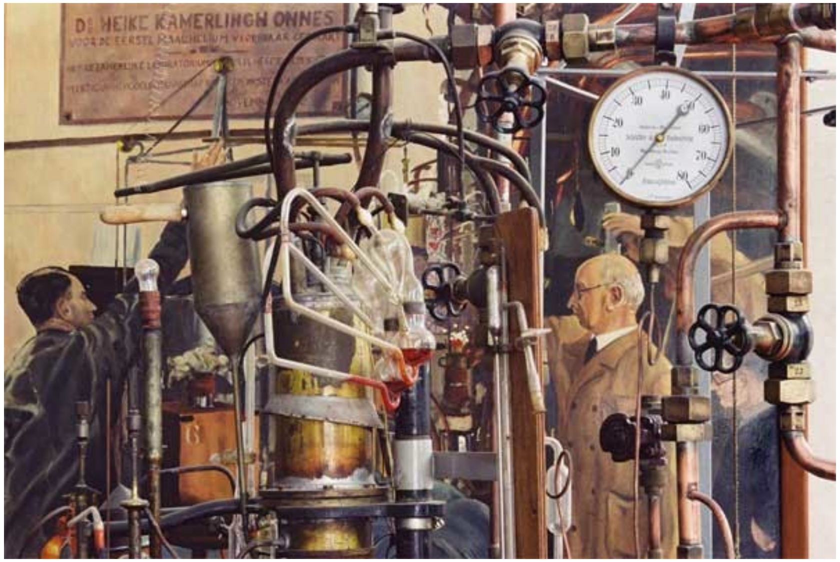
Ohm's law: $U = R * I, \qquad P = R * I^2$

Problem:

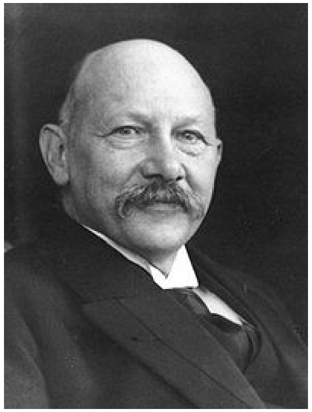
reduce ohmic losses to the absolute minimum

The Solution:

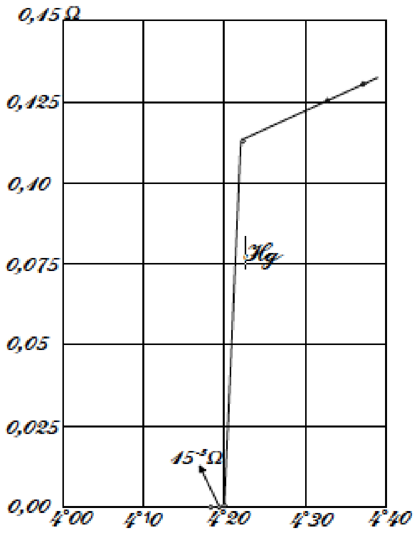
super conductivity



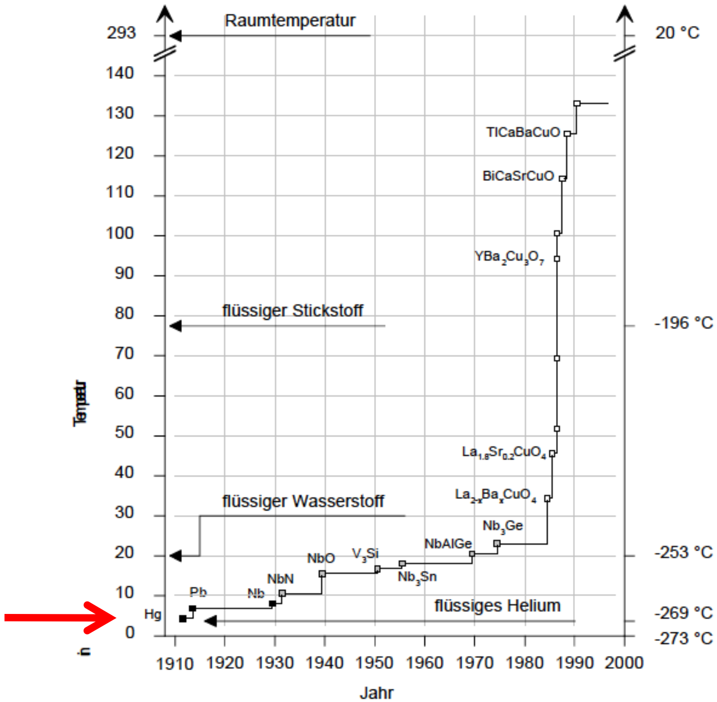
Super Conductivity



discovery of sc. by H. Kammerling Onnes, Leiden 1911

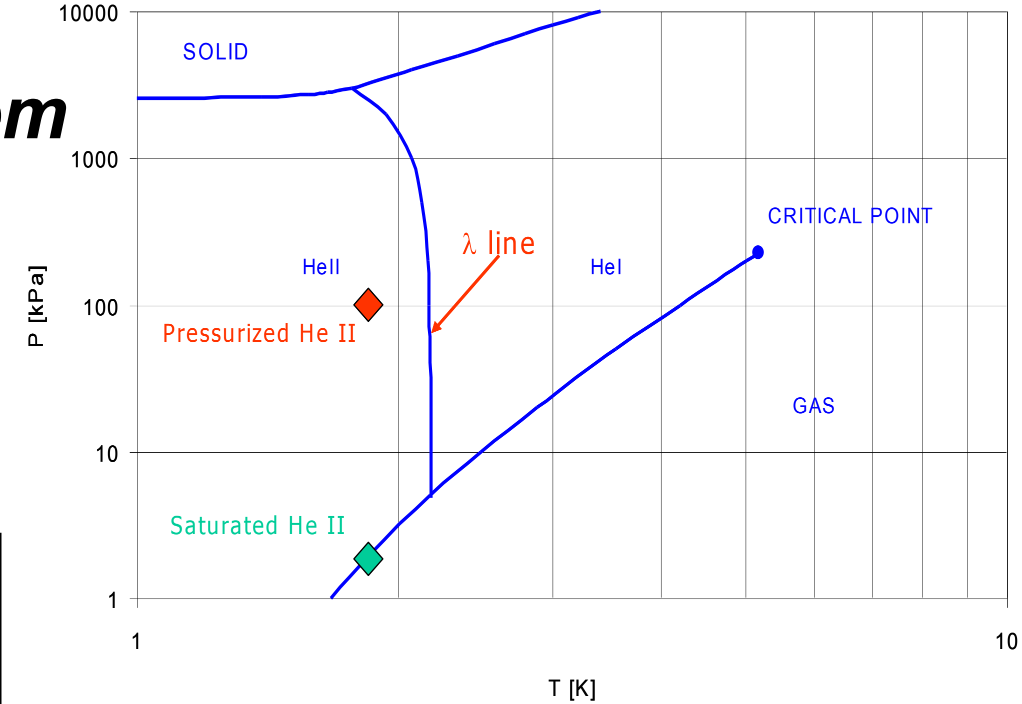


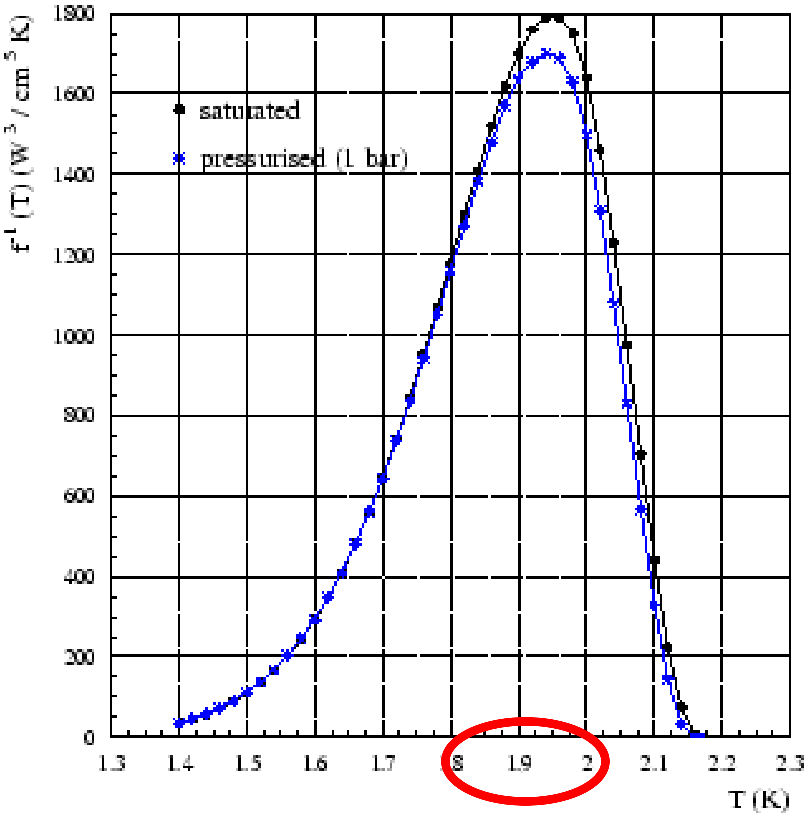
LHC 1.9 K cryo plant



Superfluid helium: 1.9 K cryo system

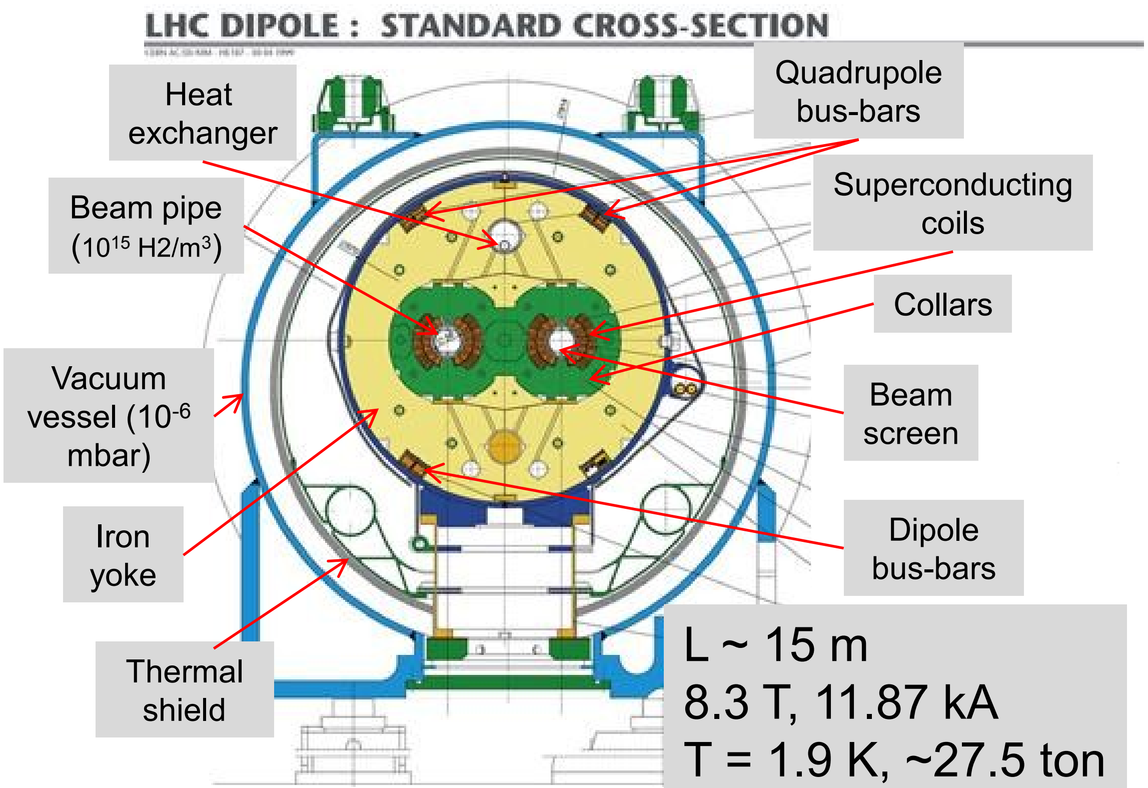
Phase diagramm of Helium

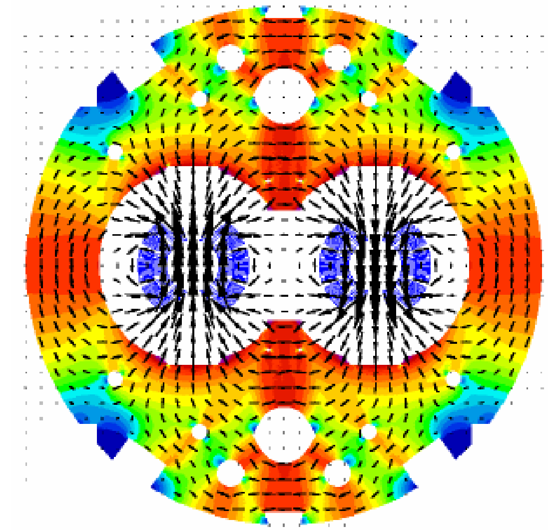




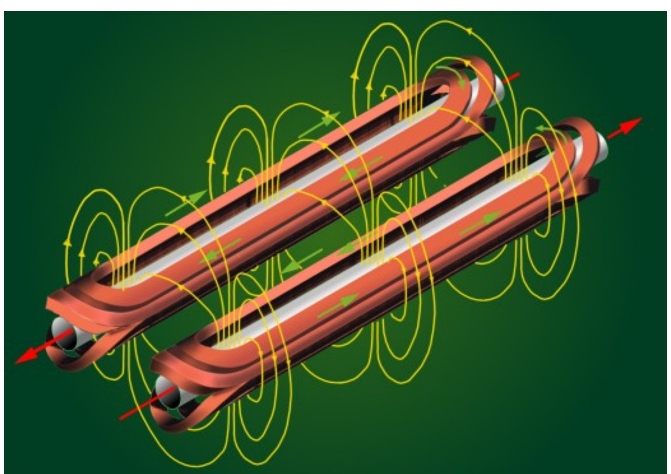
thermal conductivity of fl. Helium in supra fluid state

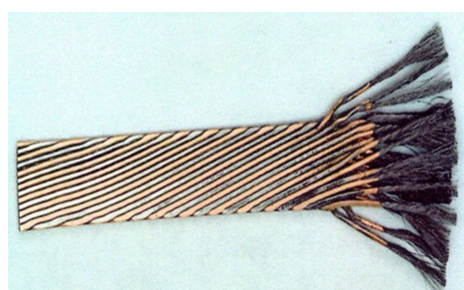
LHC: The -1232- Main Dipole Magnets



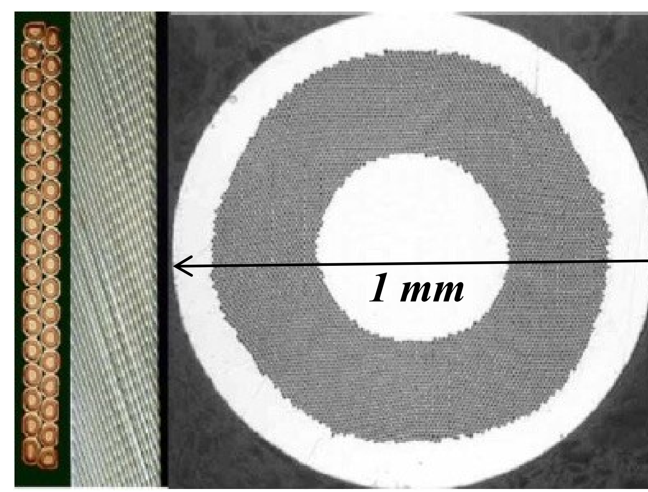


required field quality:
$\Delta B/B=10^{-4}$





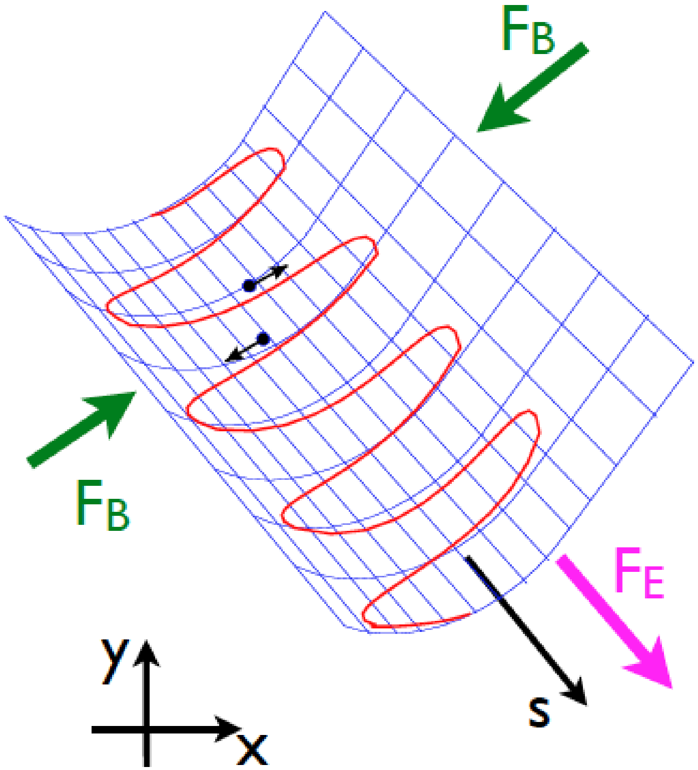
6 µm Ni-Ti filament



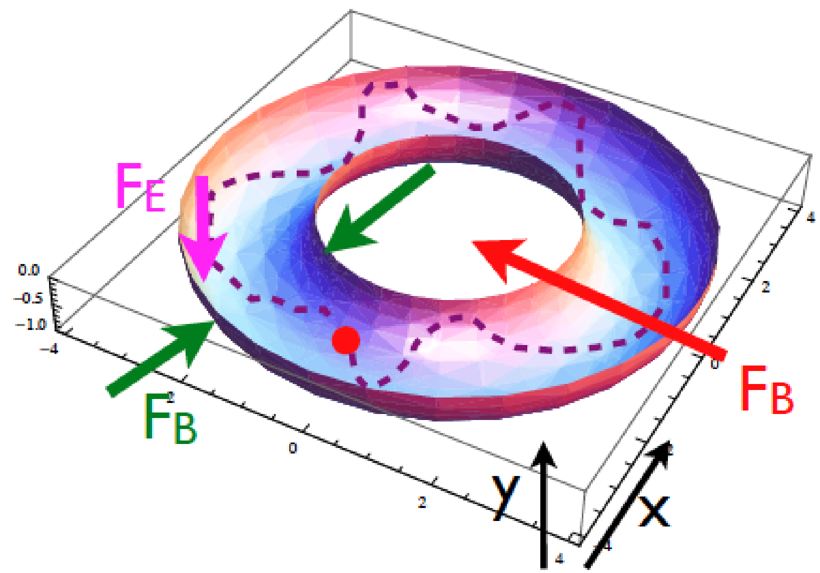
2.) Focusing Properties – Transverse Beam Optics

$$\overline{F(t)} = \underbrace{q\Big(\overline{E(t)}}_{F_E} + \underbrace{\overline{v(t)} \otimes \overline{B(t)}}_{F_B}\Big)$$

Linear Accelerator

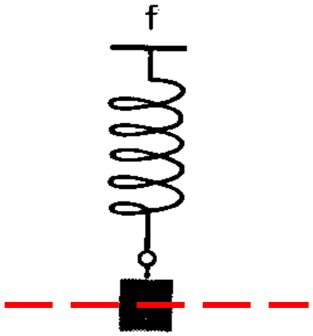


Circular Accelerator



2.) Focusing Properties – Transverse Beam Optics

classical mechanics: pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

***Storage Ring:** we need a Lorentz force that rises as a function of the distance to .... ?*

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

required: *focusing forces to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

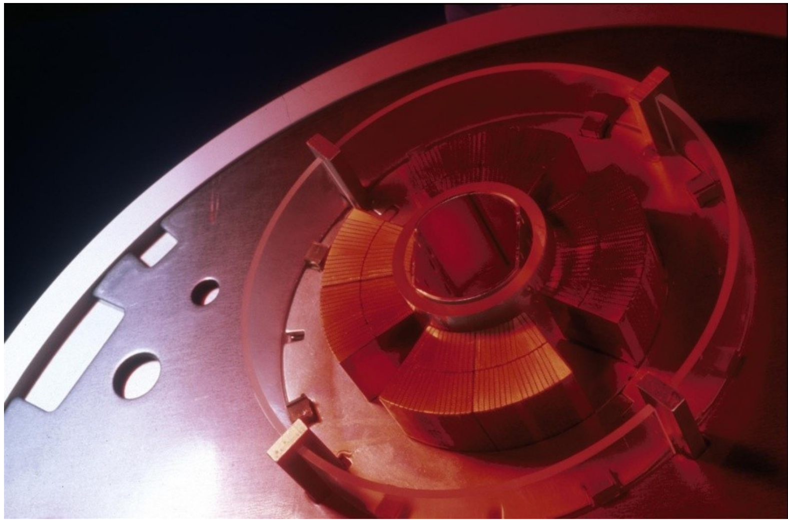
linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

→ $$k = \frac{g}{p/e}$$

simple rule:

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$



LHC main quadrupole magnet

$$g \approx 25 \,...\, 220 \quad T/m$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

Focusing forces and particle trajectories:

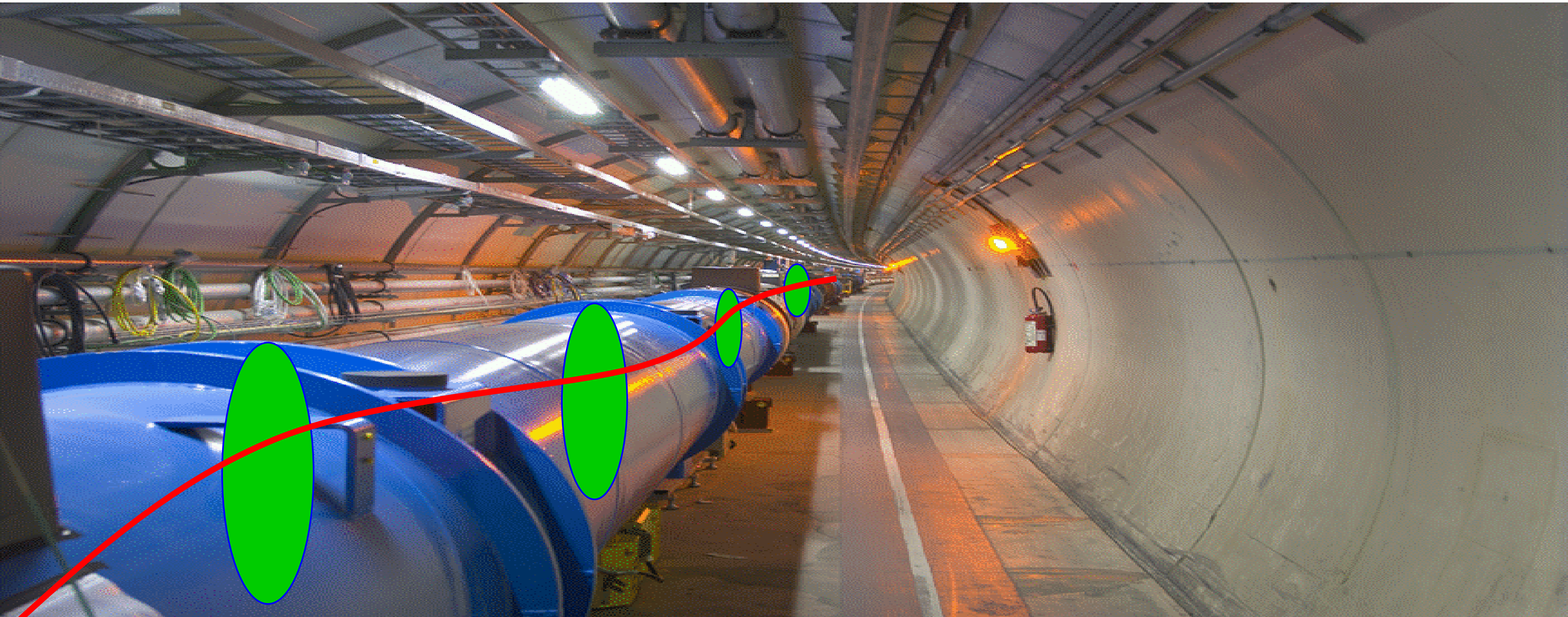
normalise magnet fields to momentum
(remember: $B*\rho = p/q$ *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

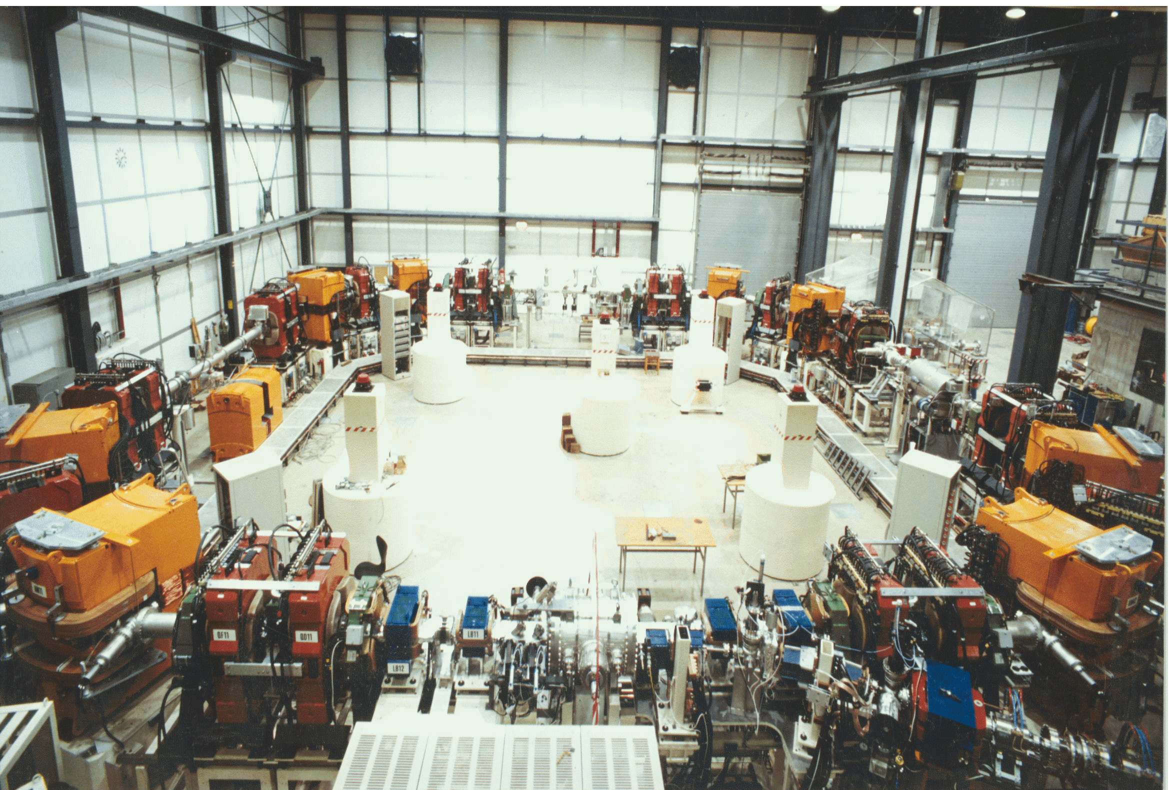
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \frac{1}{2!}m\,x^2 + \frac{1}{3!}n\,x^3 + \ldots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

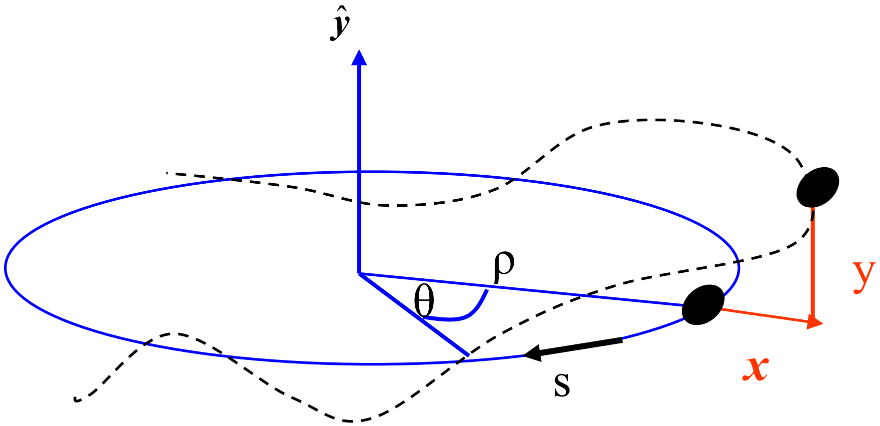
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* ***man sieht nur dipole und quads → linear***

The Equation of Motion:



* *Equation for the* ***horizontal motion:***

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

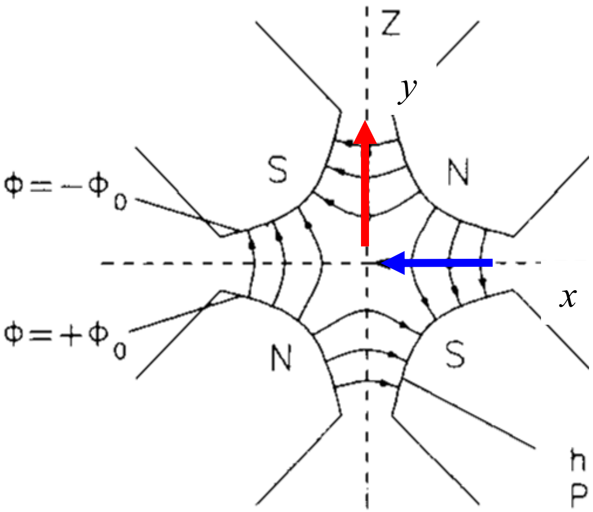
***x* = particle amplitude**
***x'* = angle of particle trajectory (wrt ideal path line)**

* *Equation for the* ***vertical motion:***

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$k \leftrightarrow -k$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

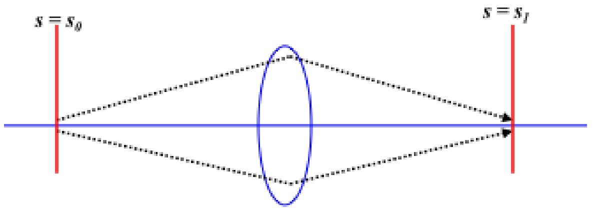
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



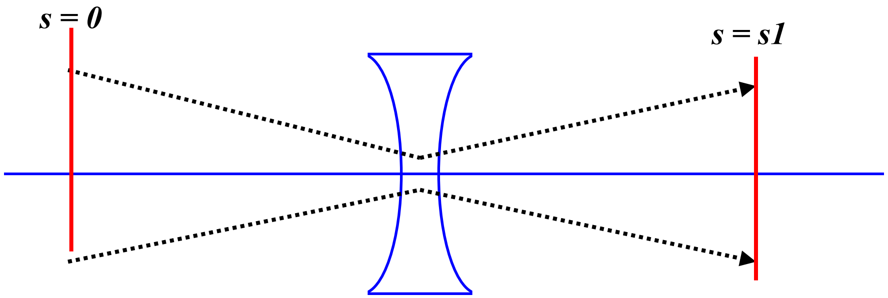
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\, x = 0$$



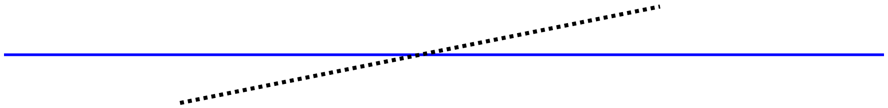
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

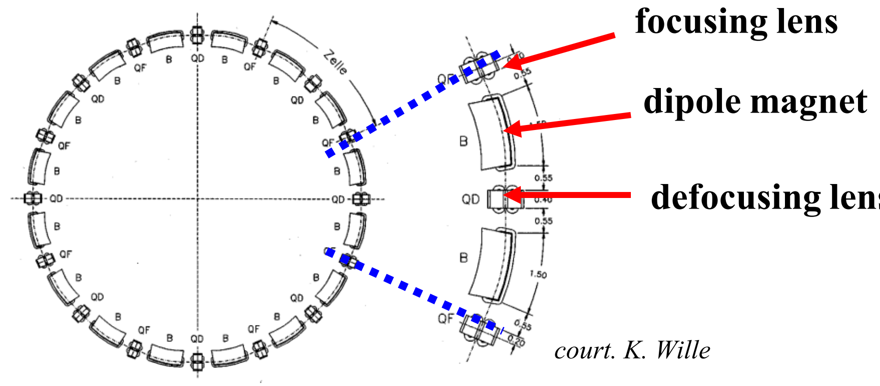
! ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“***

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

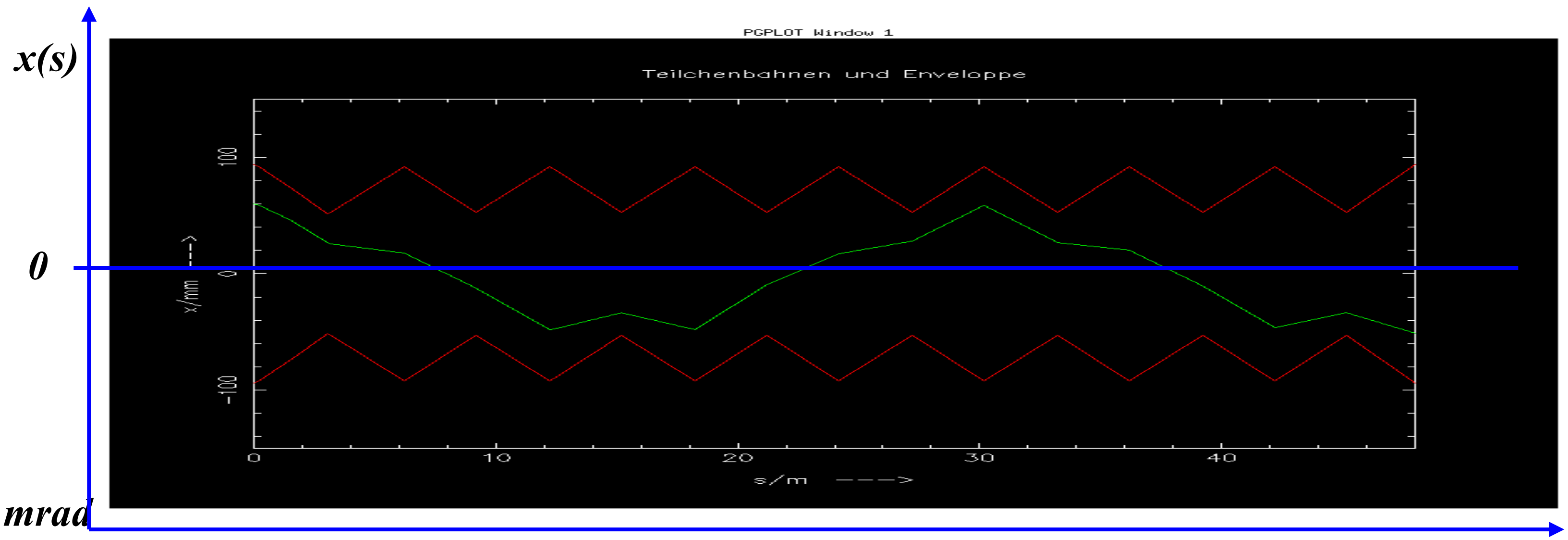
$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

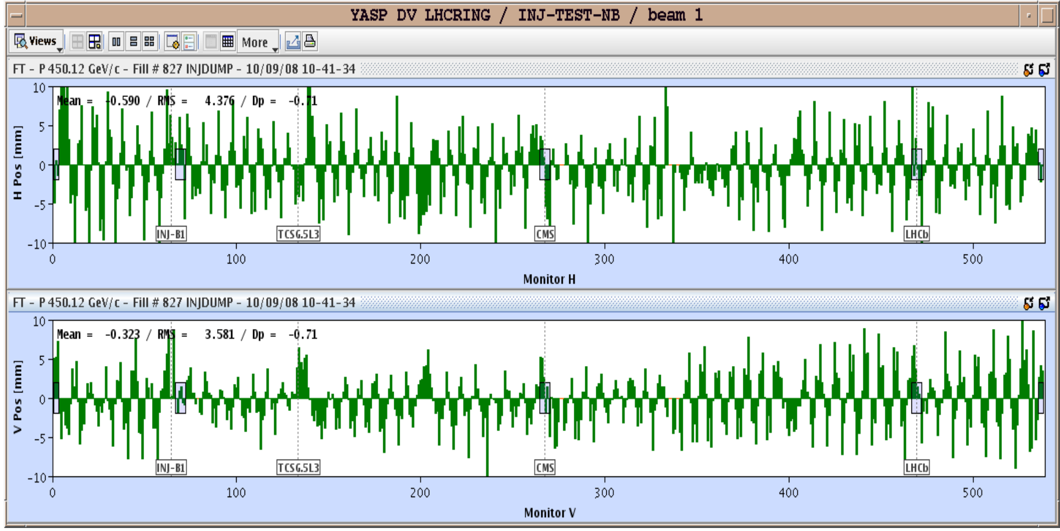
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine:

x ≈ mm, x´ ≤ mrad

5.) Orbit & Tune:



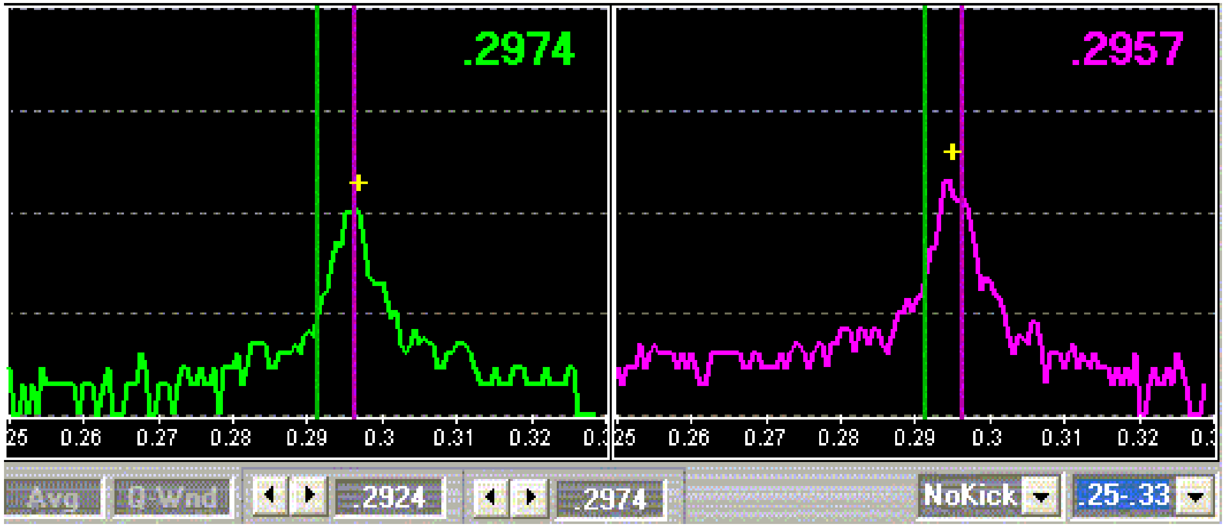
Tune: number of oscillations per turn

64.31
59.32

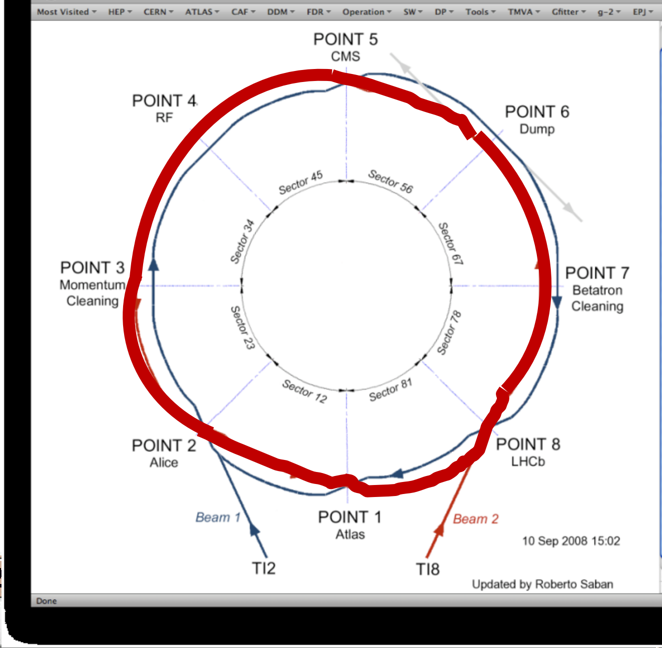
***Relevant** for beam stability:*
non integer part

LHC revolution frequency: ***11.3 kHz***

$$0.31 * 11.3 = 3.5\, kHz$$



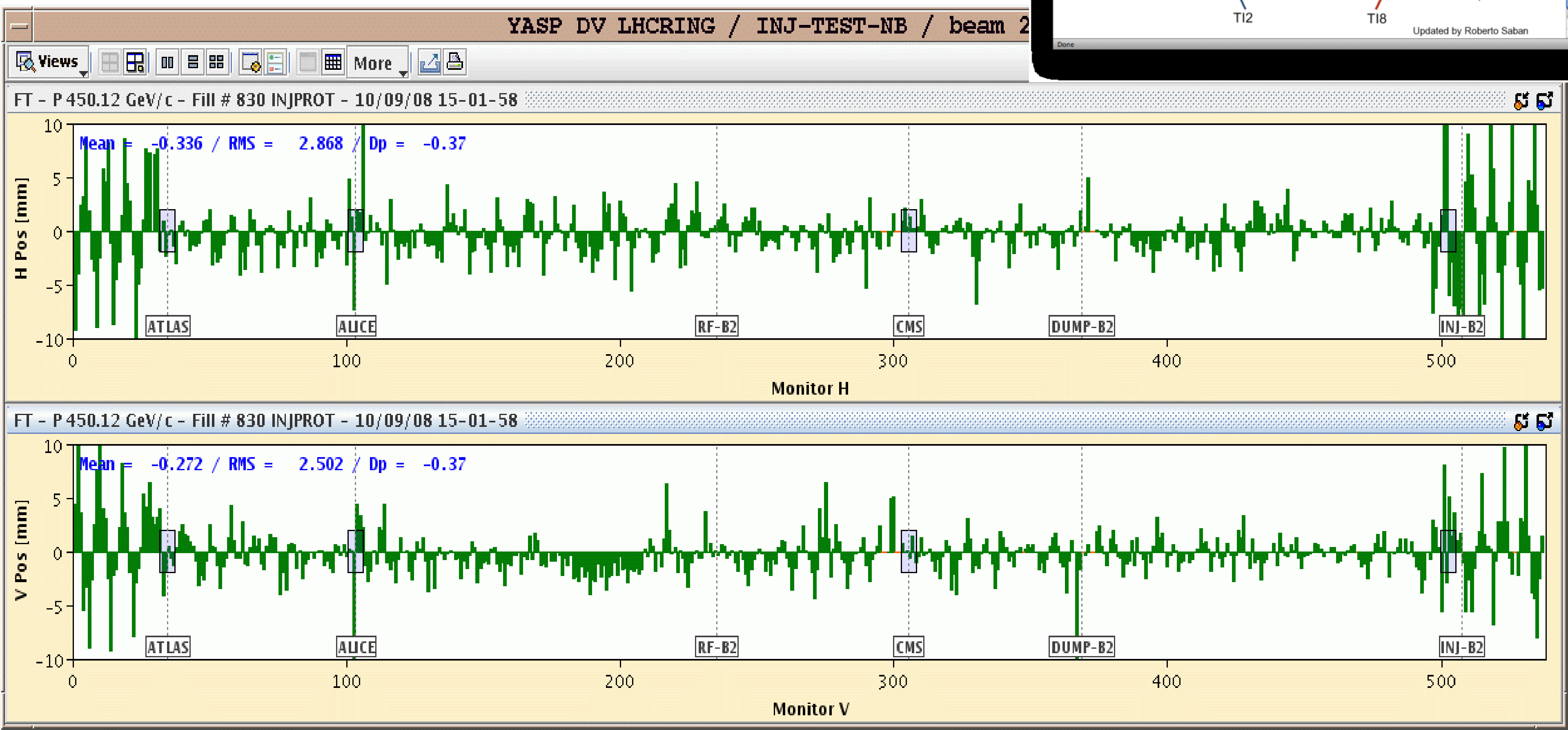
LHC Operation: Beam Commissioning



First turn steering "by sector:"

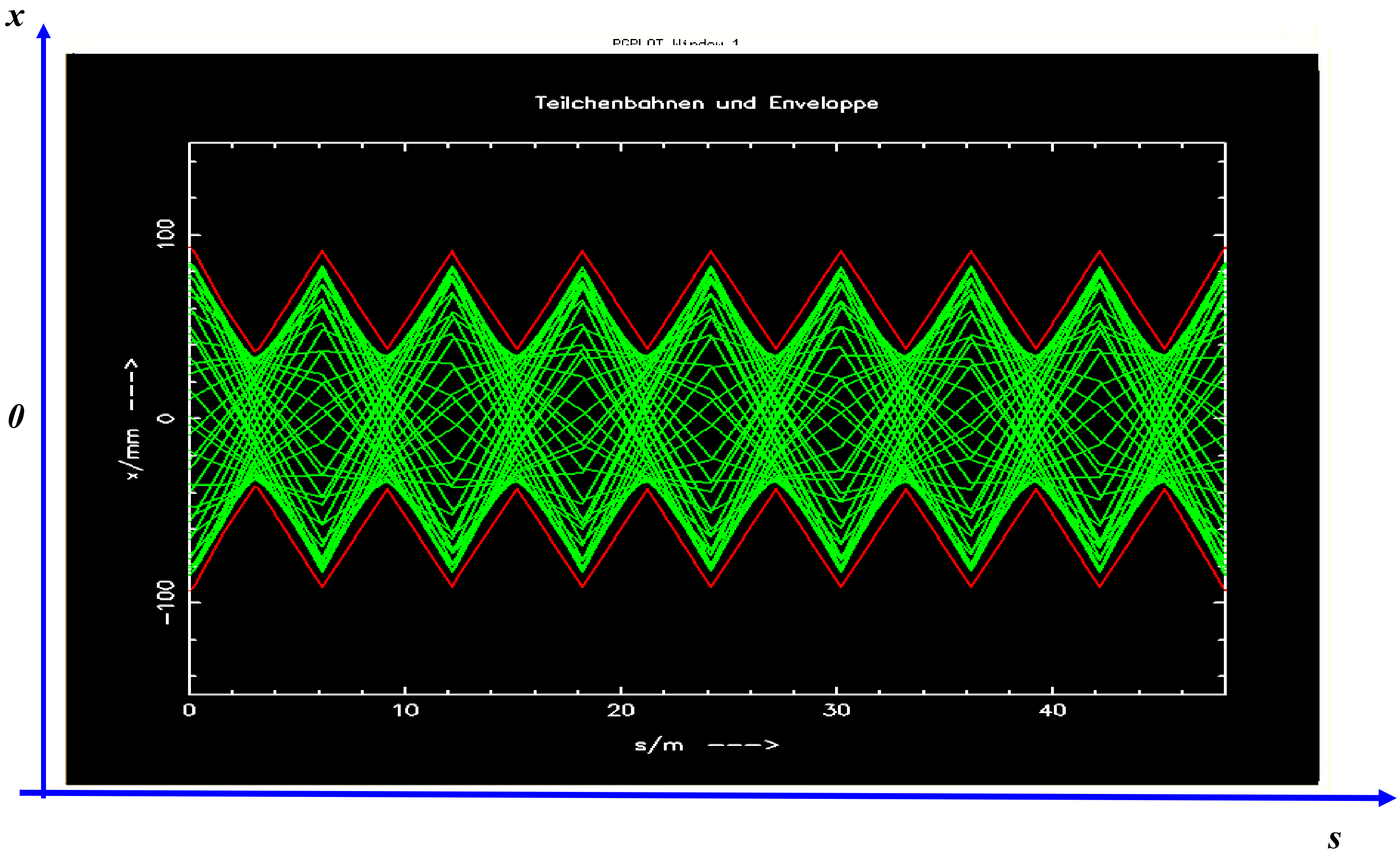
- ❑One beam at the time
- ❑Beam through 1 sector (1/8 ring),

correct trajectory, open collimator and move on.



Question: ***what will happen, if the particle performs a second turn ?***

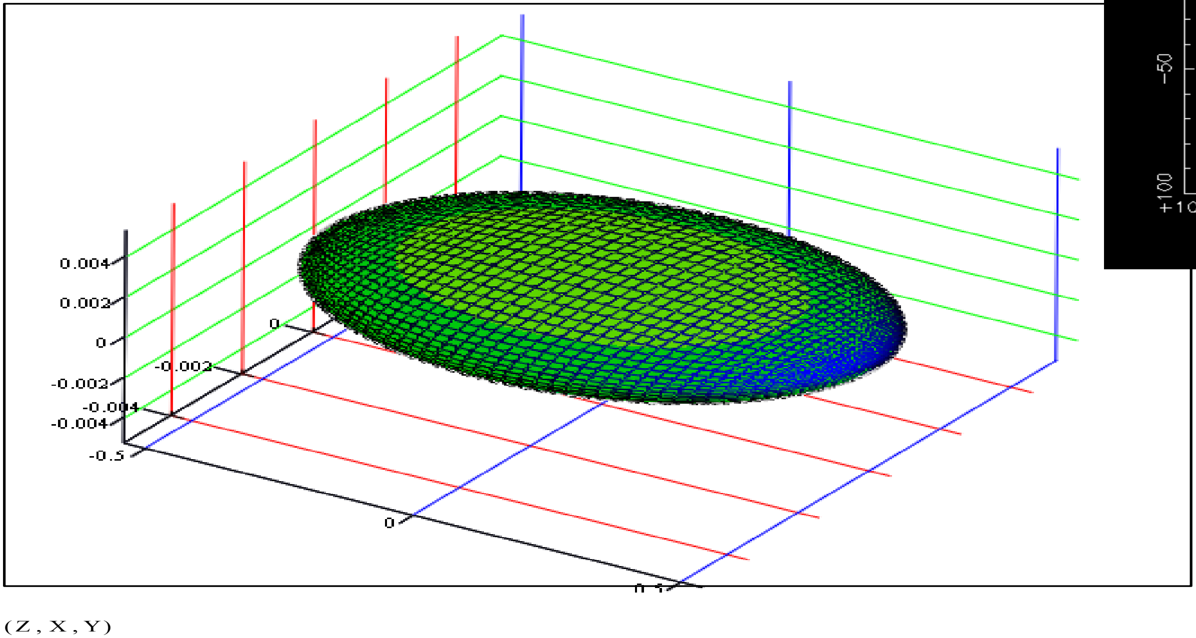
... or a third one or ... 10^{10} turns

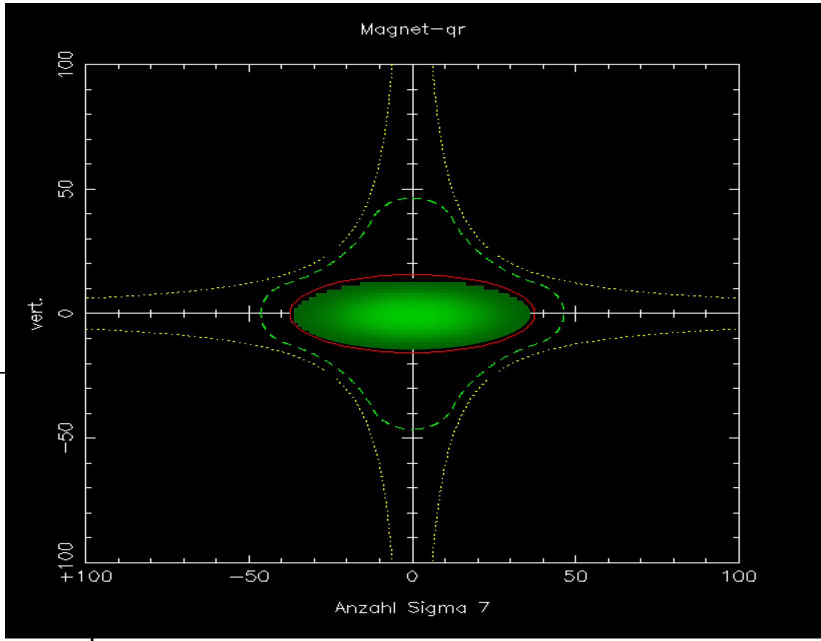


10 Seconds ... to forget everything that I said about single partilce trajectories

II.) The Ideal World:

Particle Trajectories, Beams & Bunches



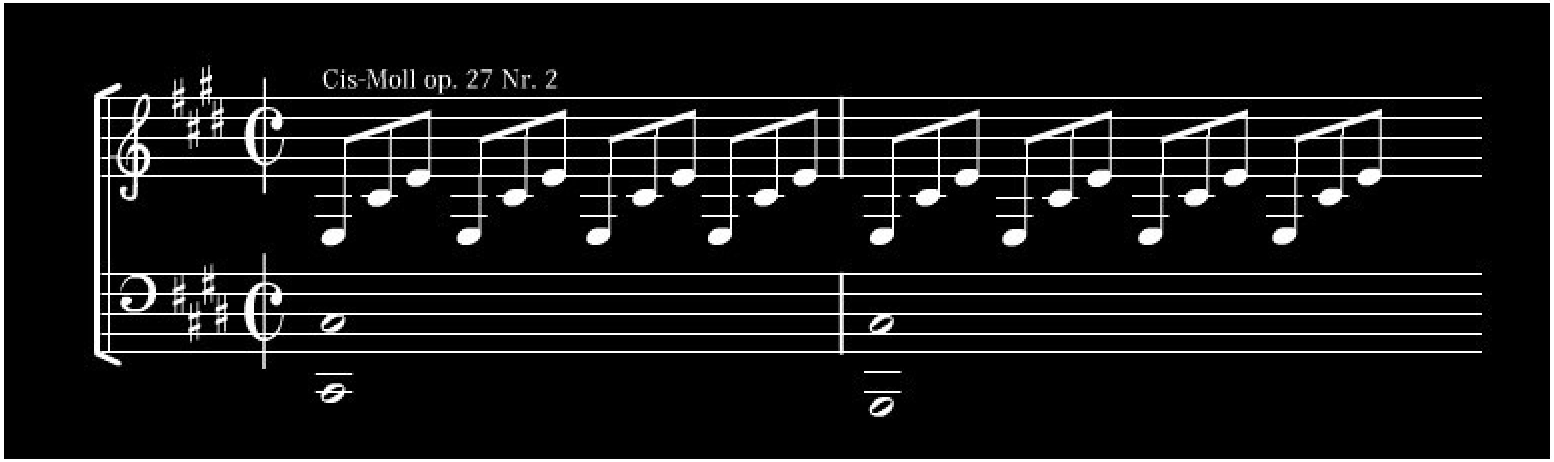


Bunch in a Storage Ring

19th century:

Ludwig van Beethoven: „Mondschein Sonate“

Sonate Nr. 14 in cis-Moll (op. 27/II,   1801)



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force $\neq$ ***const,***
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

6.) The Beta Function

„it is convenient to see“

... after some beer ... general solution of Mr Hill can be written in the form:

Ansatz:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

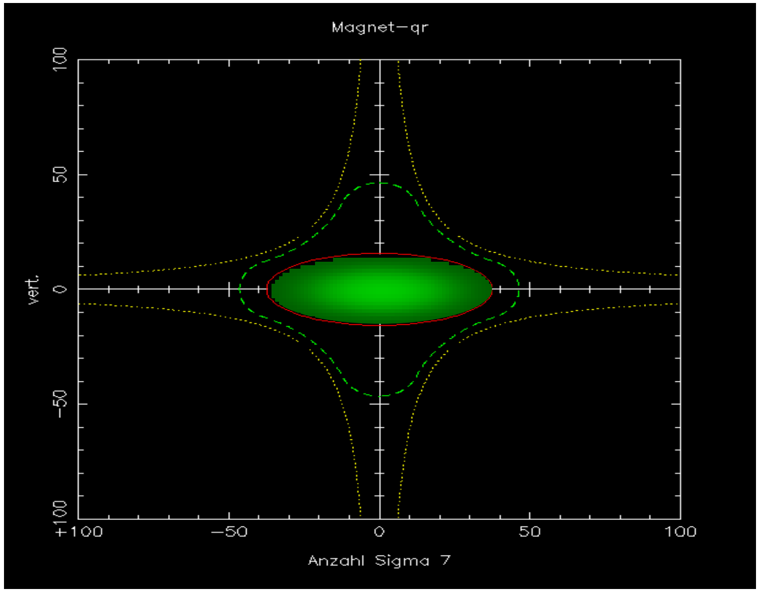
$$\beta(s+L) = \beta(s)$$

ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
scientifiquely spoken: area covered in transverse x, x´ phase space … and it is
constant !!!

Ψ(s) = „phase advance“ of the oscillation between point „0“ and „s“ in the lattice.
For one complete revolution: number of oscillations per turn „Tune“

$$Q_y = \frac{1}{2\pi} \cdot \int \frac{ds}{\beta(s)}$$

6.) The Beta Function

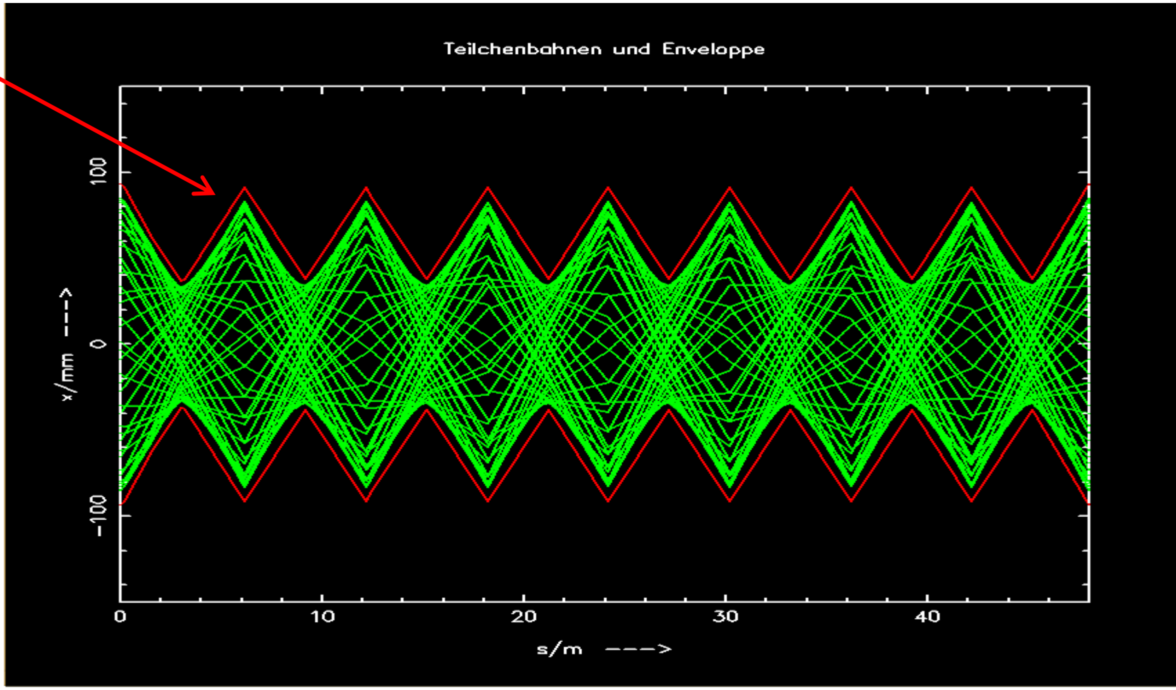


Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$

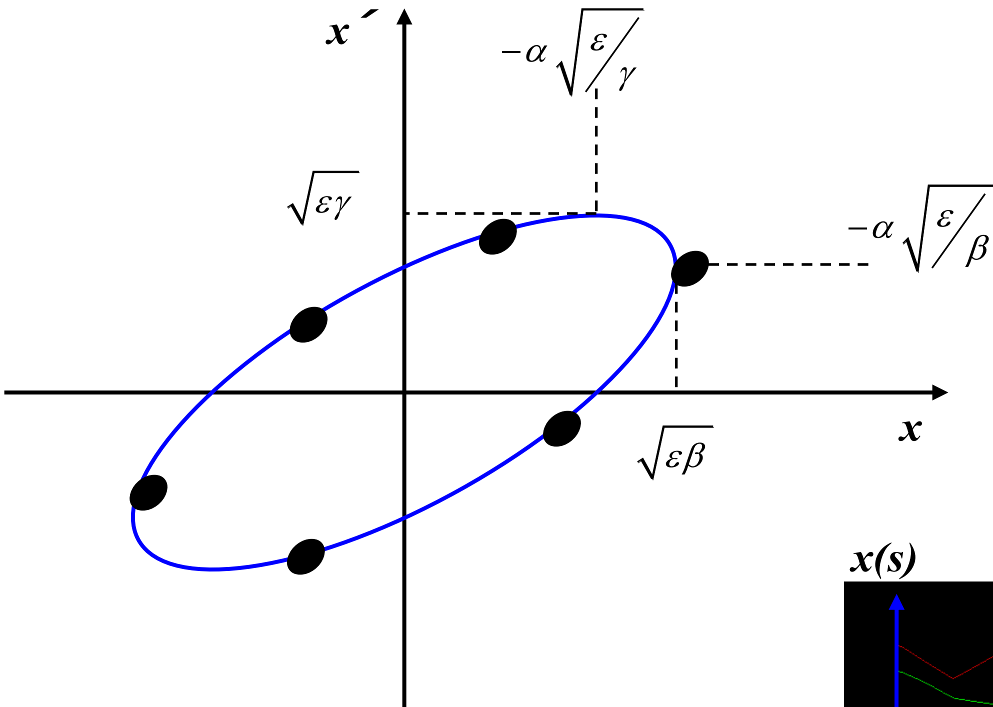


β determines the beam size
(... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.

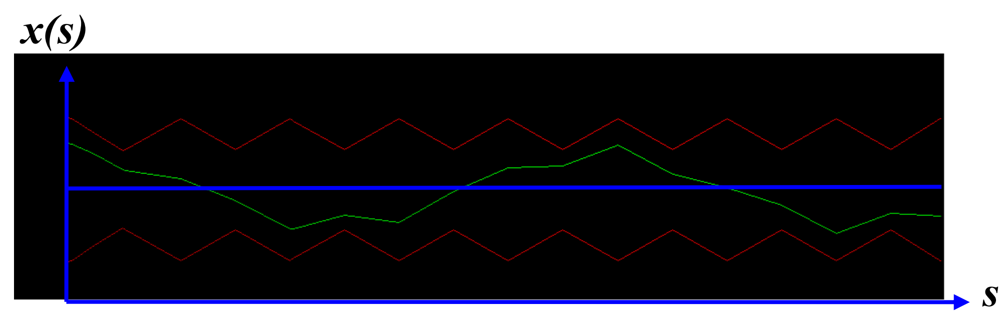
7.) Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



Liouville: ***in reasonable storage rings***
area in phase space is constant.

$$A = \pi * \varepsilon = const$$



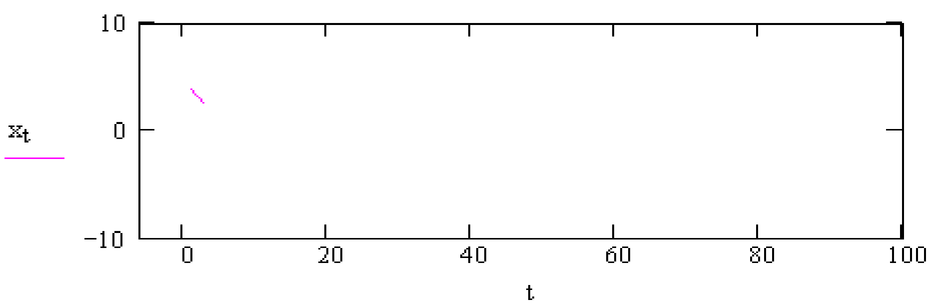
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.

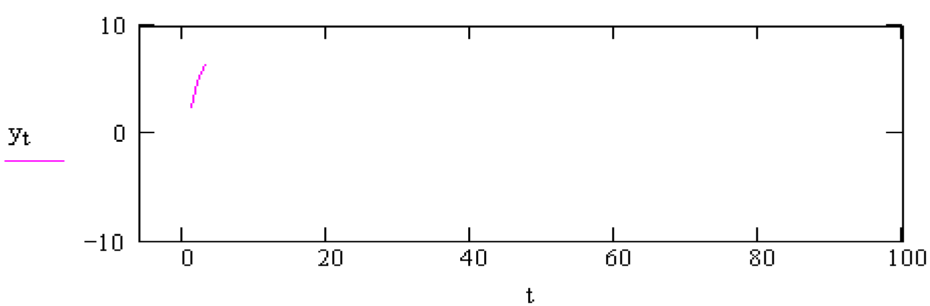
Scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!

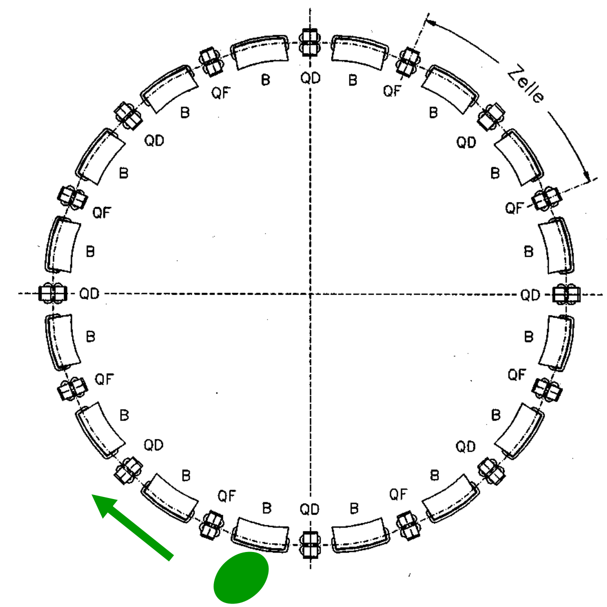
Particle Tracking in a Storage Ring

Calculate x, x´ for each linear accelerator element according to matrix formalism

plot x, x´as a function of „s"

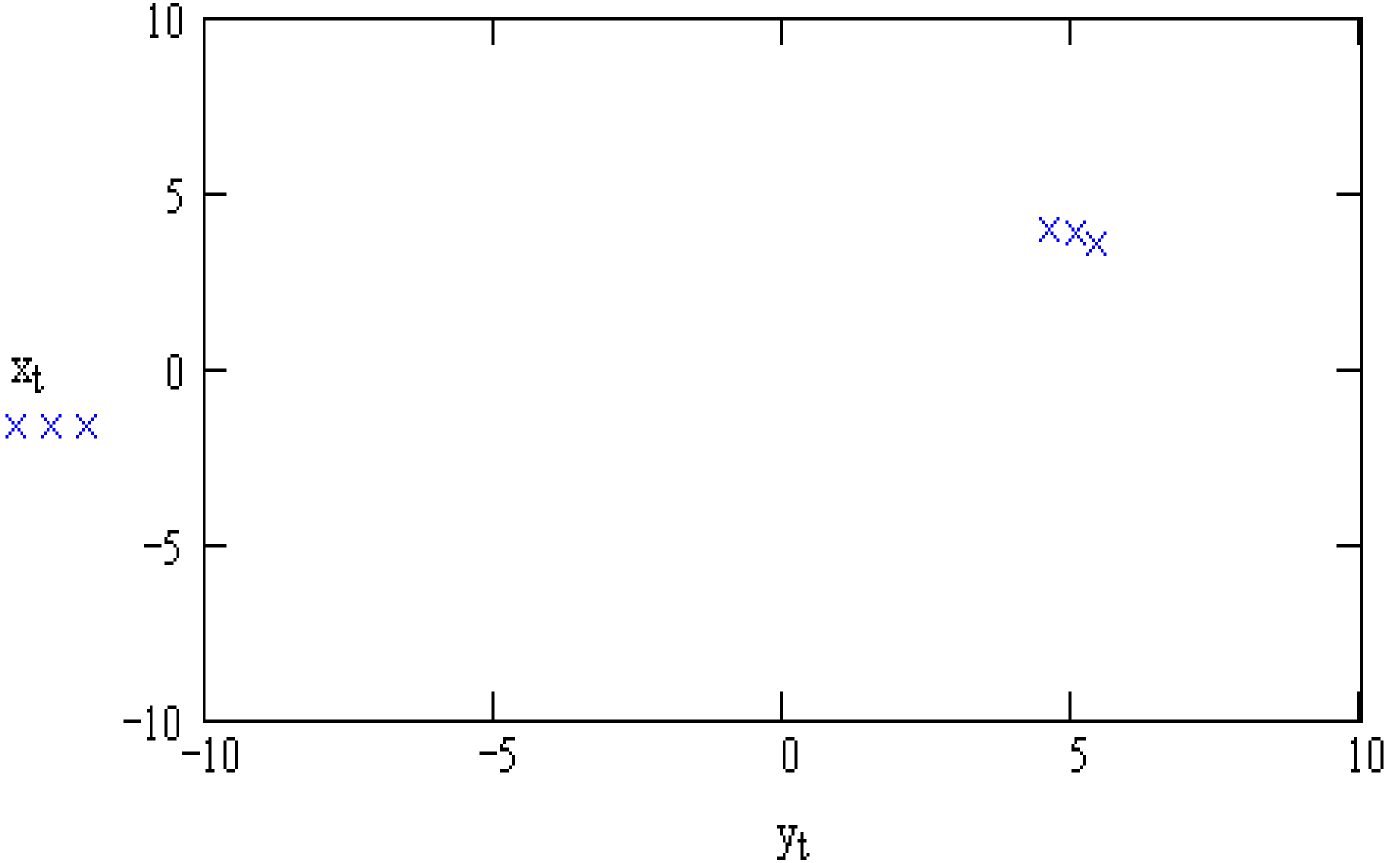






… and now the ellipse:

note for each turn x, x´at a given position „s_1" and plot in the phase space diagram



Schluss aus fertich

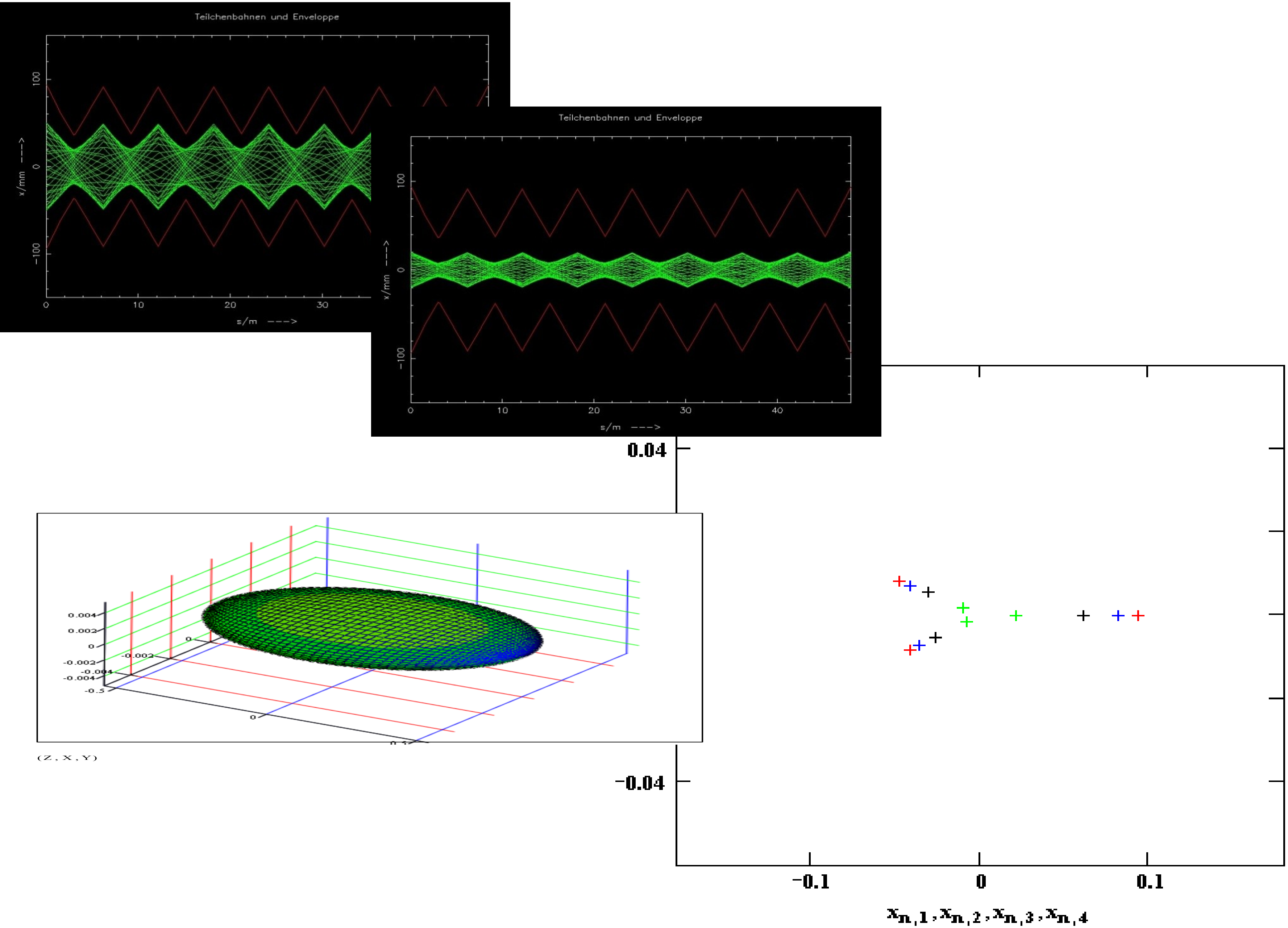
that's all folks ... for today

eso es todo por hoy ... que aproveche

fin

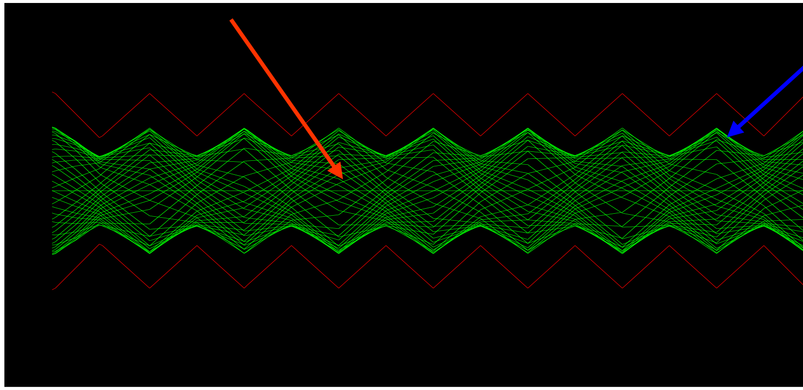
III

Emittance of the Particle Ensemble:



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

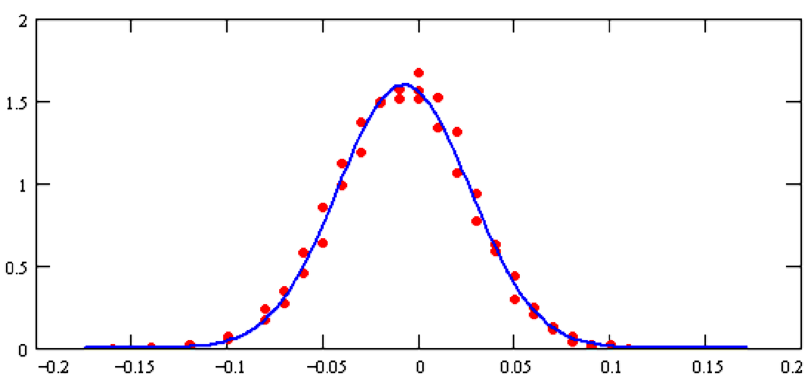
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

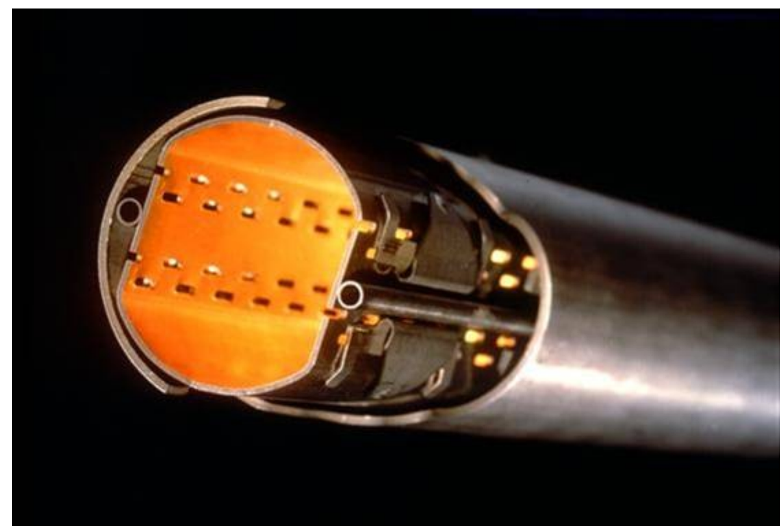
LHC:

$$\beta = 180\ m$$

$$\varepsilon = 5 * 10^{-10}\ m\ rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\ m * 180\ m} = 0.3\ mm$$

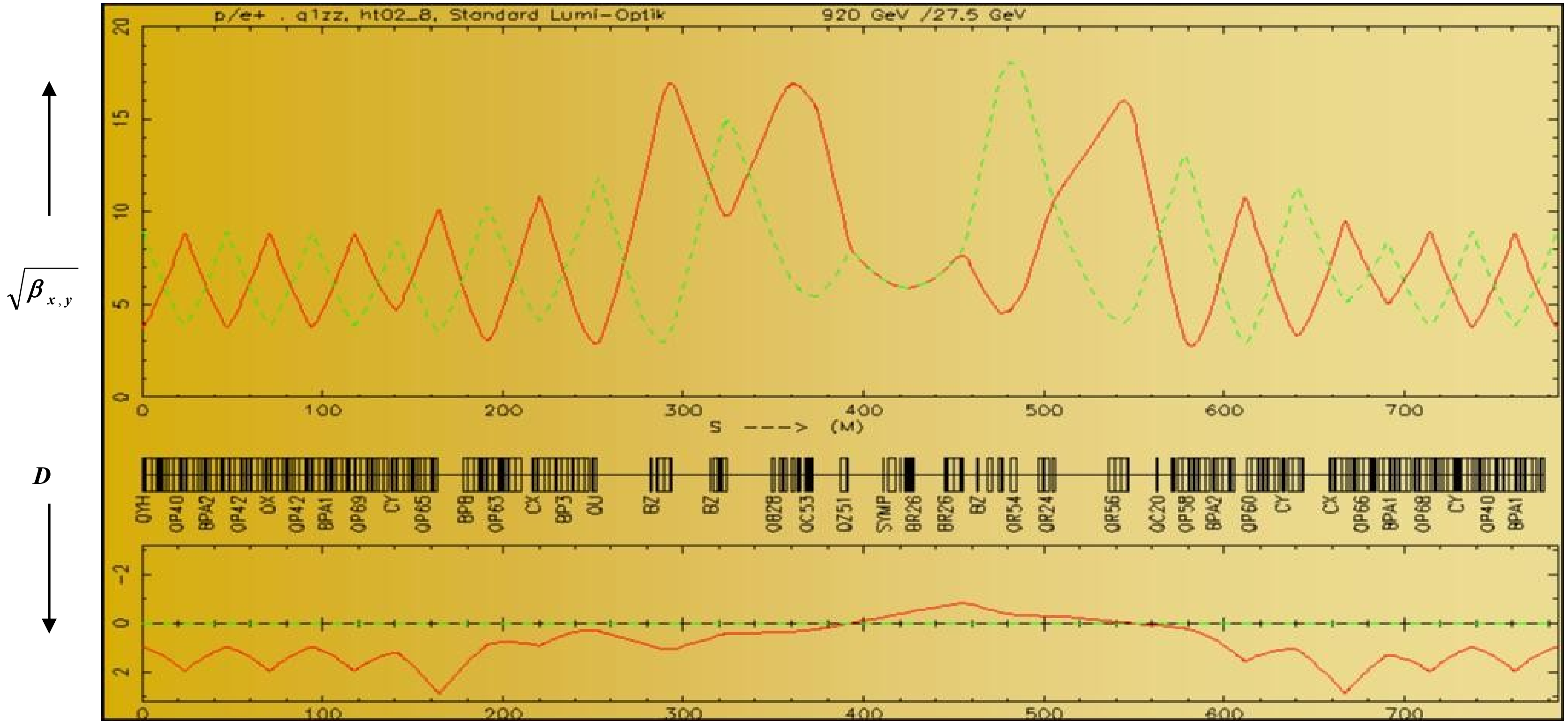




aperture requirements: $r_0 = 12 * \sigma$

III.) The „not so ideal“ World

Lattice Design in Particle Accelerators



1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Recapitulation: ...the story with the matrices !!!

Equation of Motion:

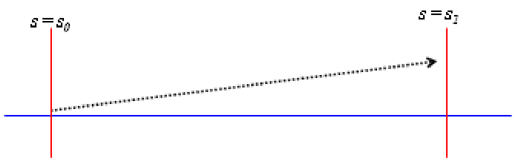
$$x'' + K\, x = 0$$

$$K = 1/\rho^2 - k \quad \text{... hor. plane:}$$

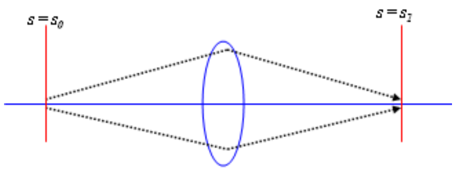
$$K = k \quad \text{... vert. Plane:}$$

Solution of Trajectory Equations

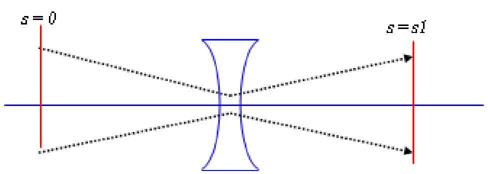
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}\,l) \\ -\sqrt{|K|}\sin(\sqrt{|K|}\,l) & \cos(\sqrt{|K|}\,l) \end{pmatrix}$$



$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}\,l) \\ \sqrt{|K|}\sinh(\sqrt{|K|}\,l) & \cosh(\sqrt{|K|}\,l) \end{pmatrix}$$

$$M_{total} = M_{QF} * M_D * M_B * M_D * M_{QD} * M_D * \quad \dots$$

8.) Lattice Design: *„… how to build a storage ring“*

Geometry of the ring: $B * \rho = p / e$

p = momentum of the particle,
ρ = curvature radius

Bρ= beam rigidity

Circular Orbit: bending angle of one dipole

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$

The angle run out in one revolution must be 2π, so for a full circle

$$\alpha = \frac{\int Bdl}{B\rho} = 2\pi$$

$$\int Bdl = 2\pi \frac{p}{q}$$

… defines the integrated dipole field around the machine.

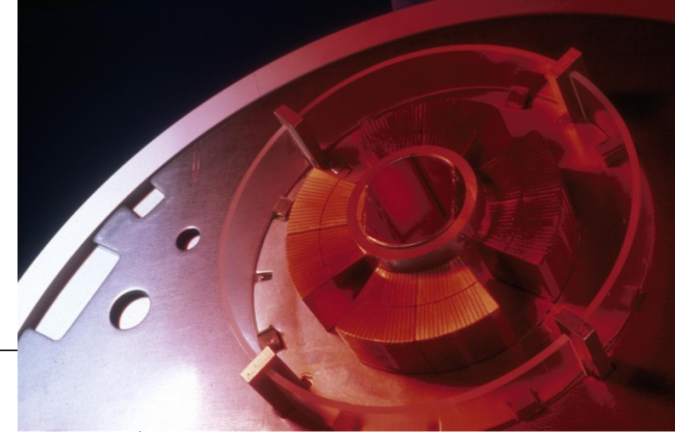
Example LHC:

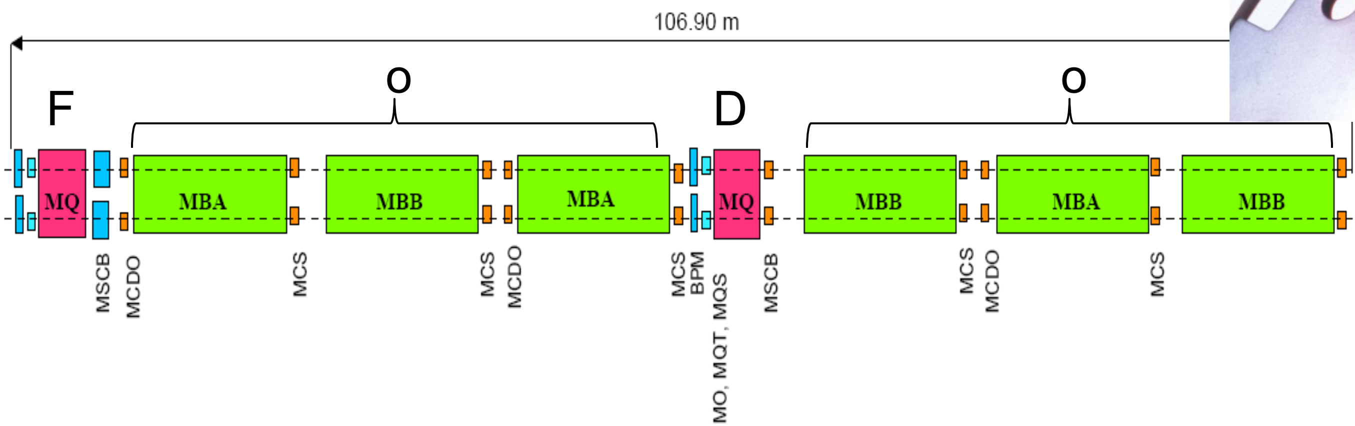
7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

$$\int B\, dl \approx N\, l\, B = 2\pi\, p / e$$

$$B \approx \frac{2\pi \quad 7000 \quad 10^{9}\, eV}{1232 \quad 15\, m \quad 3\; 10^{8}\, \frac{m}{s} \quad e} = 8.3\ \textbf{Tesla}$$

LHC: Lattice Design
the ARC 90° FoDo in both planes



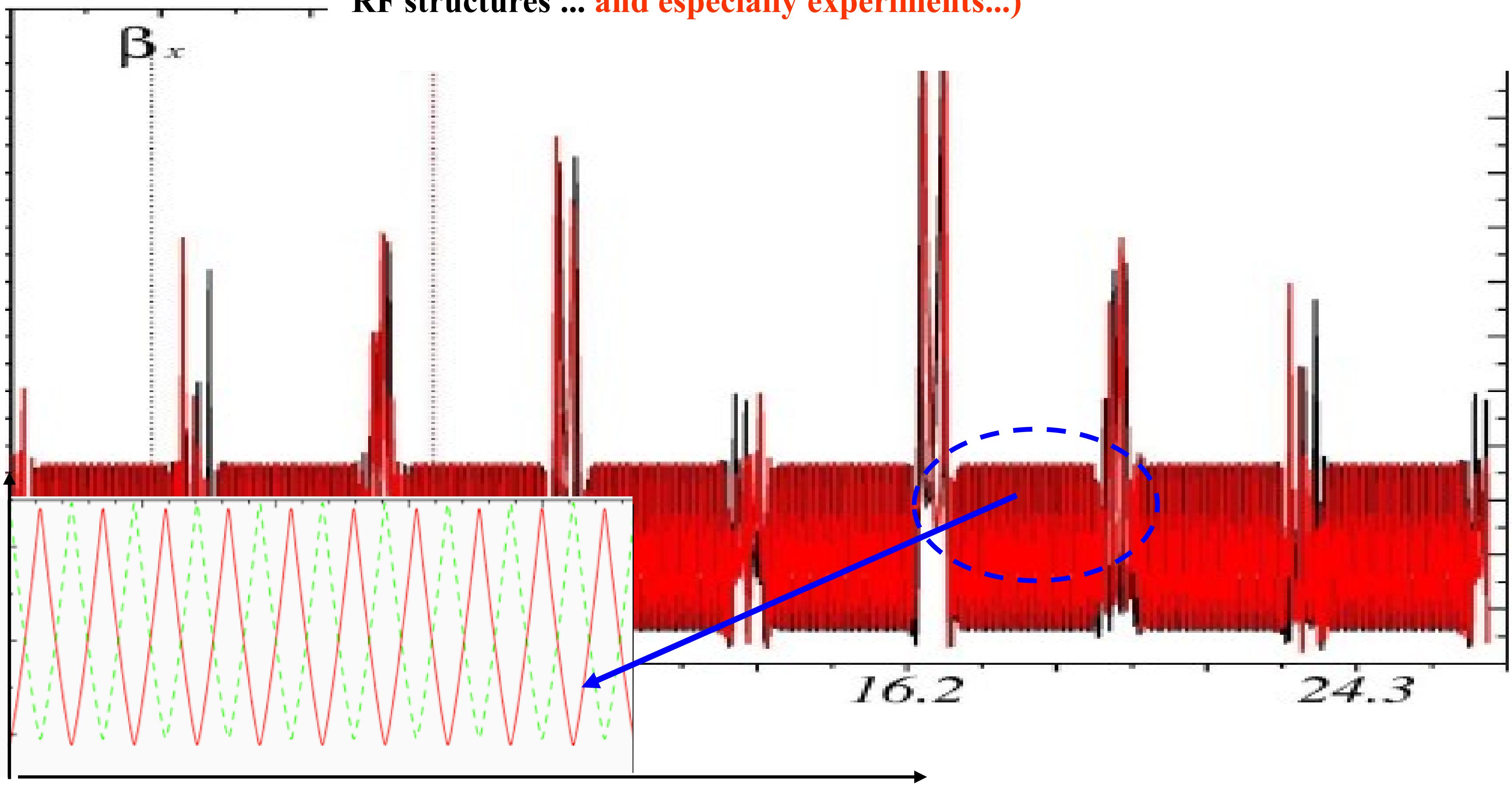


MQ: main quadrupole

Integrated gradient = 690 T
Nominal gradient = 223 T/m
Inominal = 11.87 kA
L=3.1 m

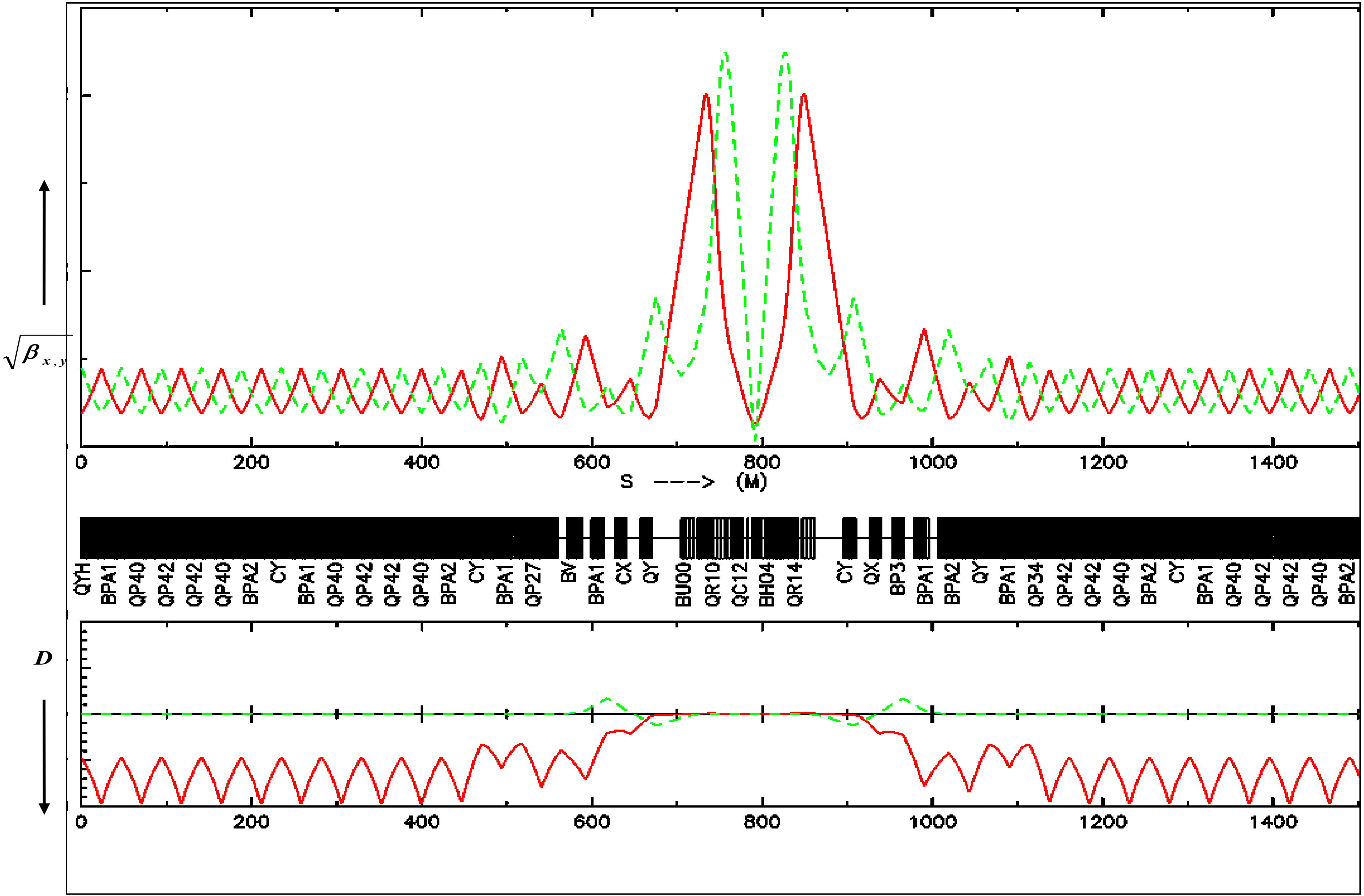
FoDo-Lattice

A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in between.
(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)



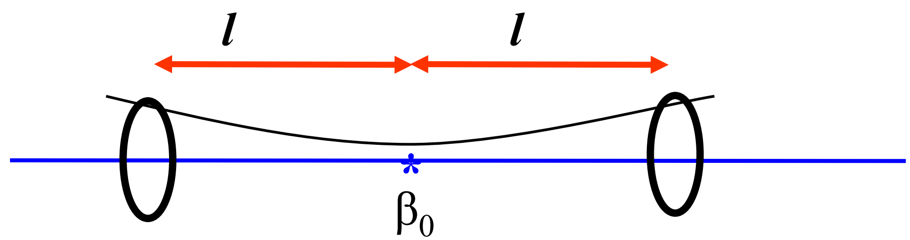
Starting point for the calculation: in the middle of a focusing quadrupole
Phase advance per cell $\mu = 45°$,
→ calculate the twiss parameters for a periodic solution

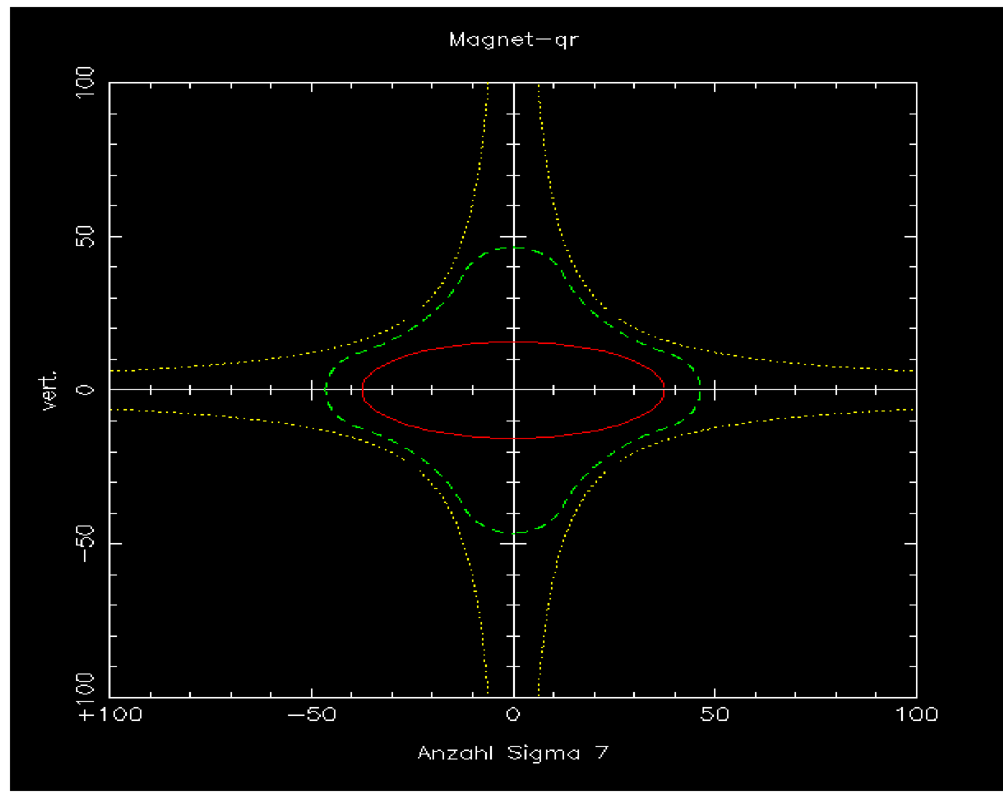
9.) Insertions



β-Function in a Drift:

$$\beta(\ell, \quad {}_0 \quad \frac{\ell}{\beta_0}$$





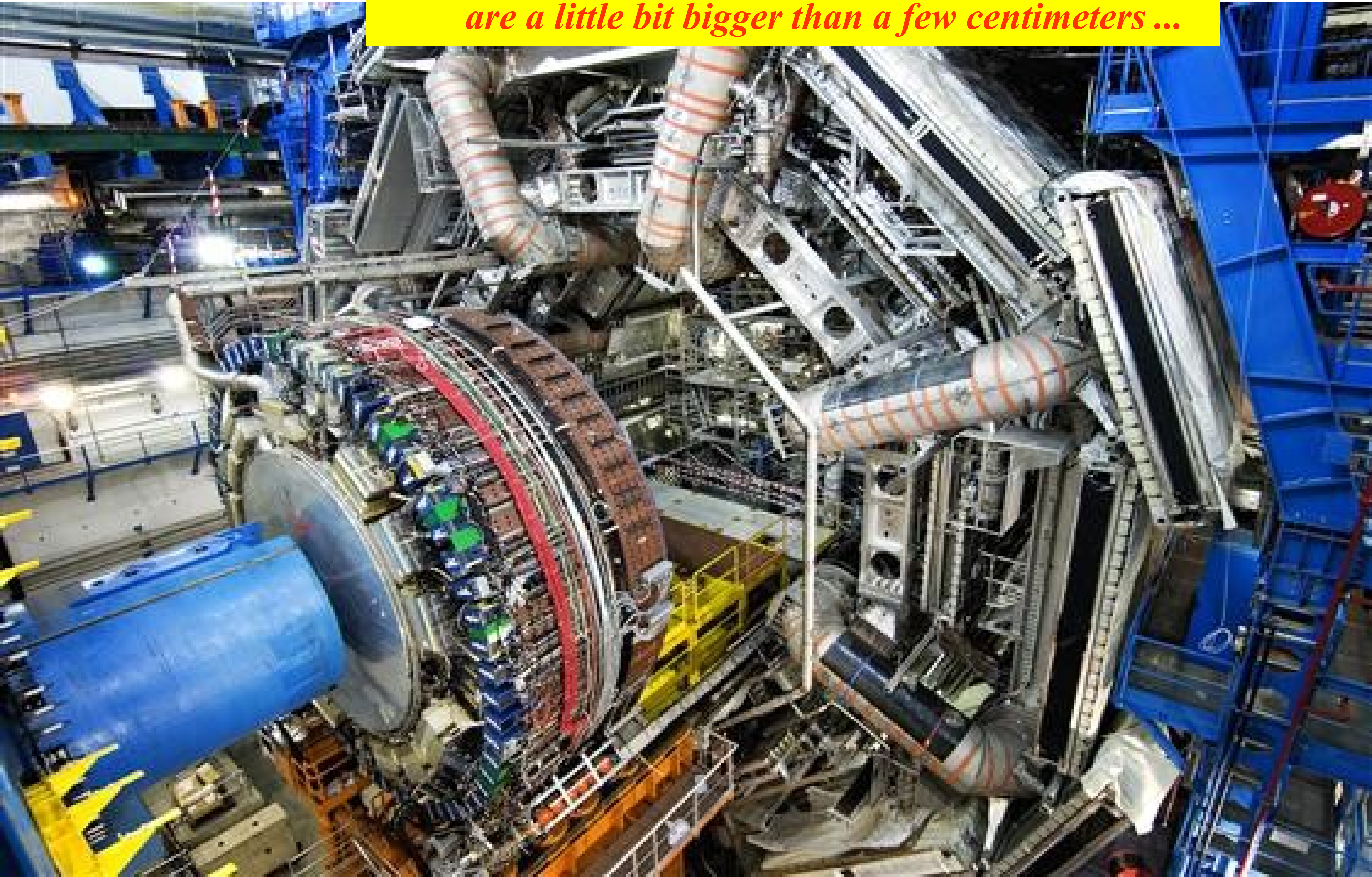
7 sigma beam size inside a mini beta quadrupole

At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

-> keep l as small as possible

... clearly there is an

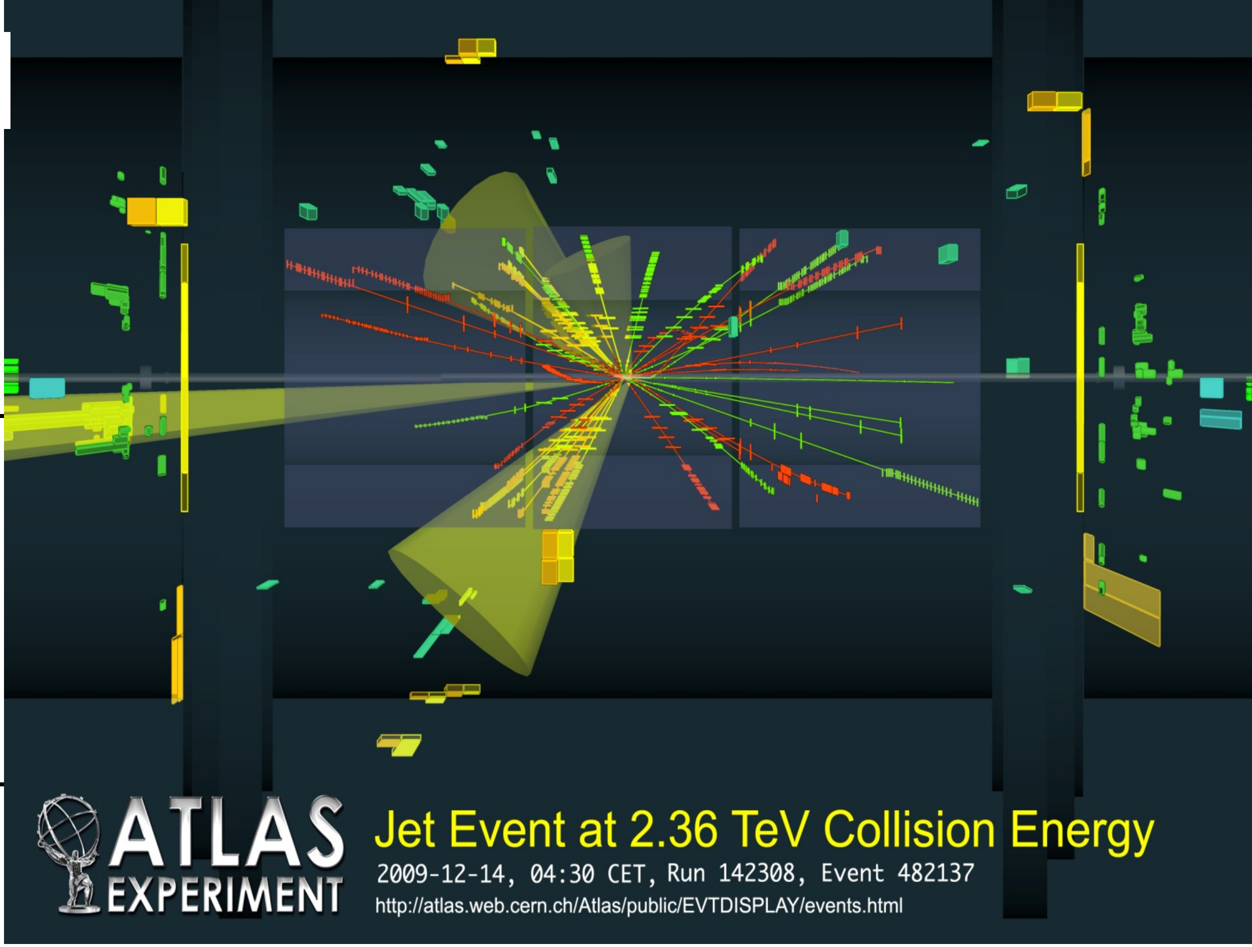
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



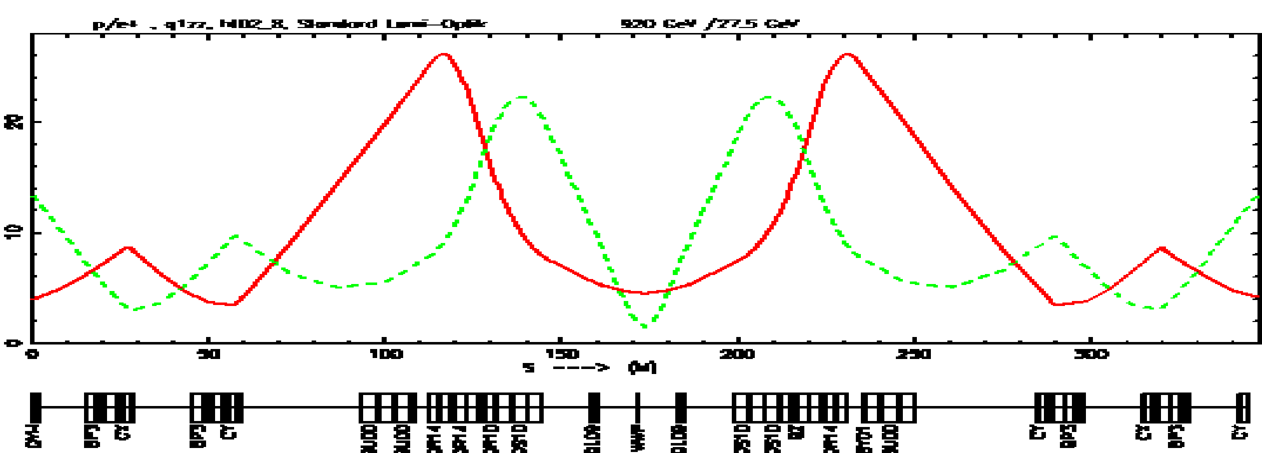
The Mini-β Insertion:

$$R = L * \Sigma_{react}$$

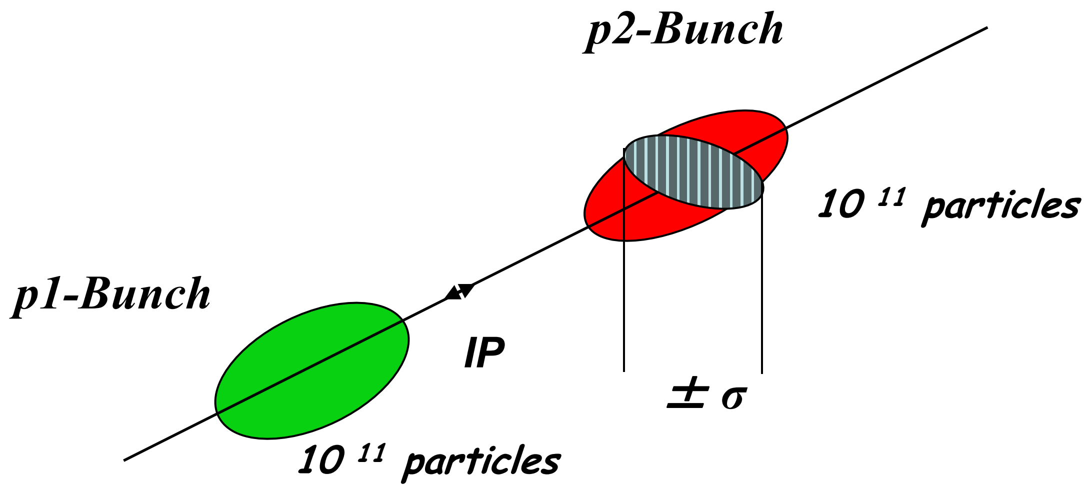
production rate of events
is determined by the
cross section Σ_{react}
and a parameter L that is given
by the design of the accelerator:
… the luminosity



$$L = \frac{1}{4\pi e^2 f_0 b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$



10.) Luminosity



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55\ m \qquad f_0 = 11.245\ kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

$$\sigma_{x,y} = 17\ \mu m$$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$I_p = 584\ mA$$

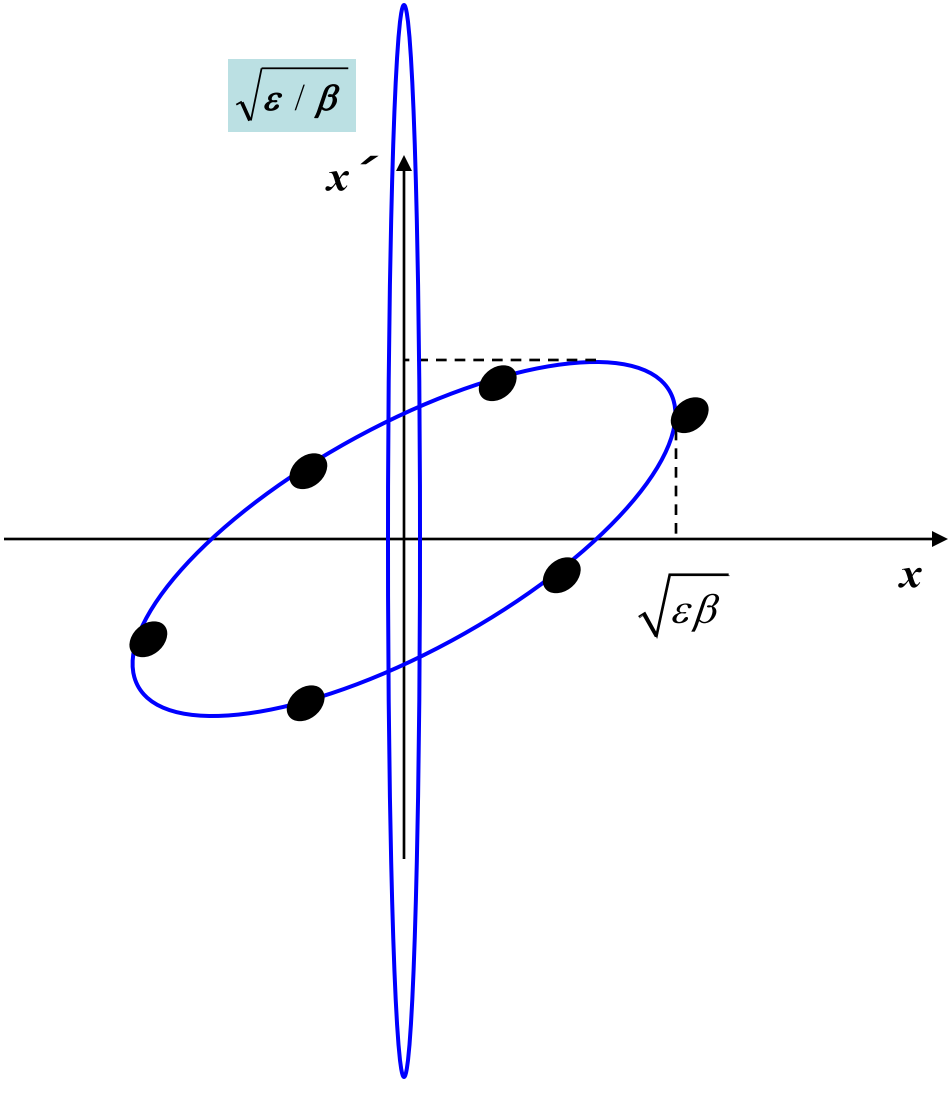
$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

beam sizes in the order of my cat's hair !!

Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.
→greetings from Liouville

the smaller the beam size
the larger the bam divergence



Mini-β Insertions: some guide lines

** calculate the periodic solution in the arc*

** introduce the drift space needed for the insertion device (detector ...)*

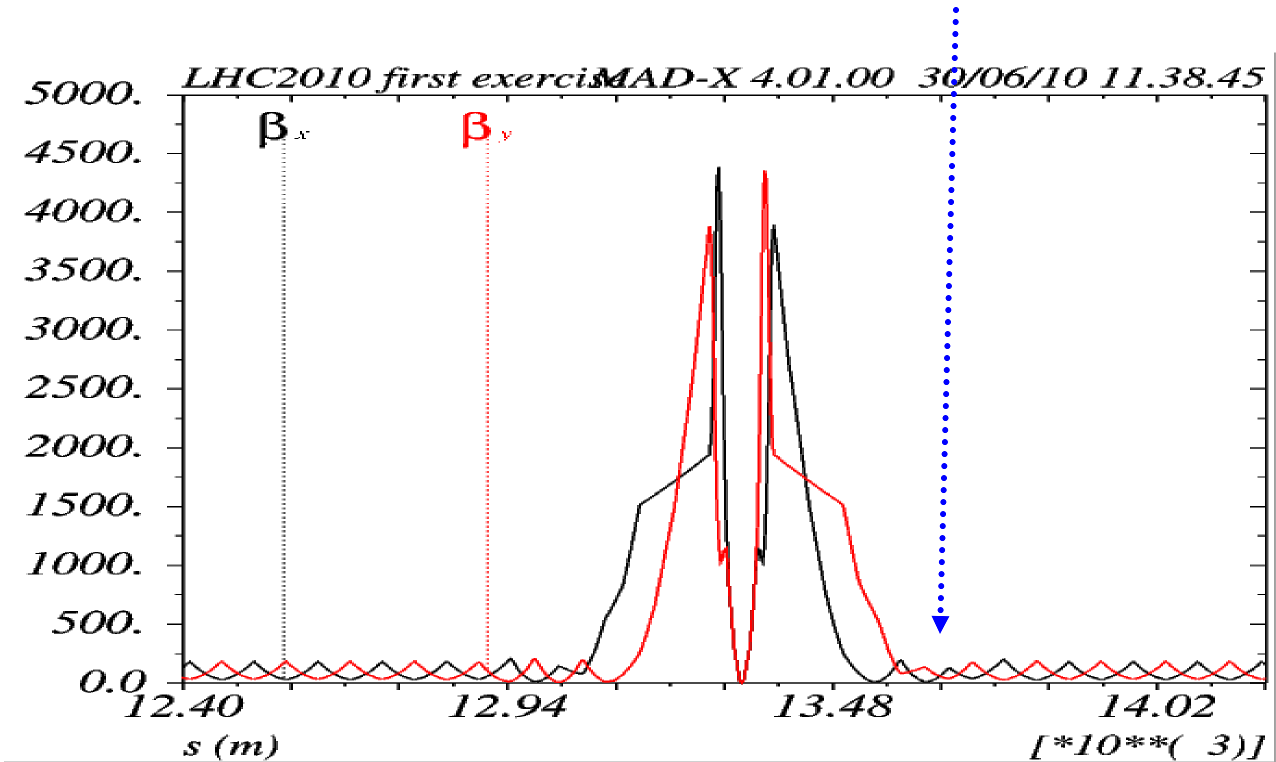
** put a quadrupole doublet (triplet ?) as close as possible*

** introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure*

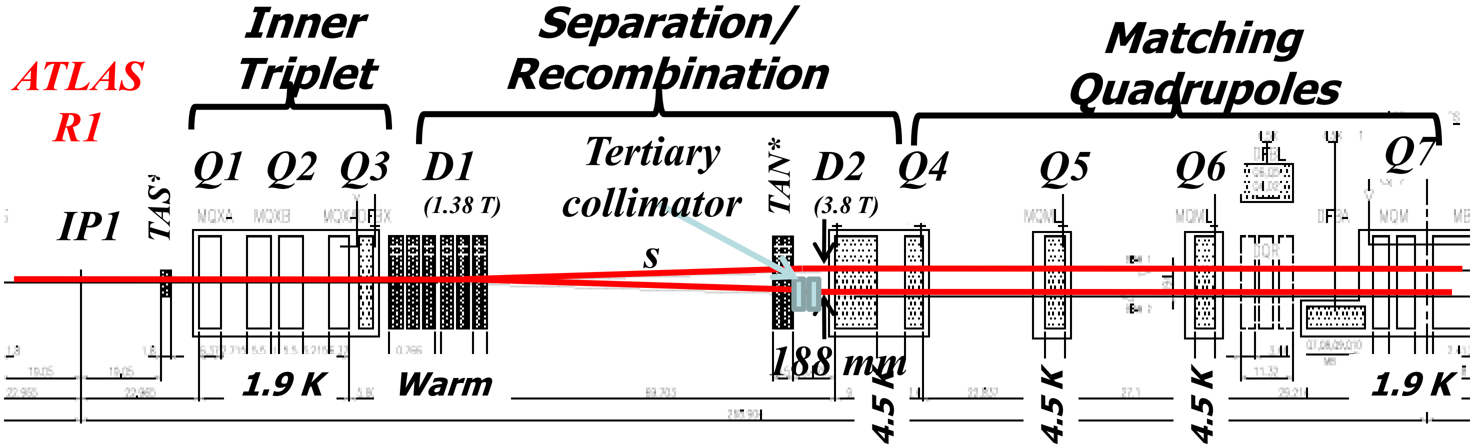
parameters to be optimised & matched to the periodic solution:

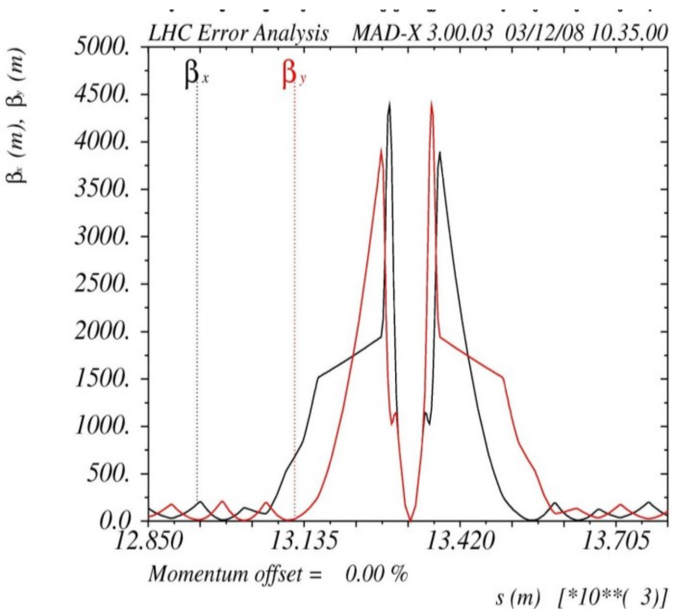
$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$
$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y$$

8 individually powered quad magnets are needed to match the insertion
(... at least)

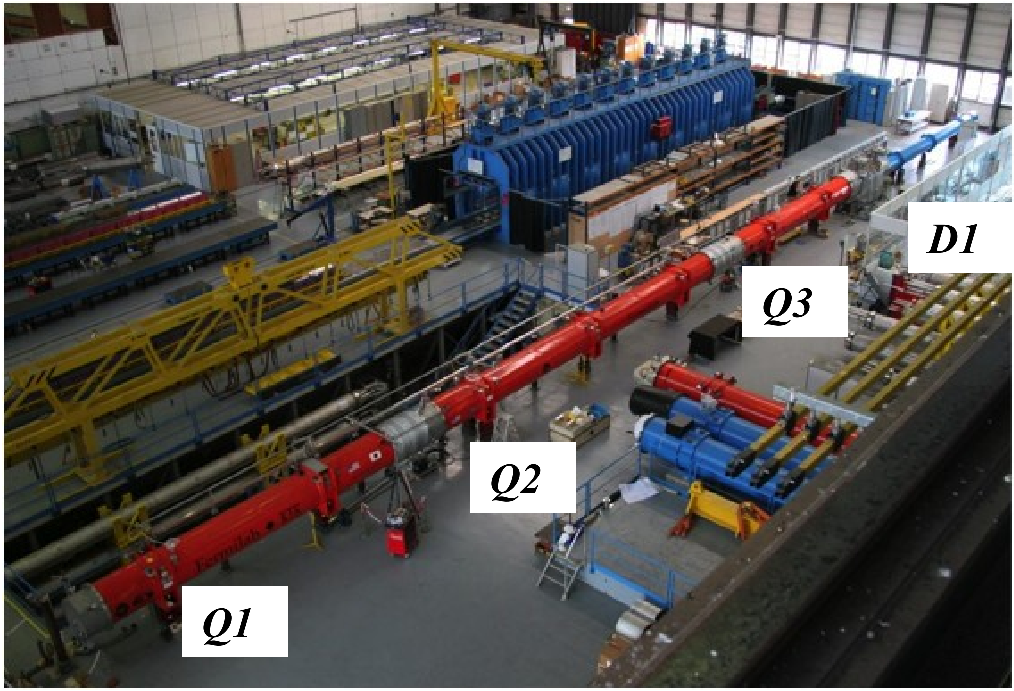


The LHC Insertions





mini β optics

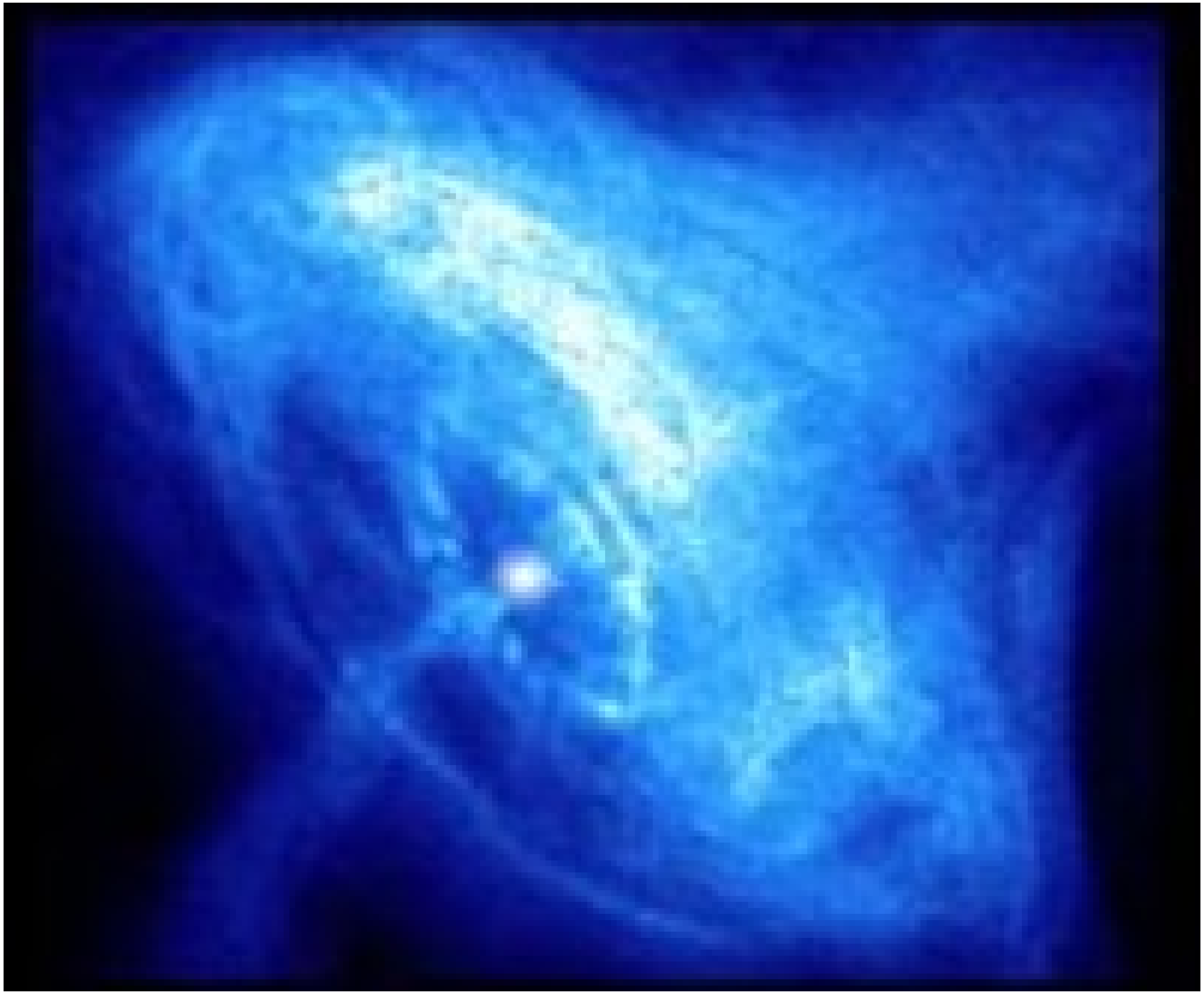


Magnets for the LHC, total budget, every magnet has a role in the optics design

Name	Quantity	Purpose
MB	1232	Main dipoles
MQ	400	Main lattice quadrupoles
MSCB	376	Combined chromaticity/ closed orbit correctors
MCS	2464	Dipole spool sextupole for persistent currents at injection
MCDO	1232	Dipole spool octupole/decapole for persistent currents
MO	336	Landau octupole for instability control
MQT	256	Trim quad for lattice correction
MCB	266	Orbit correction dipoles
MQM	100	Dispersion suppressor quadrupoles
MQY	20	Enlarged aperture quadrupoles

In total 6628 cold magnets ...

IV) … let´s talk about acceleration



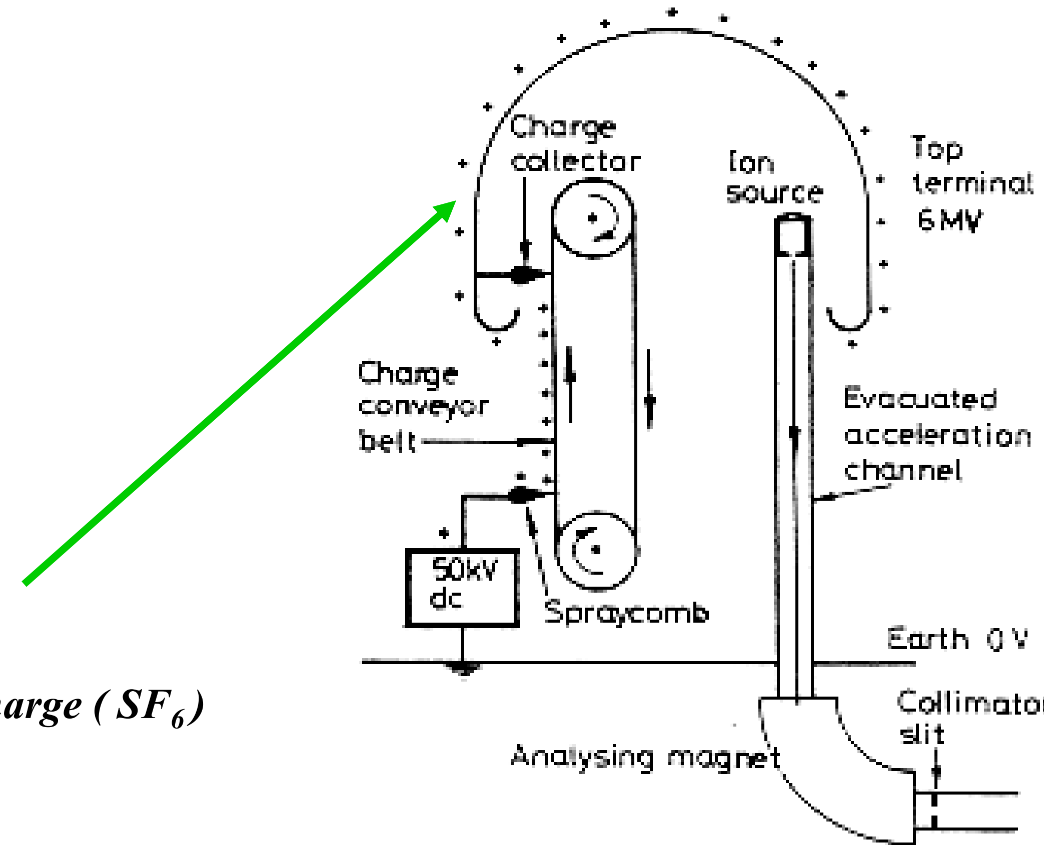
crab nebula,

burst of charged particles $E = 10^{20}$ eV

11.) Electrostatic Machines

(Tandem -) van de Graaff Accelerator

***creating high voltages by** mechanical transport of charges*



**** Terminal Potential:** $U \approx 12 \ldots 28$ **MV***
using high pressure gas to suppress discharge (** SF_6 **)

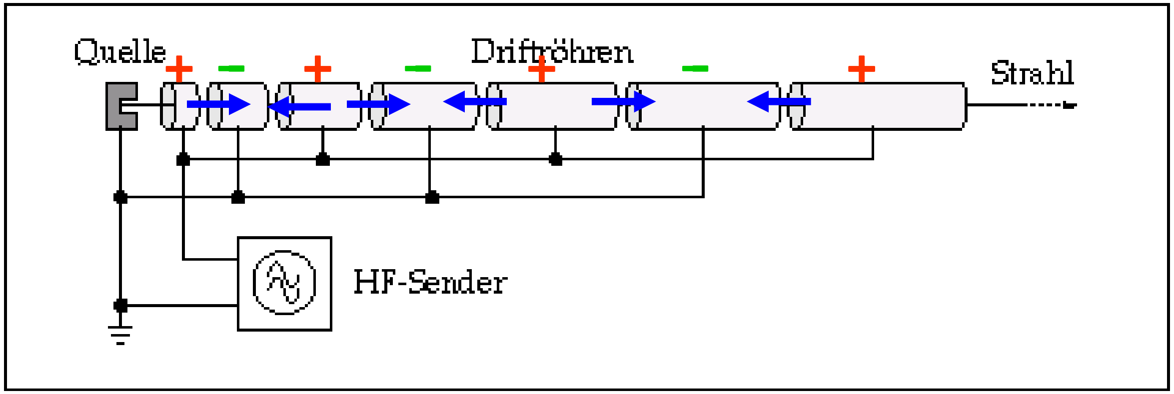
Problems: * Particle energy limited by high voltage discharges
**** high voltage can only be applied once per particle ...***
... or twice ?

* The „Tandem principle": Apply the accelerating voltage twice ...

... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

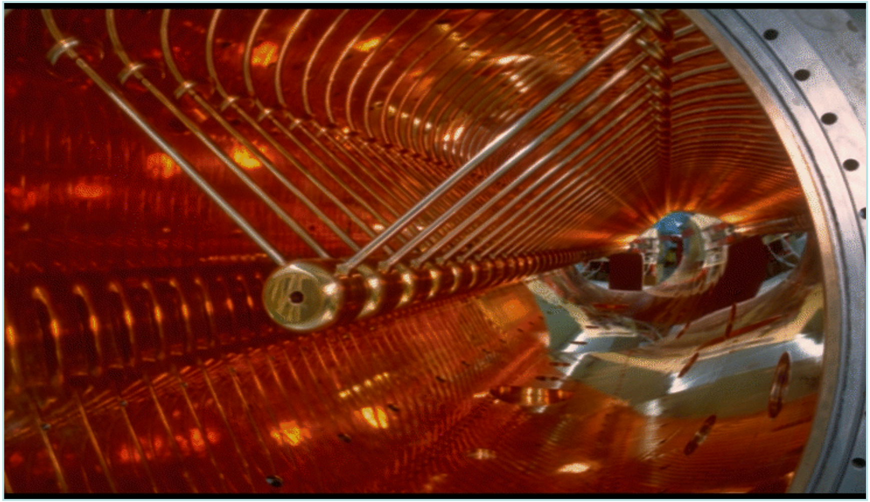
12.) Linear Accelerator 1928, Wideroe



Energy Gain per „Gap":

$$W = q\, U_0 \sin \omega_{RF}\, t$$

drift tube structure at a proton linac (GSI Unilac)



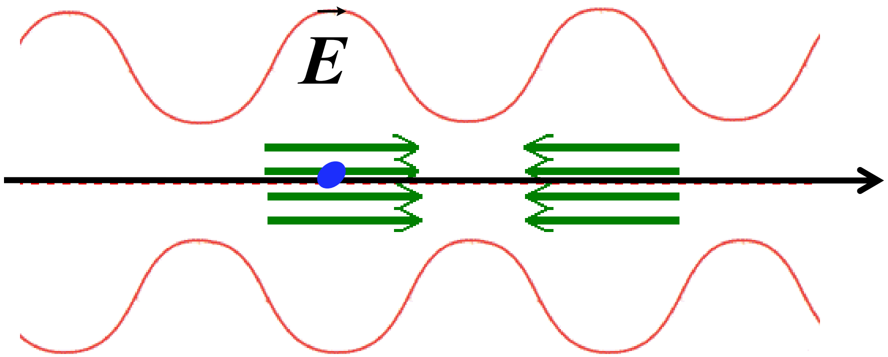
500 MHz cavities in an electron storage ring

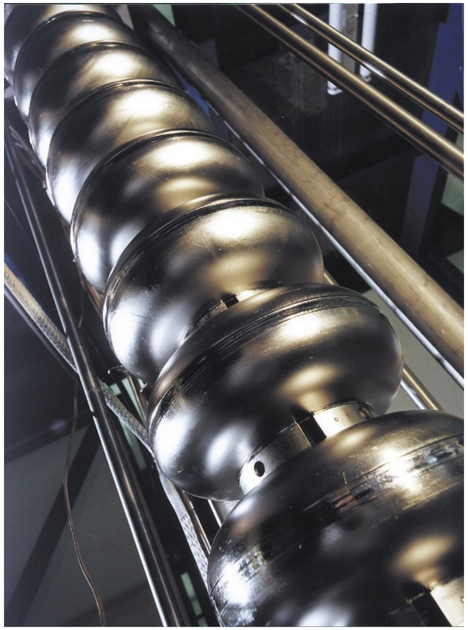
* *RF Acceleration:* *multiple application of the same acceleration voltage; brillant idea to gain higher energies*

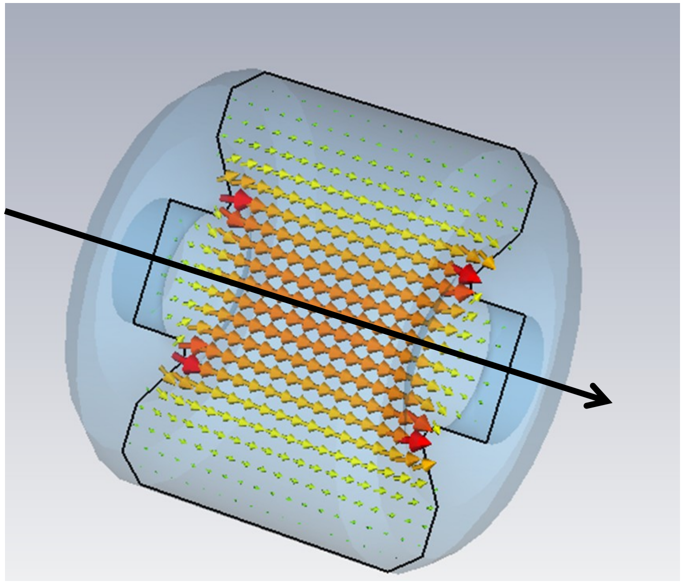
13.) The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:







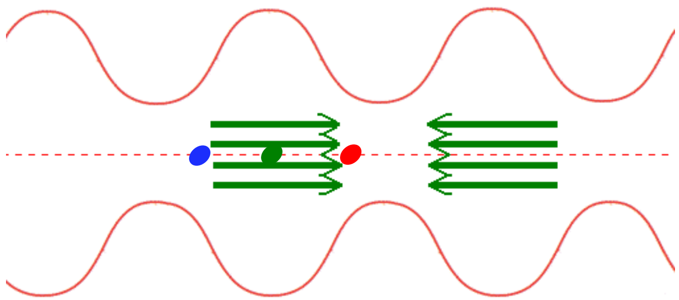
B. Salvant
N. Biancacci

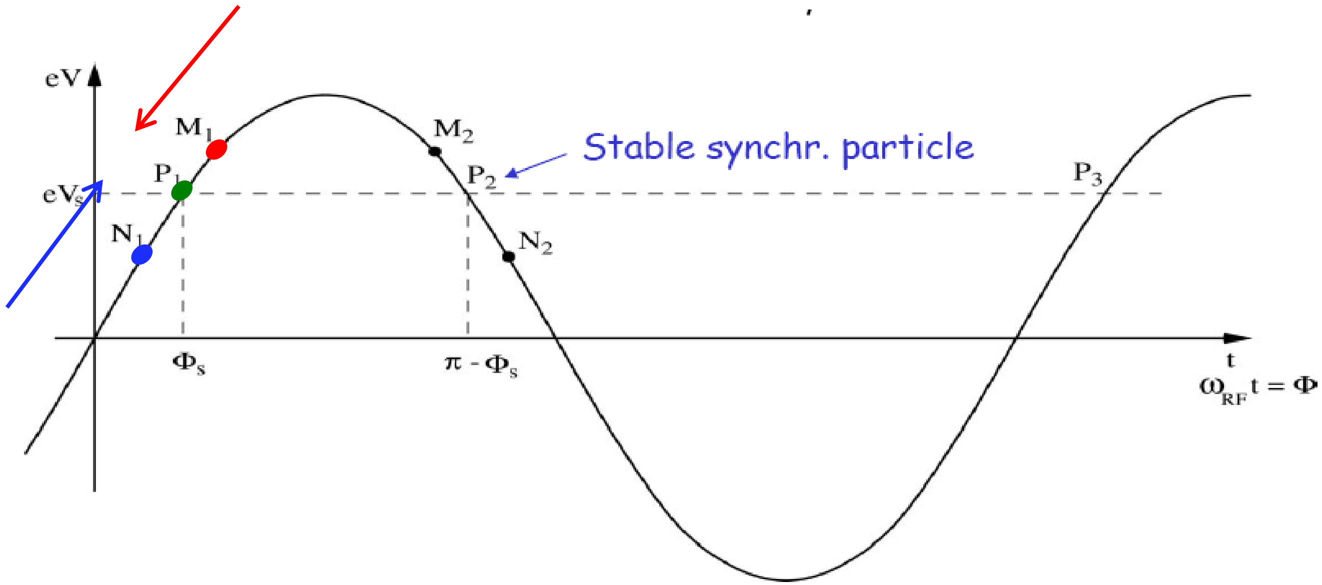
14.) The Acceleration for Δp/p≠0
"Phase Focusing" below transition

ideal particle •

particle with Δp/p > 0 • *faster*

particle with Δp/p < 0 • *slower*



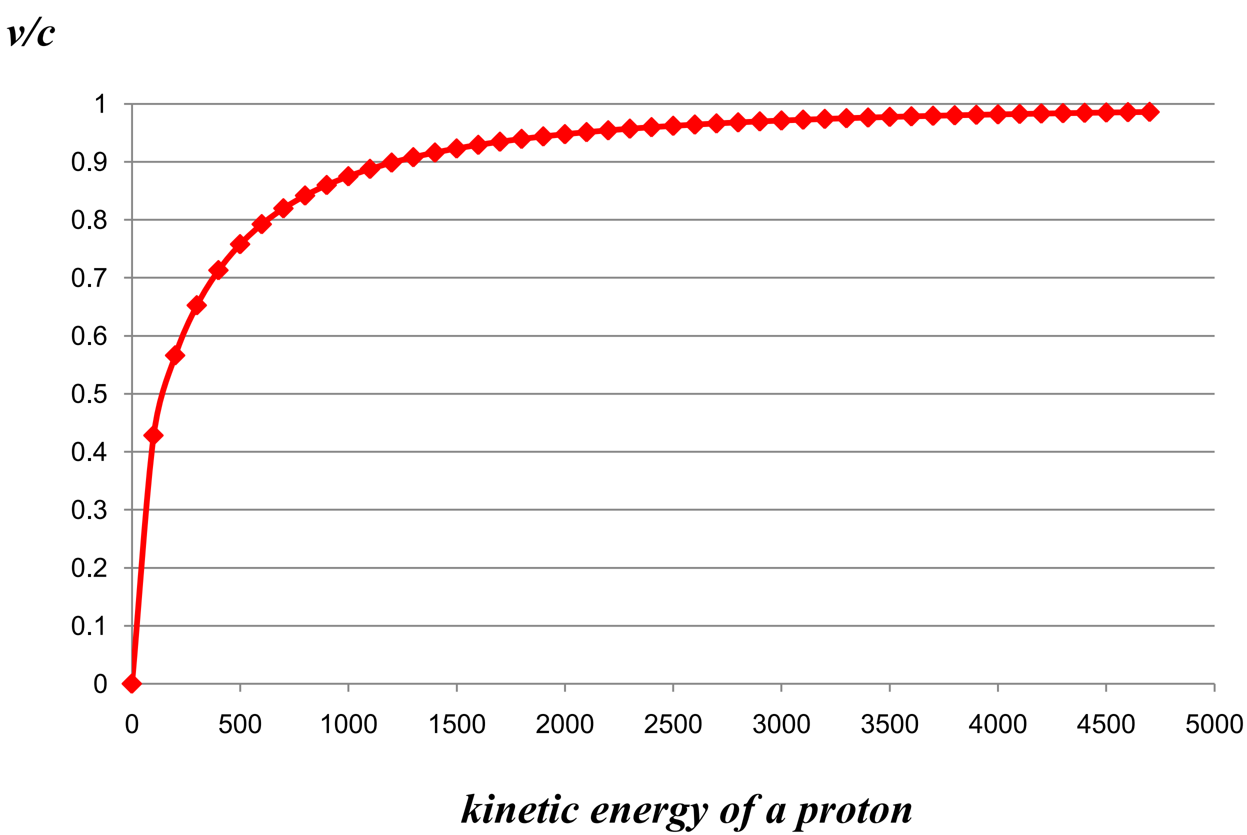


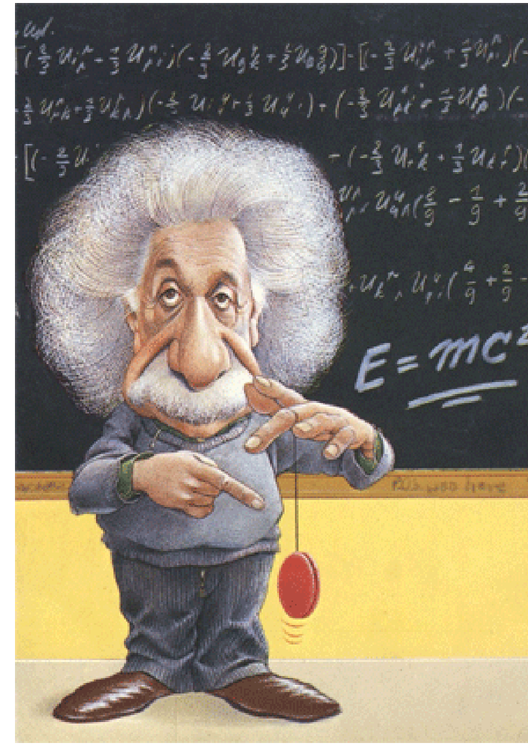
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ **≈ some Hz**

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \longrightarrow \quad \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

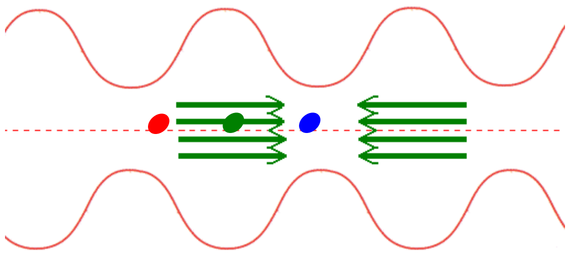
.... but heavier !

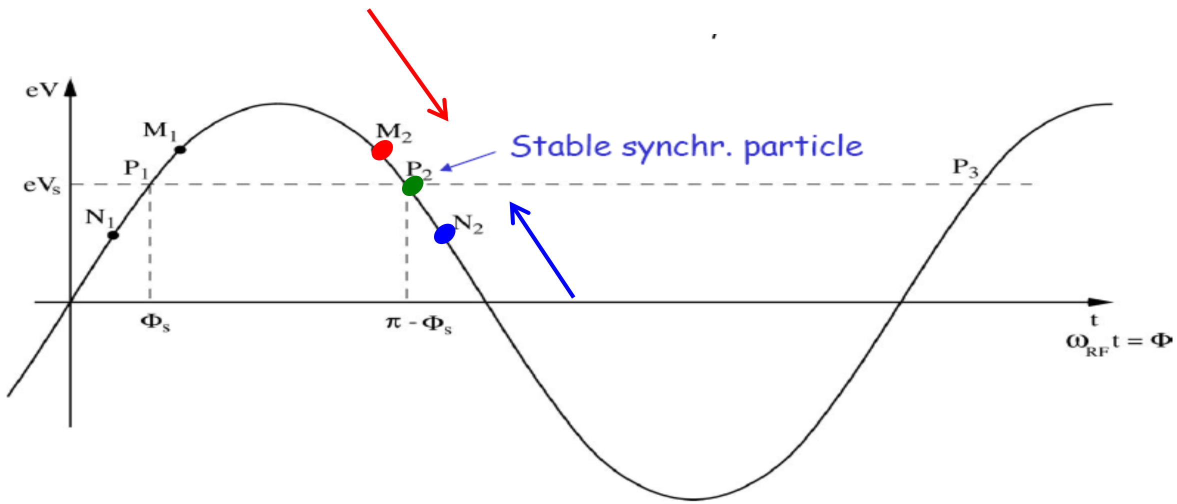
15.) The Acceleration for $\Delta p/p \neq 0$
"Phase Focusing" above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



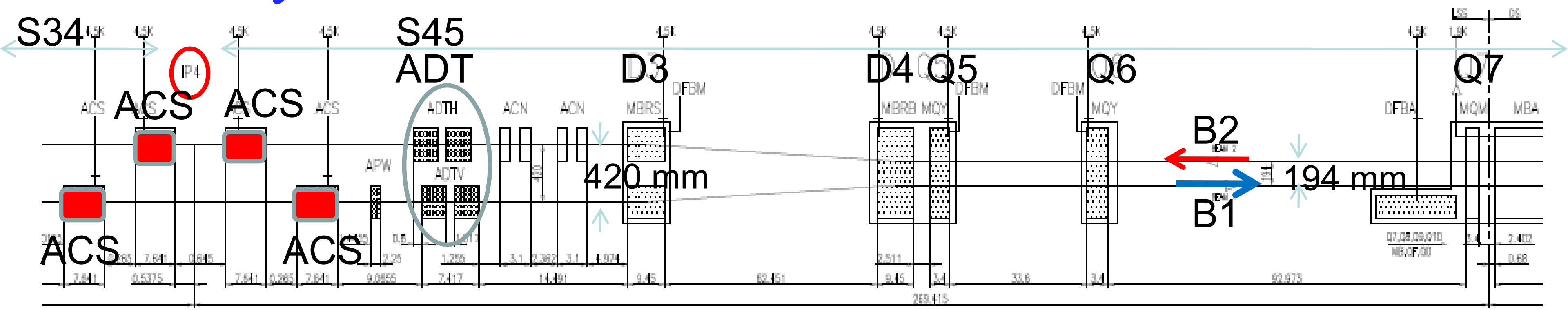


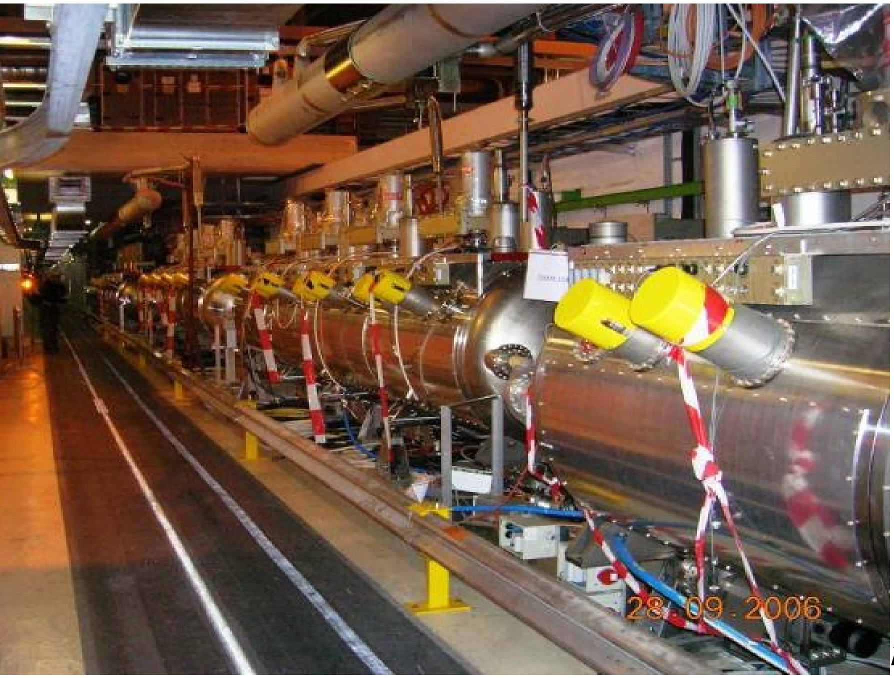
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



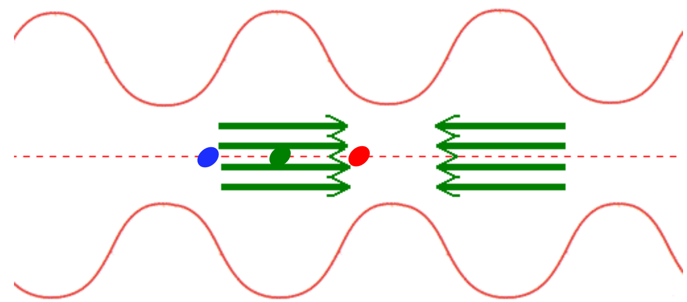


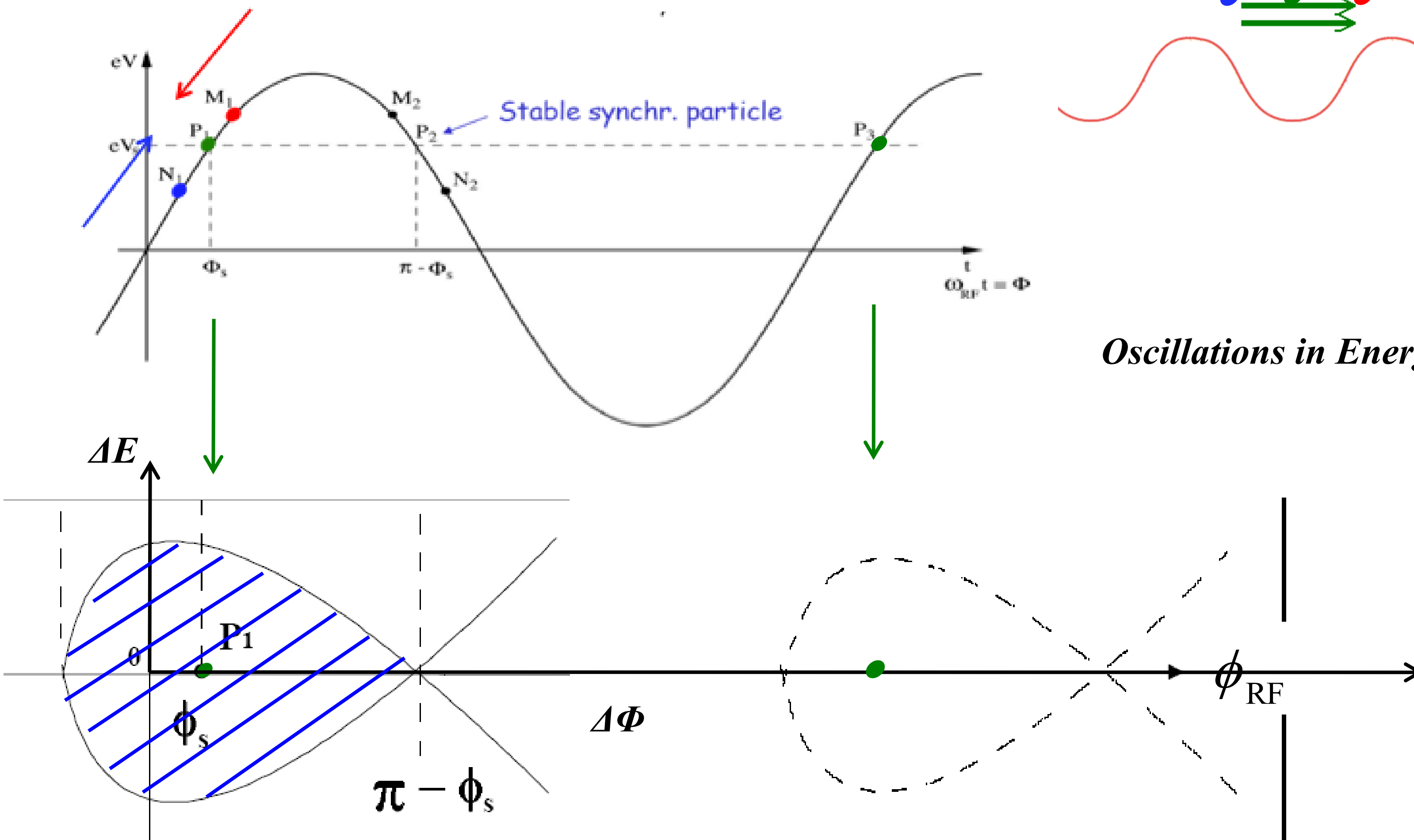
m

Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*MHz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

RF Buckets & long. dynamics in phase space



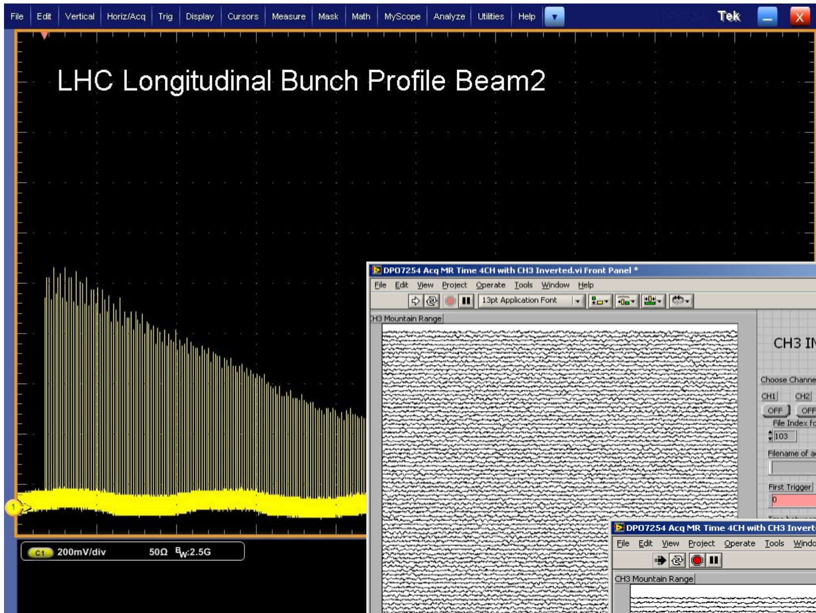


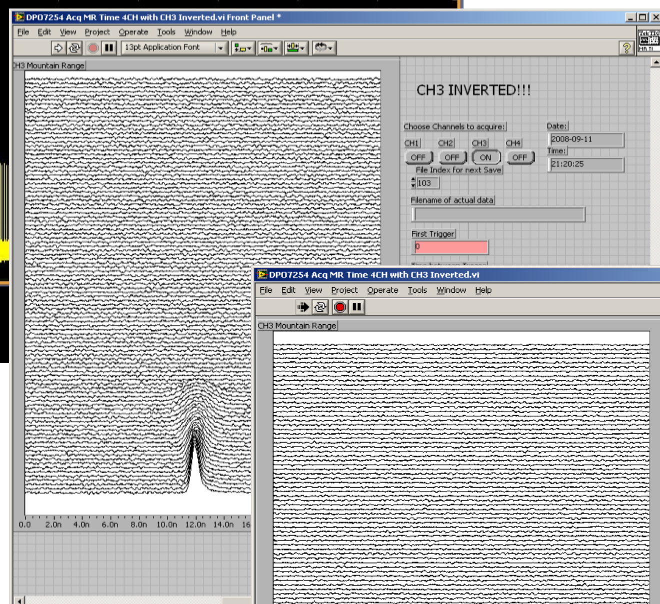
Oscillations in Energy and Phase

35640 Possible Bunch Positions (“buckets”)

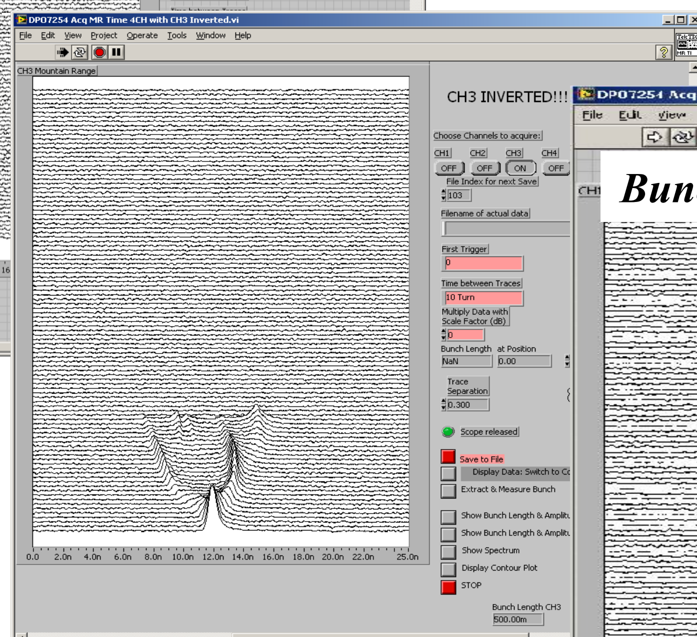
2808 Bunches

LHC Commissioning: RF

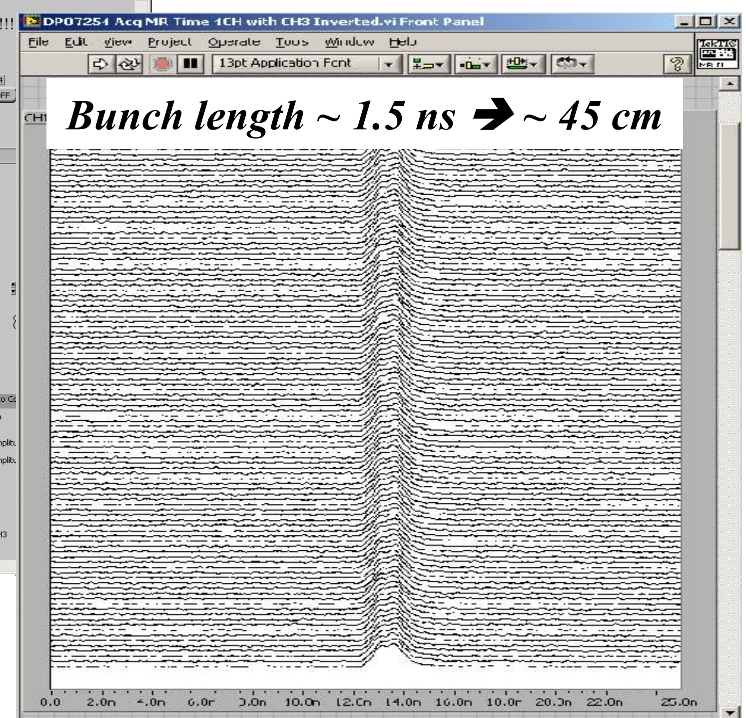




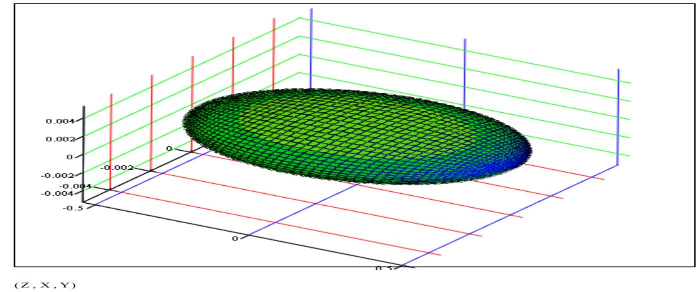
RF off



RF on,
phase optimisation



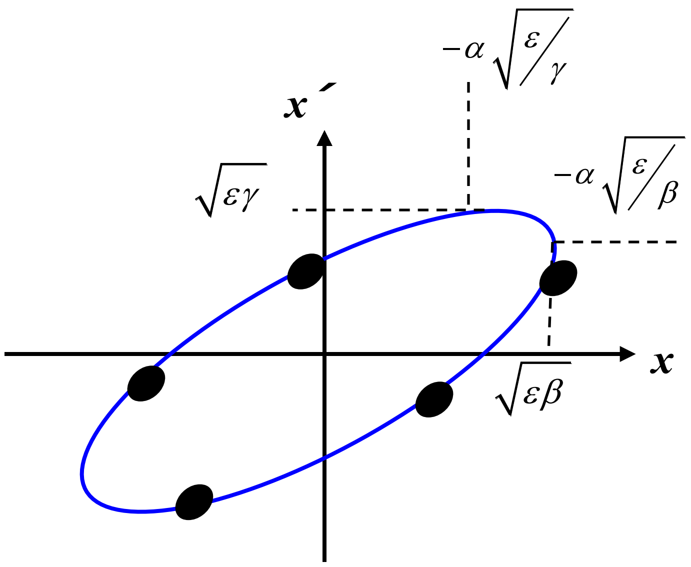
RF on, phase adjusted,
beam captured



a proton bunch: focused longitudinal by the RF field

Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\, x'^2(s)$$



Beam Emittance ***corresponds to the area covered in the x, x´ Phase Space Ellipse***

Liouville: ***Area in phase space is constant.***

But so sorry ... ε ≠ const !

Classical Mechanics:

phase space ***= diagram of the two canonical variables***
position ***&*** ***momentum***
x p_x

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad ; \quad L = T - V = kin.Energy - pot.Energy$$

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

$$q = position = x$$
$$p = momentum = \gamma m v = mc\gamma\beta_x$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad \beta_x = \frac{\dot{x}}{c}$$

Liouvilles Theorem: $\int p\,dq = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta}$$

where $\beta_x = v_x / c$

$$\int p dq = mc \int \gamma\beta_x dx$$

$$\int p dq = mc\,\gamma\beta \underbrace{\int x' dx}_{\varepsilon}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

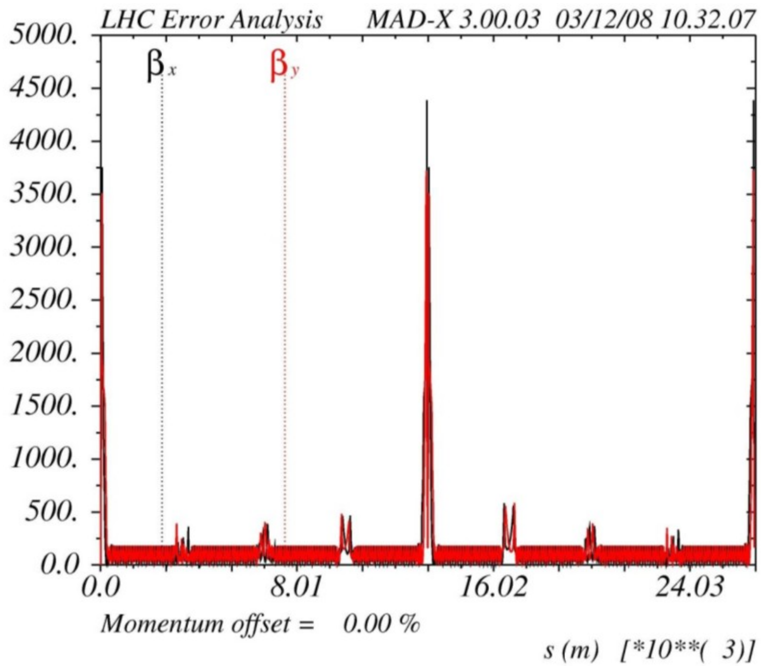
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

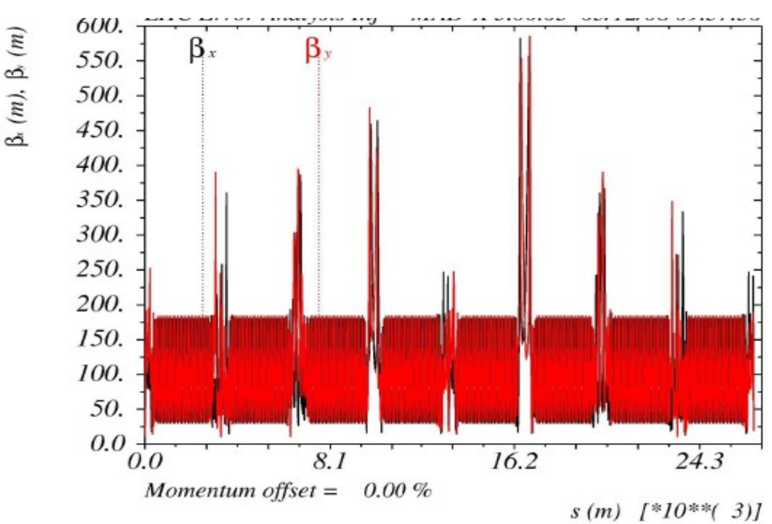
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

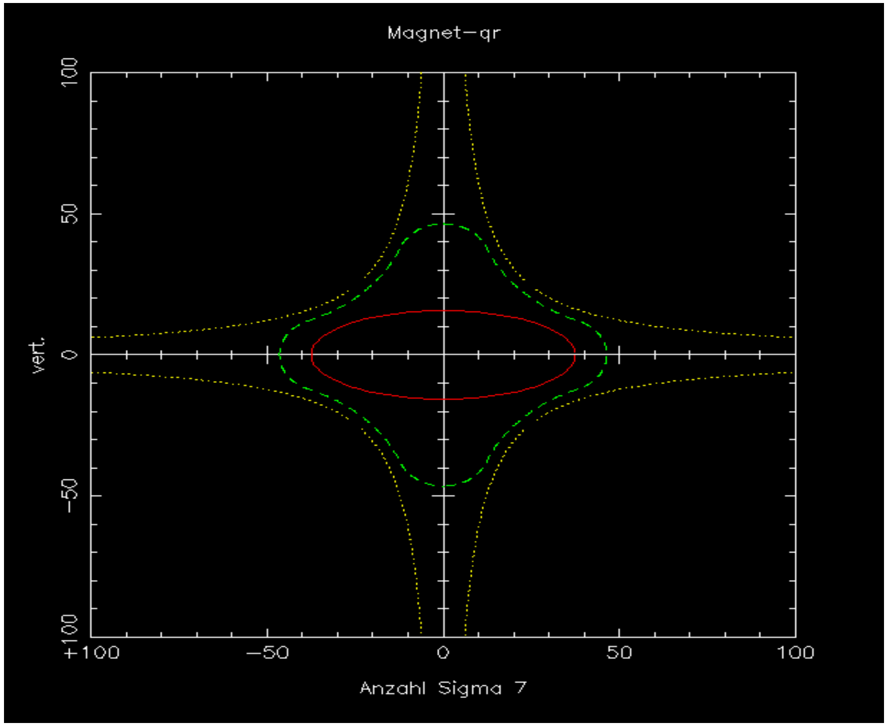


LHC injection optics at 450 GeV

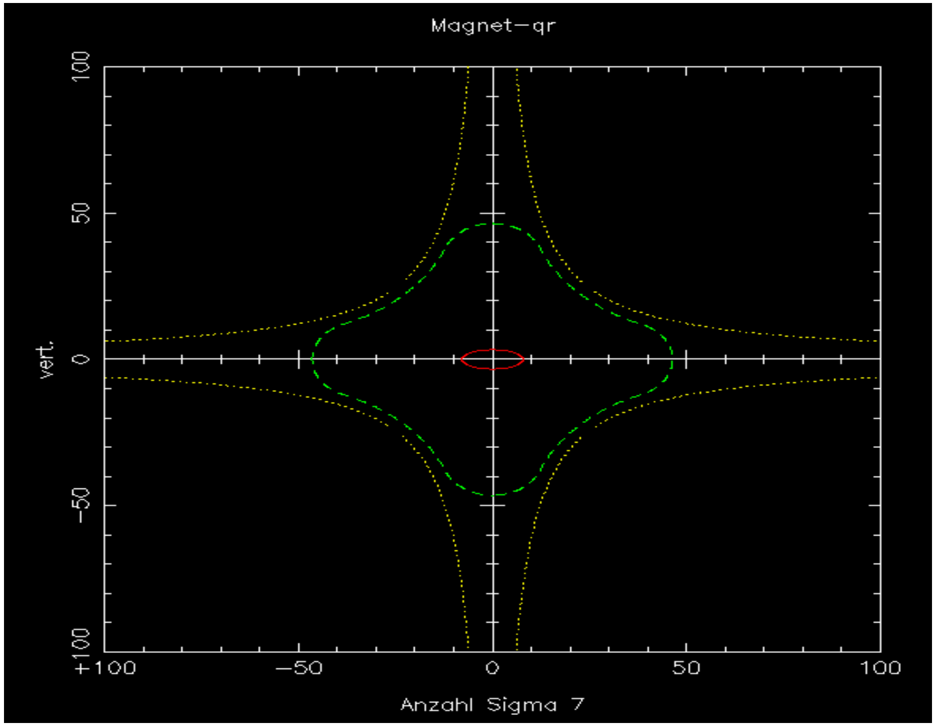
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon\ (40GeV) = 1.2 * 10^{-7}$
$\varepsilon\ (920GeV) = 5.1 * 10^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

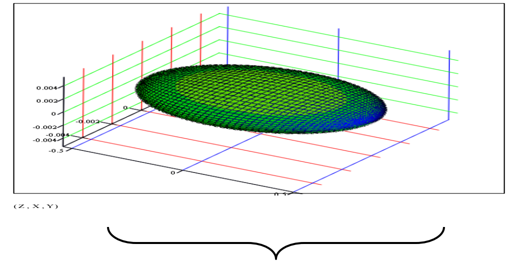
IV

V.) Are there Any Problems ???

sure there are

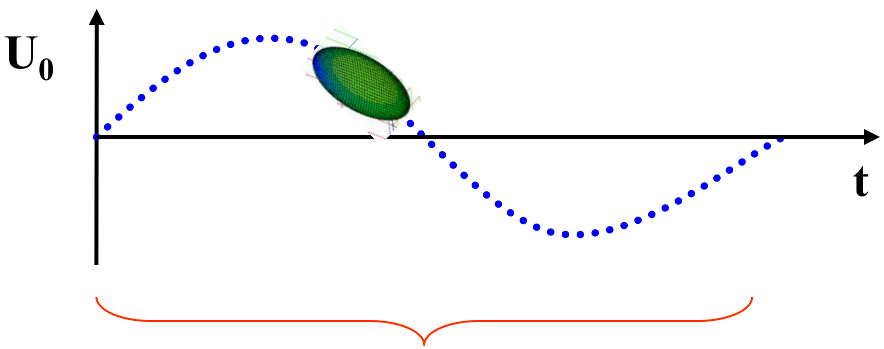
RF Acceleration-Problem: panta rhei !!!

(Heraklit: 540-480 v. Chr.)



Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$\lambda = 75\ cm$

$$\left.\begin{aligned} \nu &= 400\ MHz \\ c &= \lambda\ \nu \end{aligned}\right\}\ \lambda = 75\ cm$$

$\sin(90^o) = 1$

$\sin(84^o) = 0.994$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

Sure there are !!!

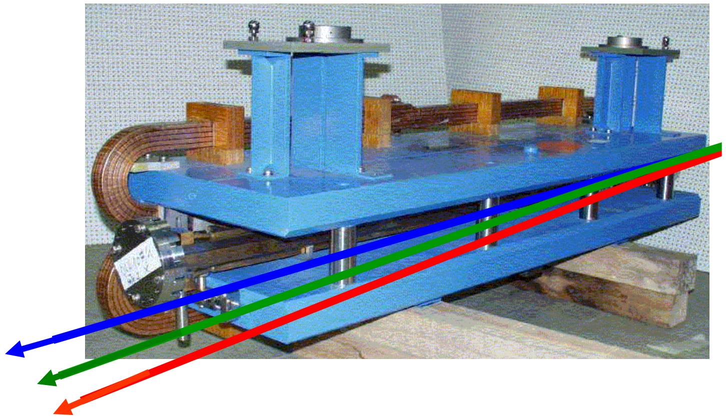
font colors due to pedagogical reasons

17.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: ***prop. to magn. field & prop. zu 1/p***

dipole magnet

$$\alpha = \frac{\int B\, dl}{p/e}$$



$$x_D(s) = D(s)\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

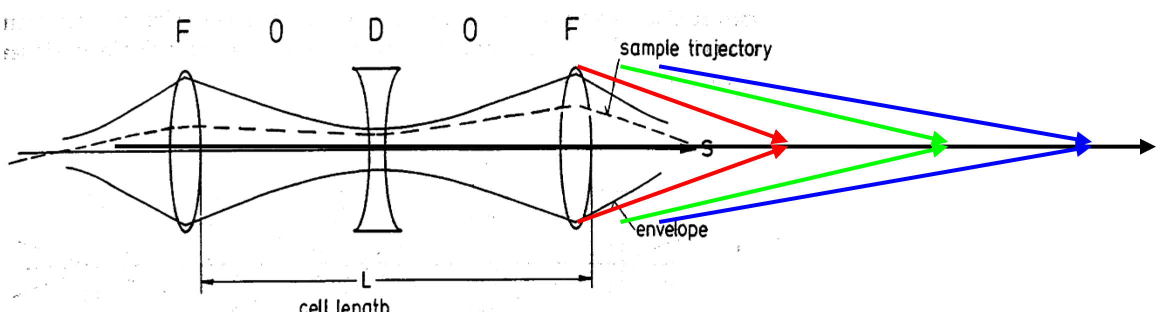


Figure 29: FODO cell

particle having ...

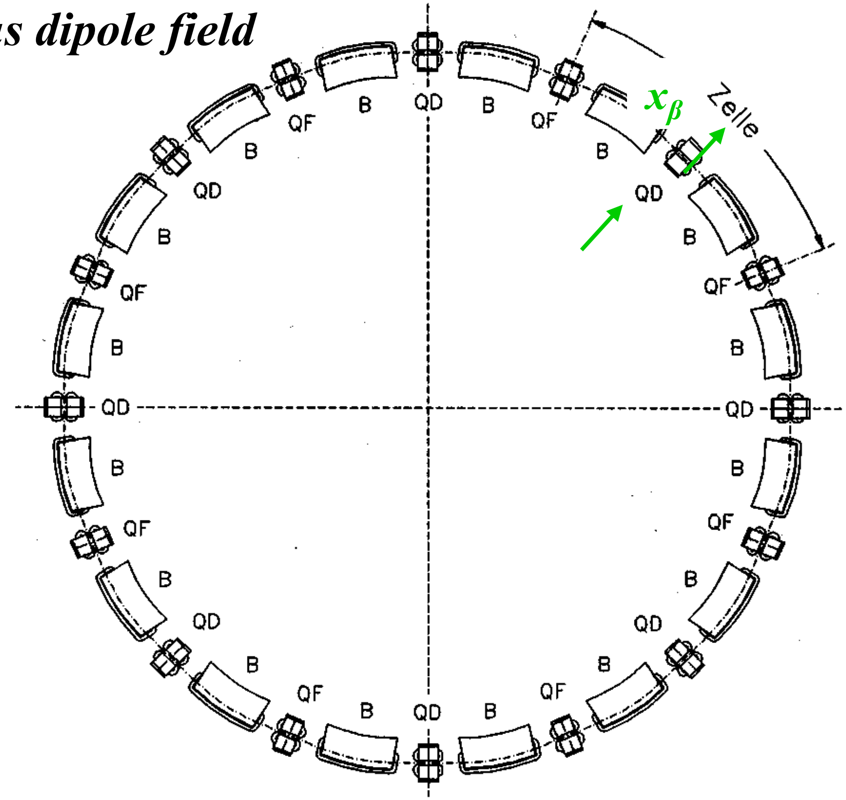
to high energy

to low energy

ideal energy

Dispersion

Example: homogeneous dipole field



ıit for Δp/p > 0

$: D(s) \cdot \frac{\Delta p}{p}$

Matrix formalism:

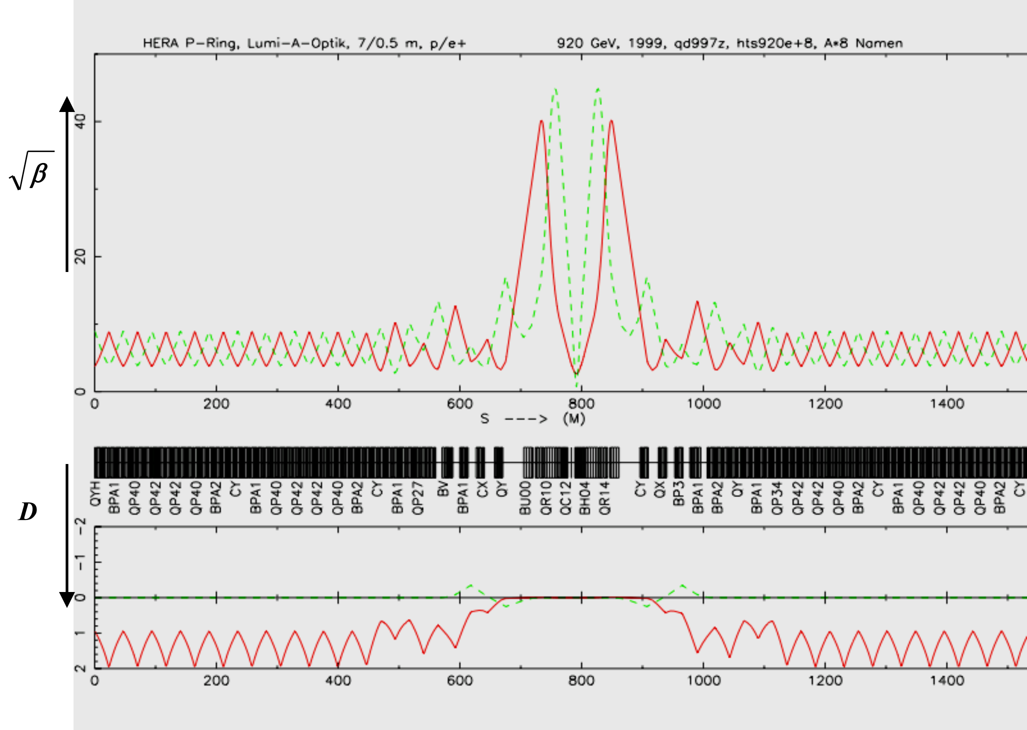
$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x'_0 + D(s) \cdot \frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \ldots 2 \; mm$$
$$D(s) \approx 1 \ldots 2 \; m$$
$$\Delta p / p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation

contribution due to Dispersion ≈ beam size

→ Dispersion must vanish at the collision point

!

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

26.) Chromaticity:
A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: ***prop. to magn. field & prop. zu 1/p***

focusing lens

$$k = \frac{g}{p/e}$$

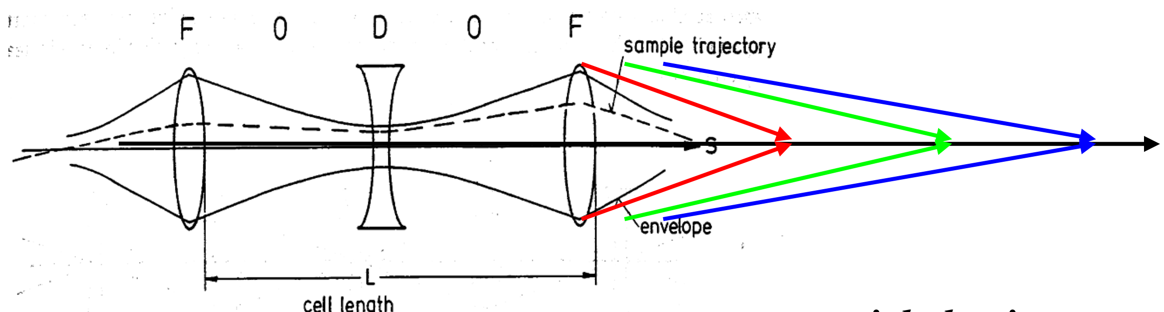


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

… which acts like a quadrupole error in the machine and leads to a tune spread:

$$\Delta Q = \frac{1}{4\pi}\frac{\Delta p}{p}\, k\beta(s)\, ds$$

definition of chromaticity:

$$\Delta Q = Q' * \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

Q' *is a* ***number*** *indicating the* ***size of the tune spot*** *in the working diagram,*
Q' *is always created if the beam is focussed*
→ *it is determined by the focusing strength* ***k*** *of all quadrupoles*

$$Q' = -\frac{1}{4\pi}\oint k(s)\,\beta(s)\,ds$$

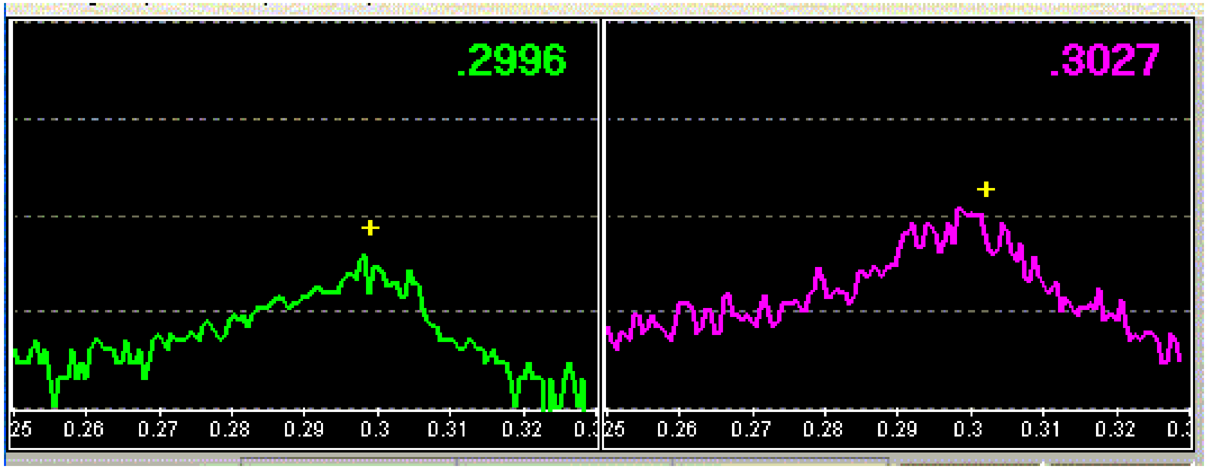
k = quadrupole strength
β = ***betafunction*** *indicates the beam size … and even more the* ***sensitivity of the beam to external fields***

Example: LHC

$Q' = 250$
$\Delta p/p = \pm 0.2 \cdot 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

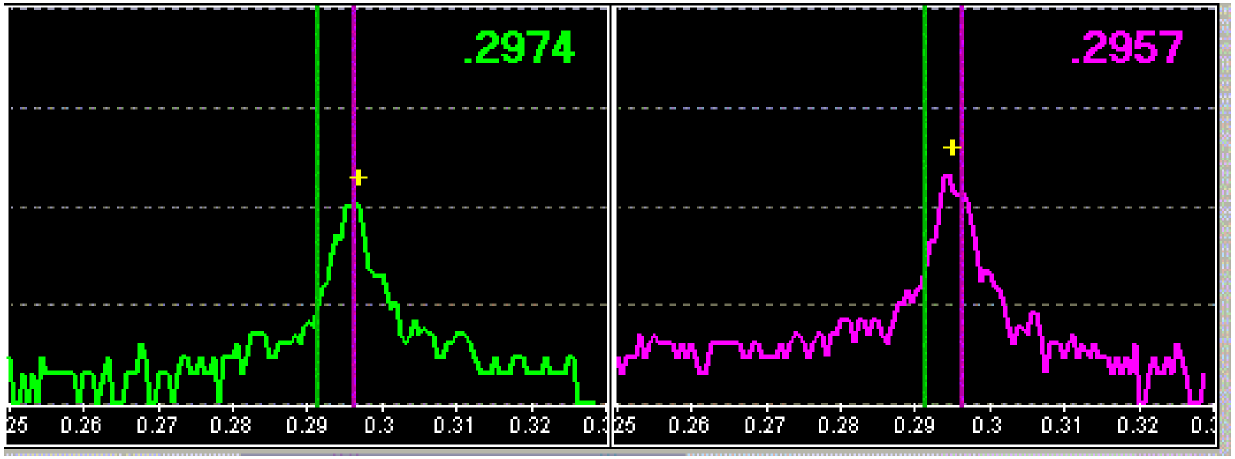
→ *Some particles get very close to resonances and are lost*

in other words: the tune is not a point it is a ***pancake***



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)



Some Golden Rules to Avoid Trouble

I.) Golden Rule number one: do not focus the beam !

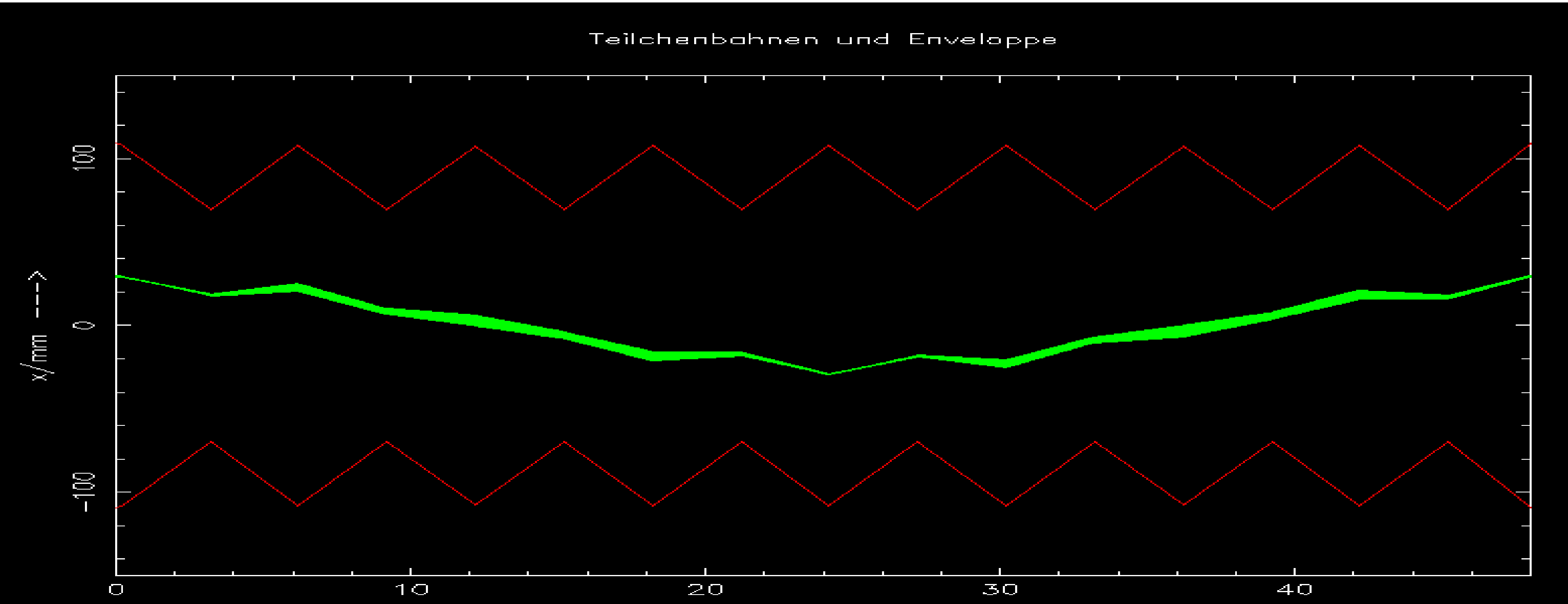
Problem: Resonances

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{s1}} \sqrt{\beta_{s1}} * \cos(\psi_{s1} - \psi_s - \pi Q)\, ds}{2 \sin \pi Q}$$

Assume: Tune = integer $Q = 1 \rightarrow 0$

Qualitatively spoken:

Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.

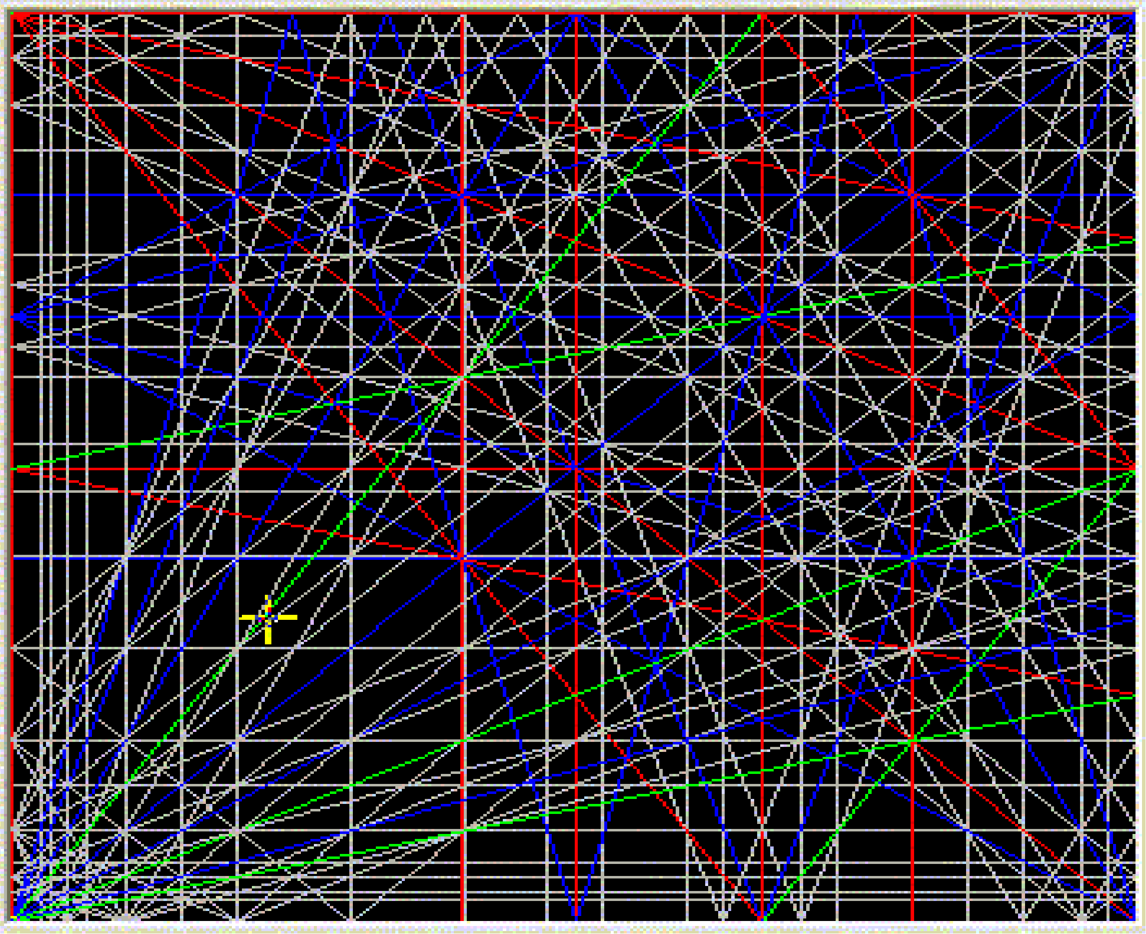


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

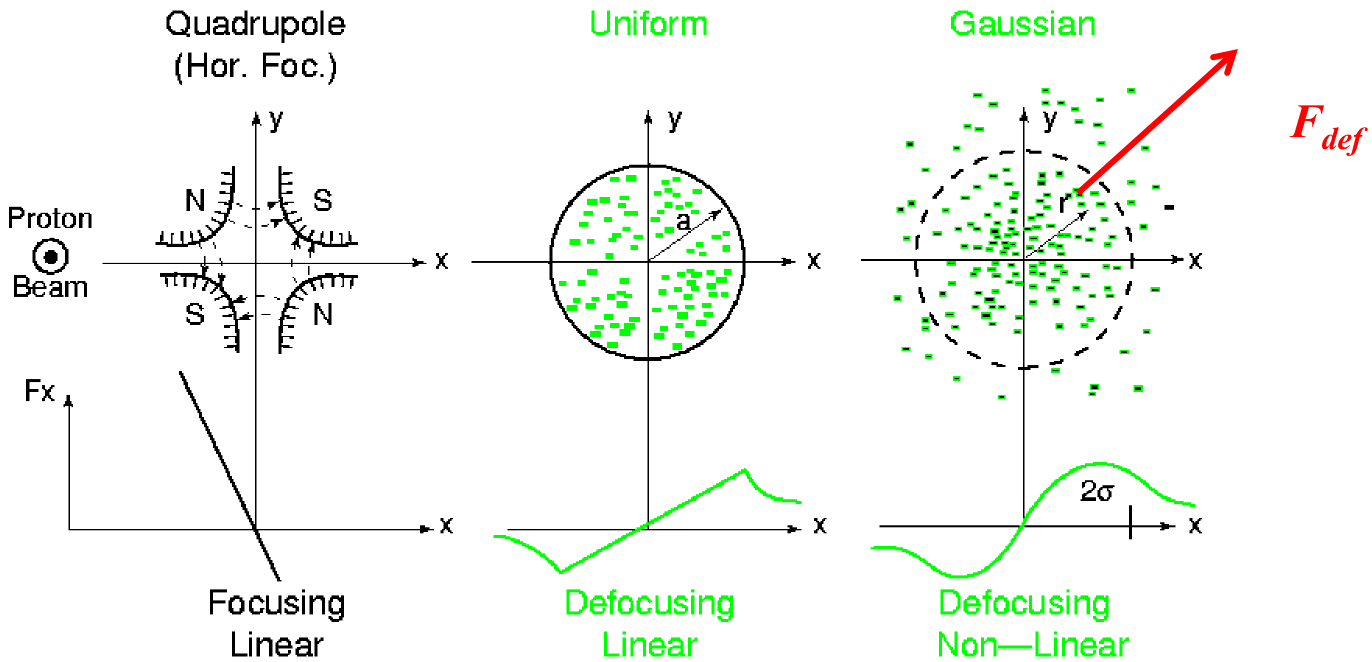
Tune diagram up to 3rd order

… and up to 7th order



Homework for the operateurs:
find a nice place for the tune
where against all probability
the beam will survive

II.) Golden Rule number two: *Never accelerate charged particles !*



Transport line with quadrupoles

$$x'' + K(s)x = 0$$

Transport line with quadrupoles and space charge

$$x'' + (K(s) + K_{SC}(s))x = 0$$

$$x'' + \left(K(s) - \frac{2r_0 I}{ea^2\beta^3\gamma^3 c} \right)x = 0$$

K_{SC}

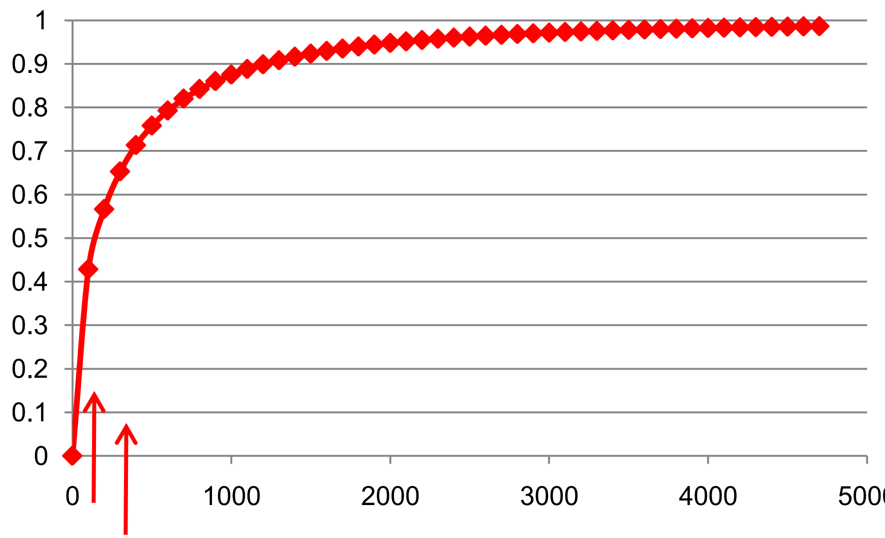
Golden Rule number two: ***Never accelerate charged particles !***

Tune Shift due to Space Charge Effect

Problem at low energies

$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi \varepsilon_{x,y} \beta \gamma^2}$$

v/c



Linac 2 E_{kin}*=60 MeV*

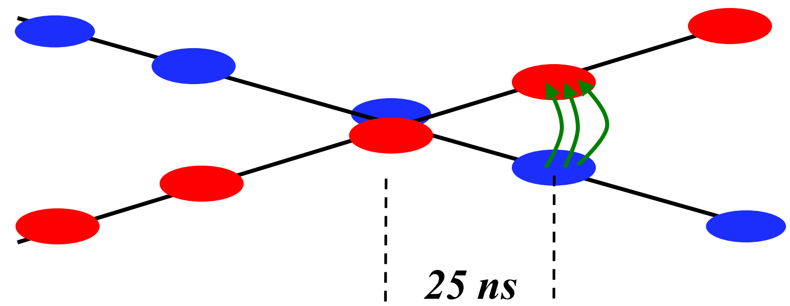
Linac 4 E_{kin}*=150 MeV*

E_{kin} *of a proton*

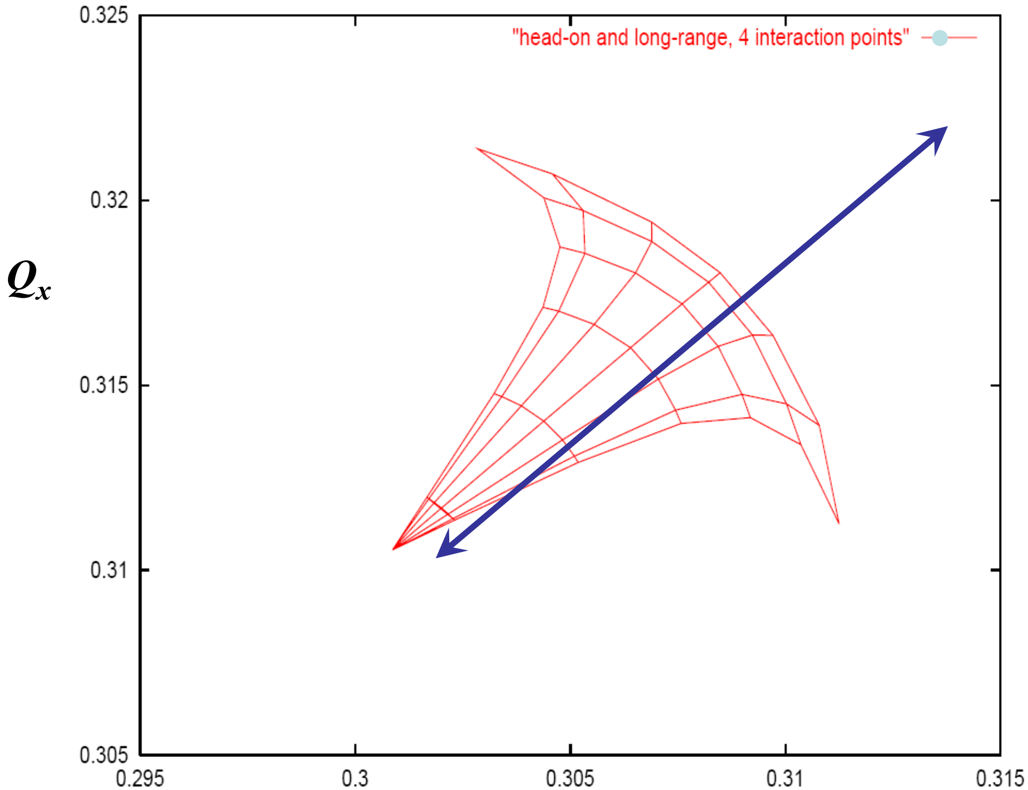
... at low speed the particles repel each other

III.) Golden Rule number three:

Never Collide the Beams !



the colliding bunches influence each other
→ change the focusing properties of the ring !!

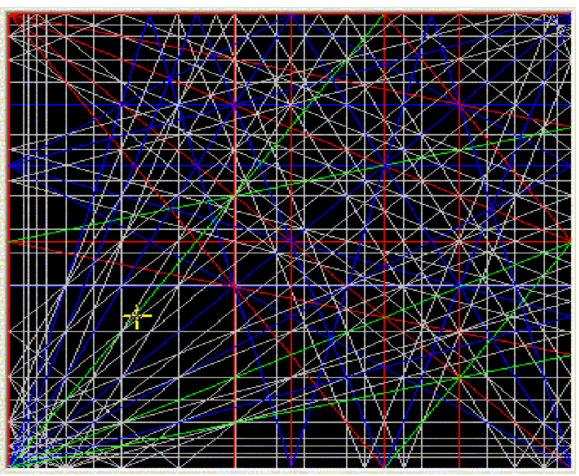


Courtesy W. Herr Q_x

most simple case:
linear beam beam tune shift

$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi\gamma_p(\sigma_x + \sigma_y) * \sigma_x}$$

and again the resonances !!!



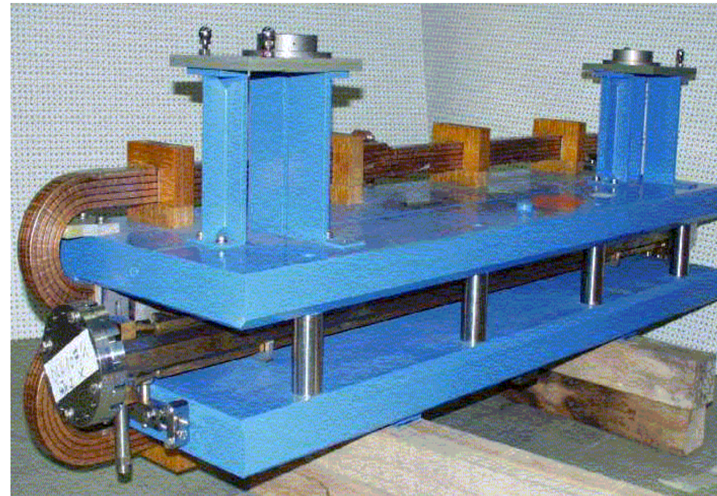
! --

IV.) Golden Rule Number 4: Never use Magnets

! *************
! -------------
! bn at injection

```
b1M_MQXCD_inj   :=  0.0000 ; b1U_MQXCD_inj   :=   0000
b2M_MQXCD_inj   :=  0.0000 ; b2U_MQXCD_inj   :=   0000
b3M_MQXCD_inj   :=  0.0000 ; b3U_MQXCD_inj   :=   8900
b4                                           :=   6400
b5                                           :=   4600
b6                                           :=   2800
b7                                           :=   2100
b8                                           :=   1600
b9M_MQXCD_inj   :=  0.0000 ; b9U_MQXCD_inj   :=   0800
b10M_MQXCD_inj  :=  0.5000 ; b10U_MQXCD_inj  :=   0600
b11M_MQXCD_inj  :=  0.0000 ; b11U_MQXCD_inj  :=   0300
b12M                         12U_MQXCD_inj   :=   0200
b13M                         13U_MQXCD_inj   :=   0100
b14M_MQXCD_inj  := -0.2700 ; b14U_MQXCD_inj  :=  0.0300 ; b14R_MQXCD_inj  :=  0.0100
b15M_MQXCD_                  b15U_MQXCD_inj  :=  0.0000 ; b15R_MQXCD_inj  :=  0.0000
```

$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$

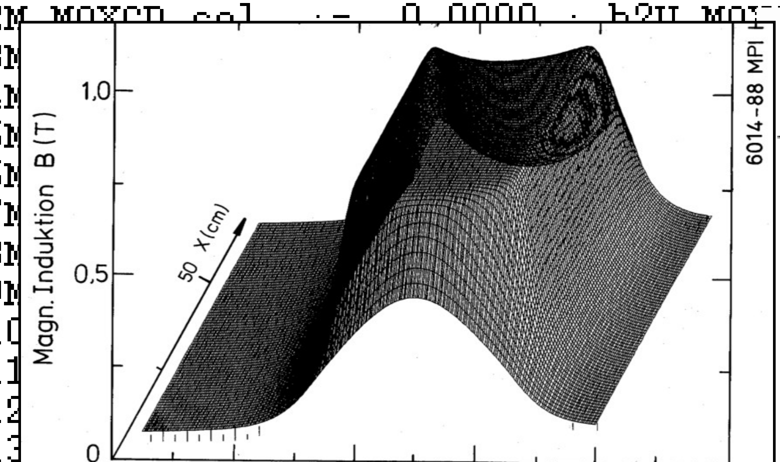


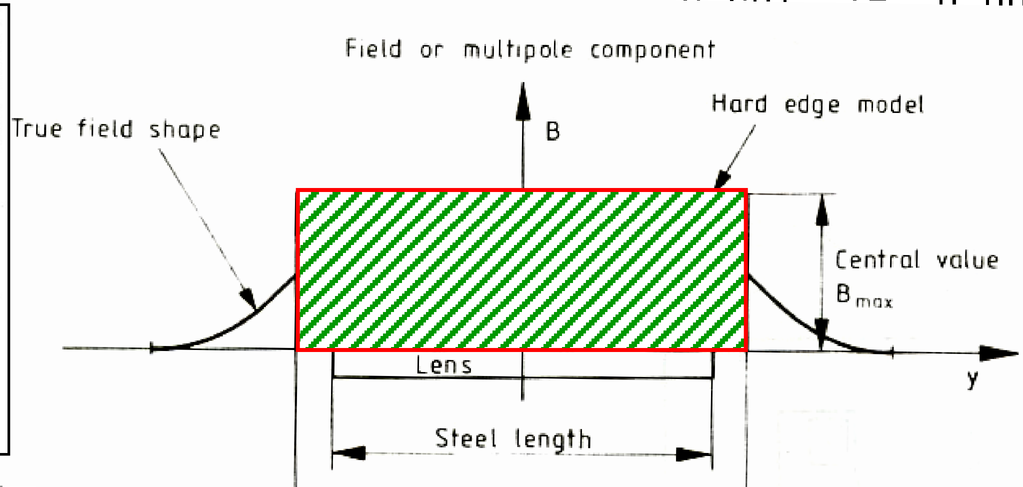
"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B ds$$

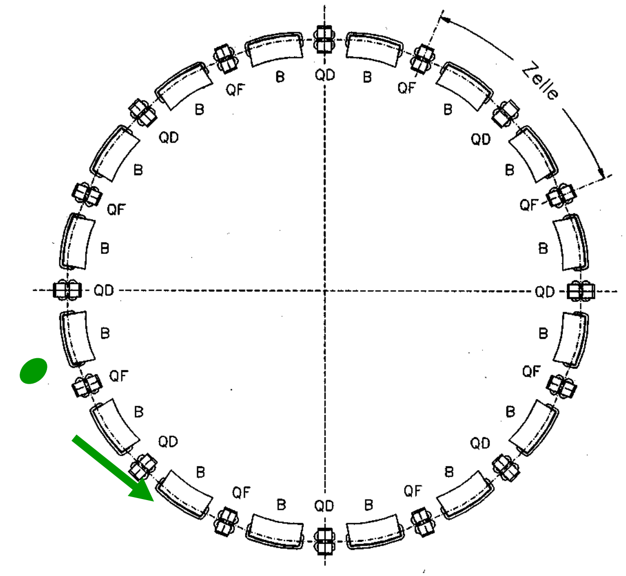
! bn in collision

```
b1M_MQXCD_c                  b1U_MQXCD_col   :=  0.0000 ; b1R_MQXCD_col   :=  0.0000
b2M_MQXCD_col   :=  0.0000 ; b2U_MQ                                       :=  0.0000
b3M
b4M
b5M
b6M
b7M
b8M
b9M
b10
b11
b12
b13
b14M_MQXCD_col  :=  0.0000 ; b14U_M
b15M_MQXCD_col  :=  0.0000 ; b15U_MQXCD_col  :=  0.0000 ; b15R_MQXCD_col  :=  U.UUUU
```





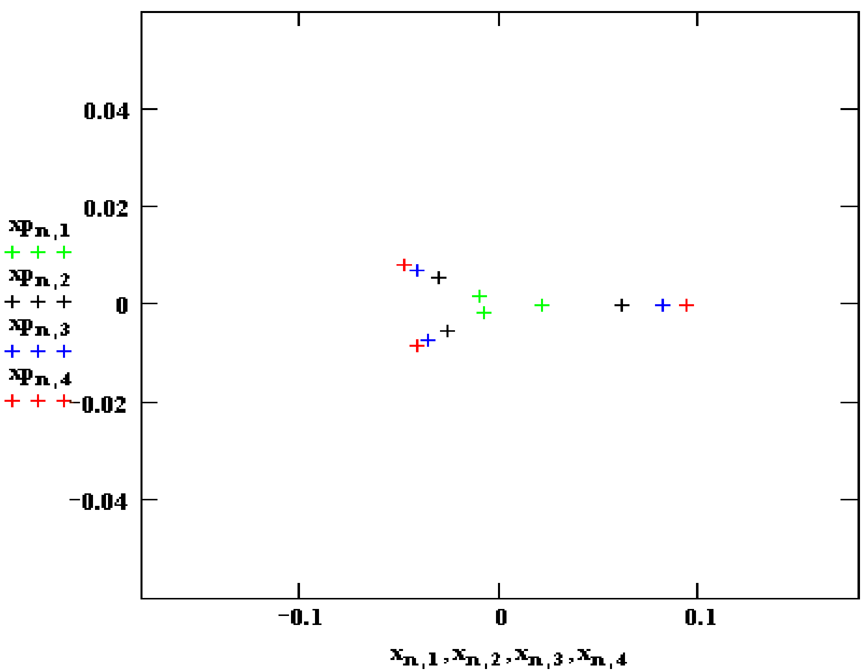
Clearly there is another problem ...
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

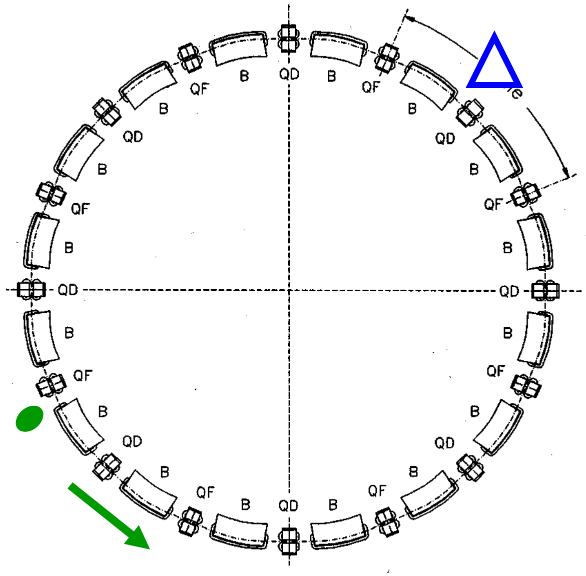


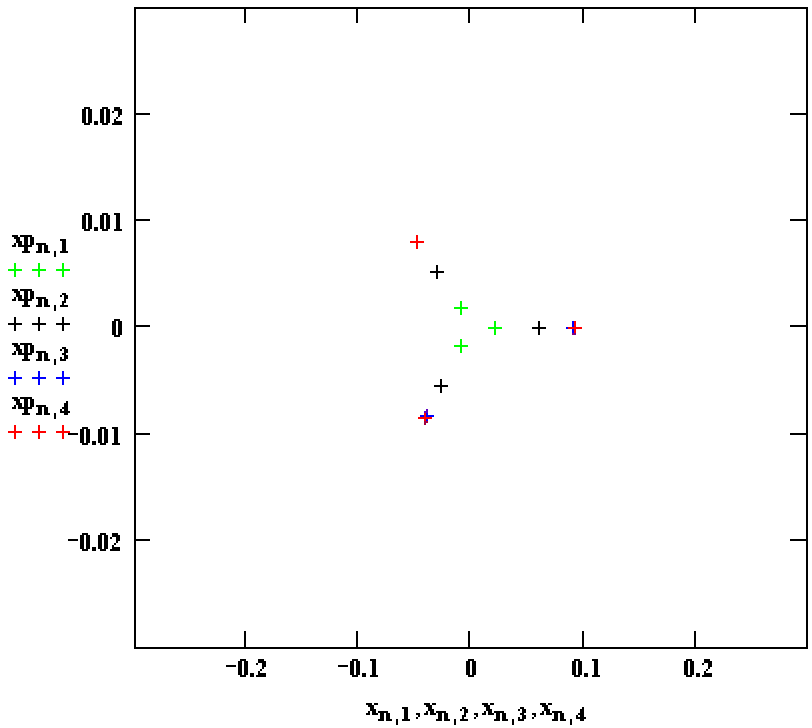
A beam of 4 particles

– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

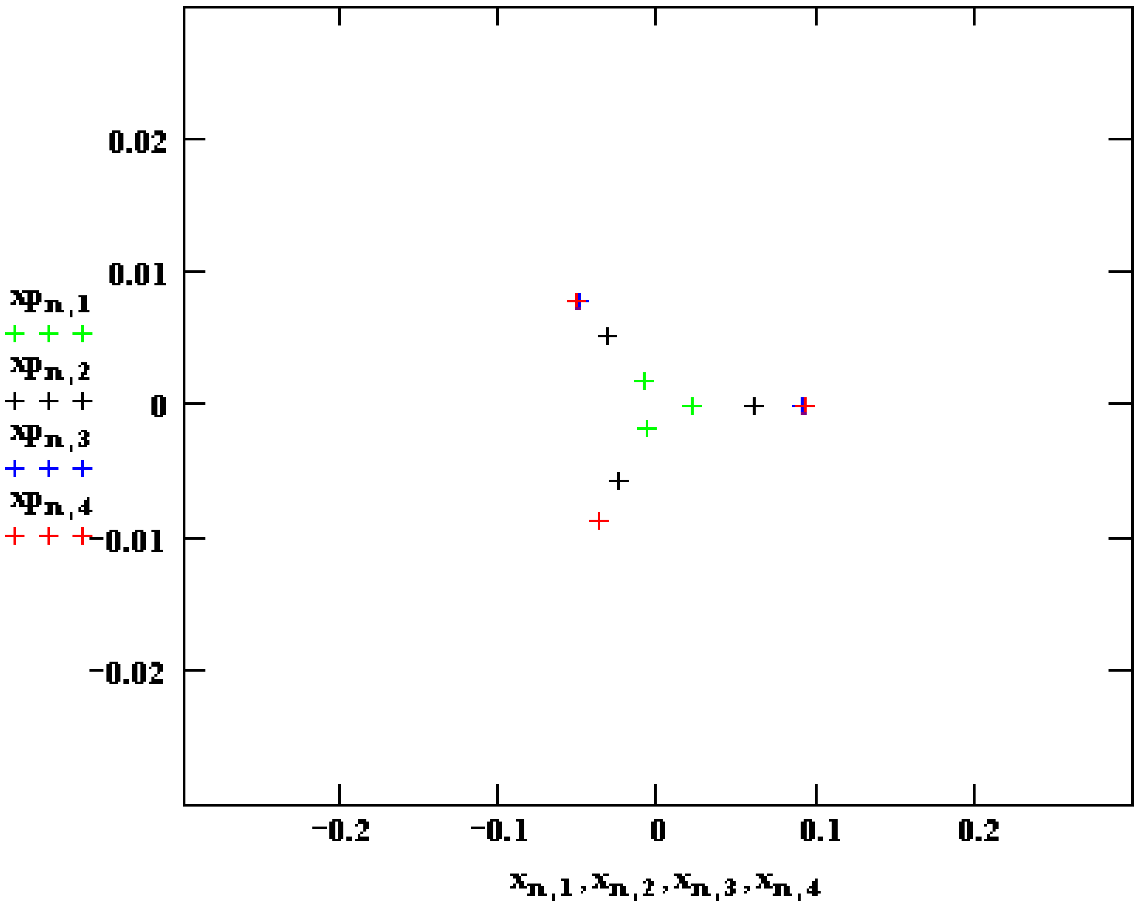
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation
„ particle tracking "

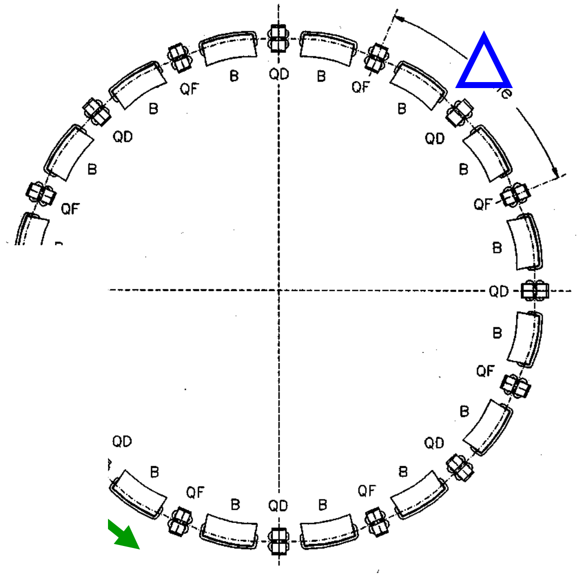




Effect of a strong (!!!) Sextupole …

→ *Catastrophy !*

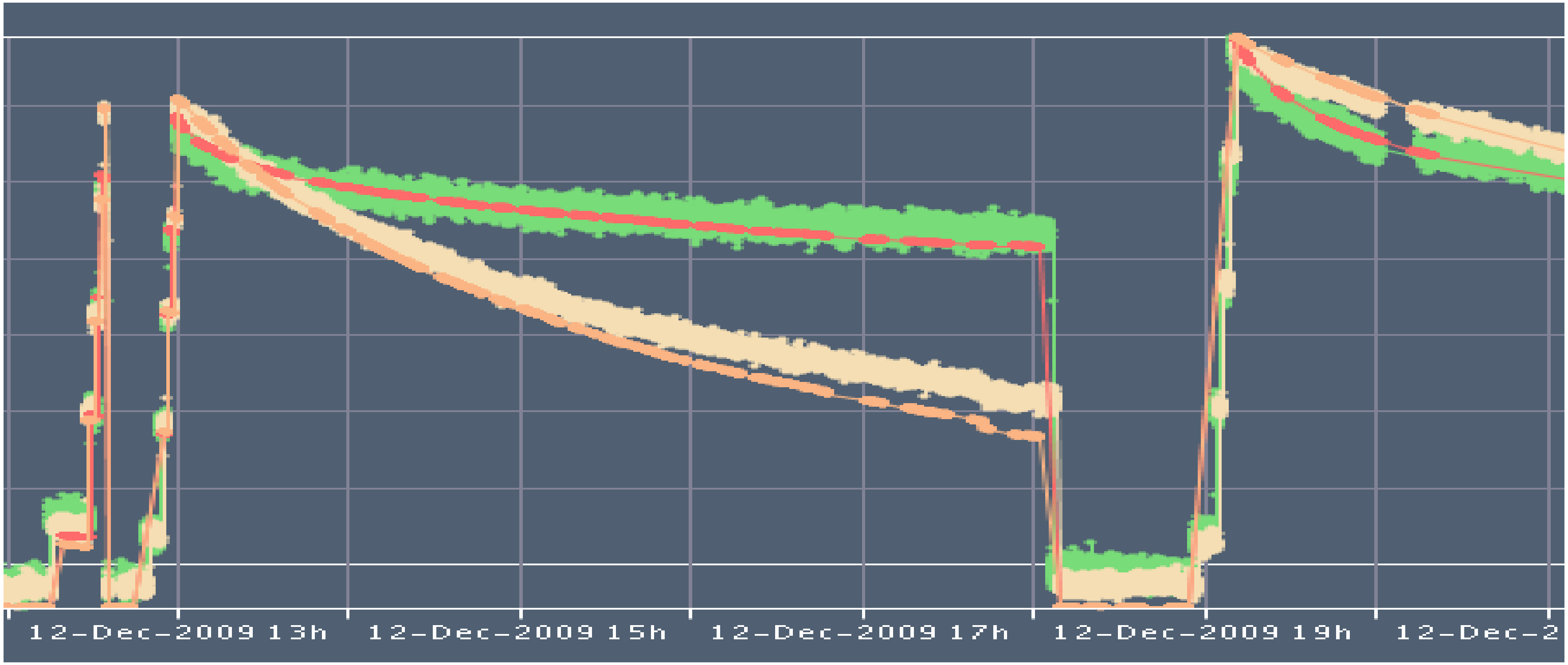




„dynamic aperture“

Golden Rule XXL: COURAGE

and with a lot of effort from Bachelor / Master / Diploma / PhD and Summer-Students the machine is running !!!



thank'x for your help and have a lot of fun

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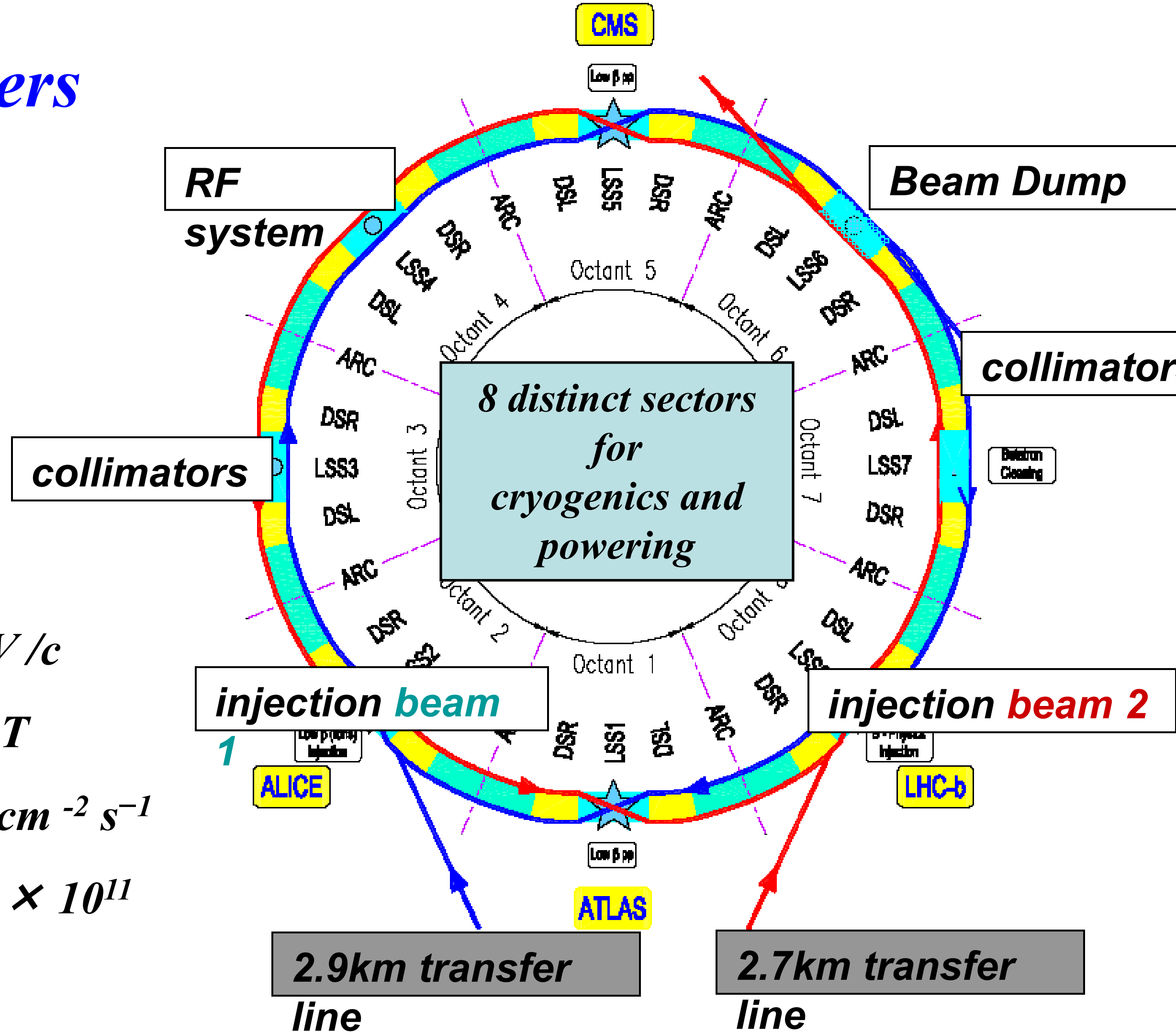
V.) Accelerator Operation

Bernhard Holzer
CERN-LHC

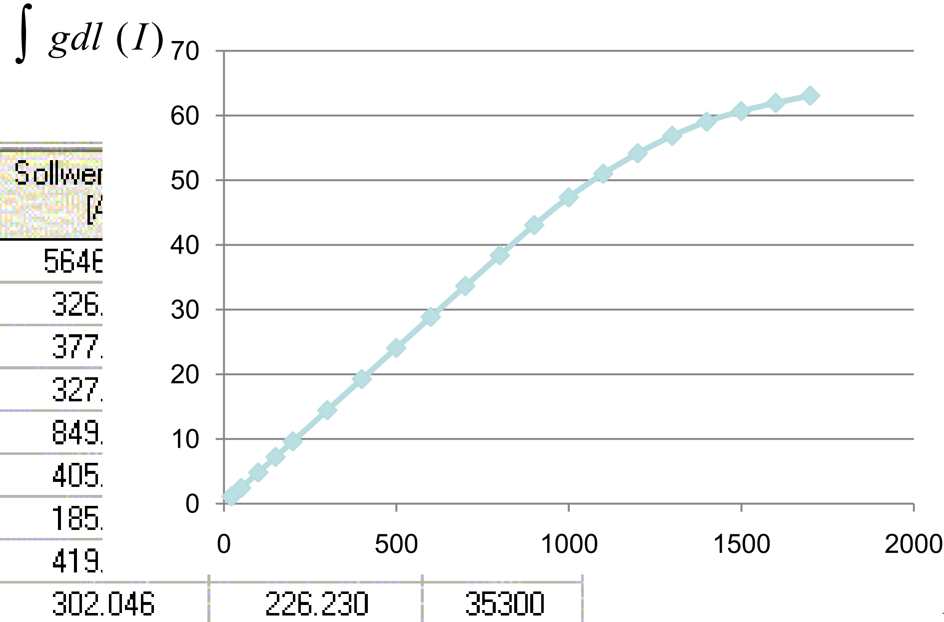
*

LHC Main Parameters

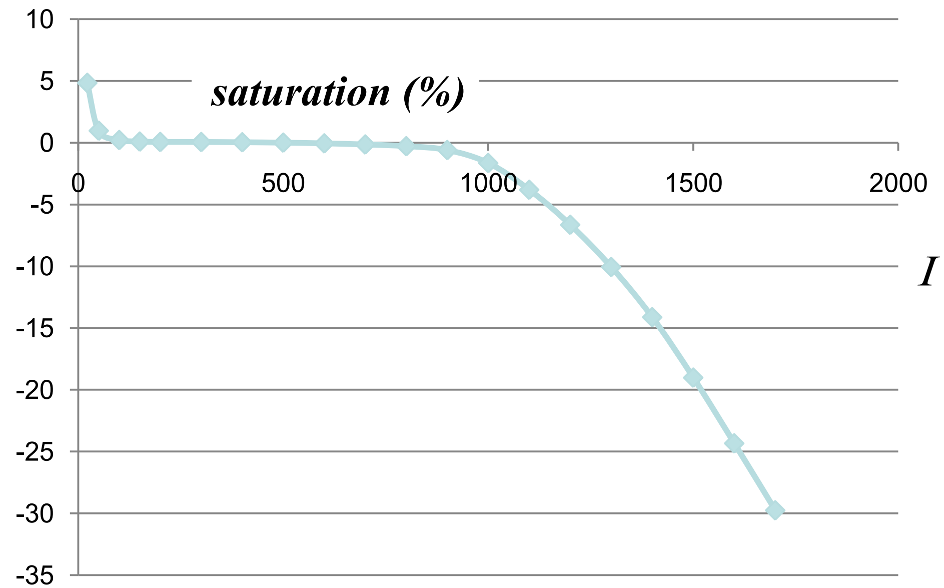


Momentum at collision	*7 TeV /c*
Dipole field for 7 TeV	*8.33 T*
Luminosity	10^{34} *cm* $^{-2}$ *s* $^{-1}$
Protons per bunch	1.15×10^{11}
Number of bunches/beam	*2808*
Nominal bunch spacing	*25 ns*
Normalized emittance	*3.75 μm*
rms beam size (7TeV, arc)	*300 μm*
beam pipe diameter	*56 mm*

Magnet Currents



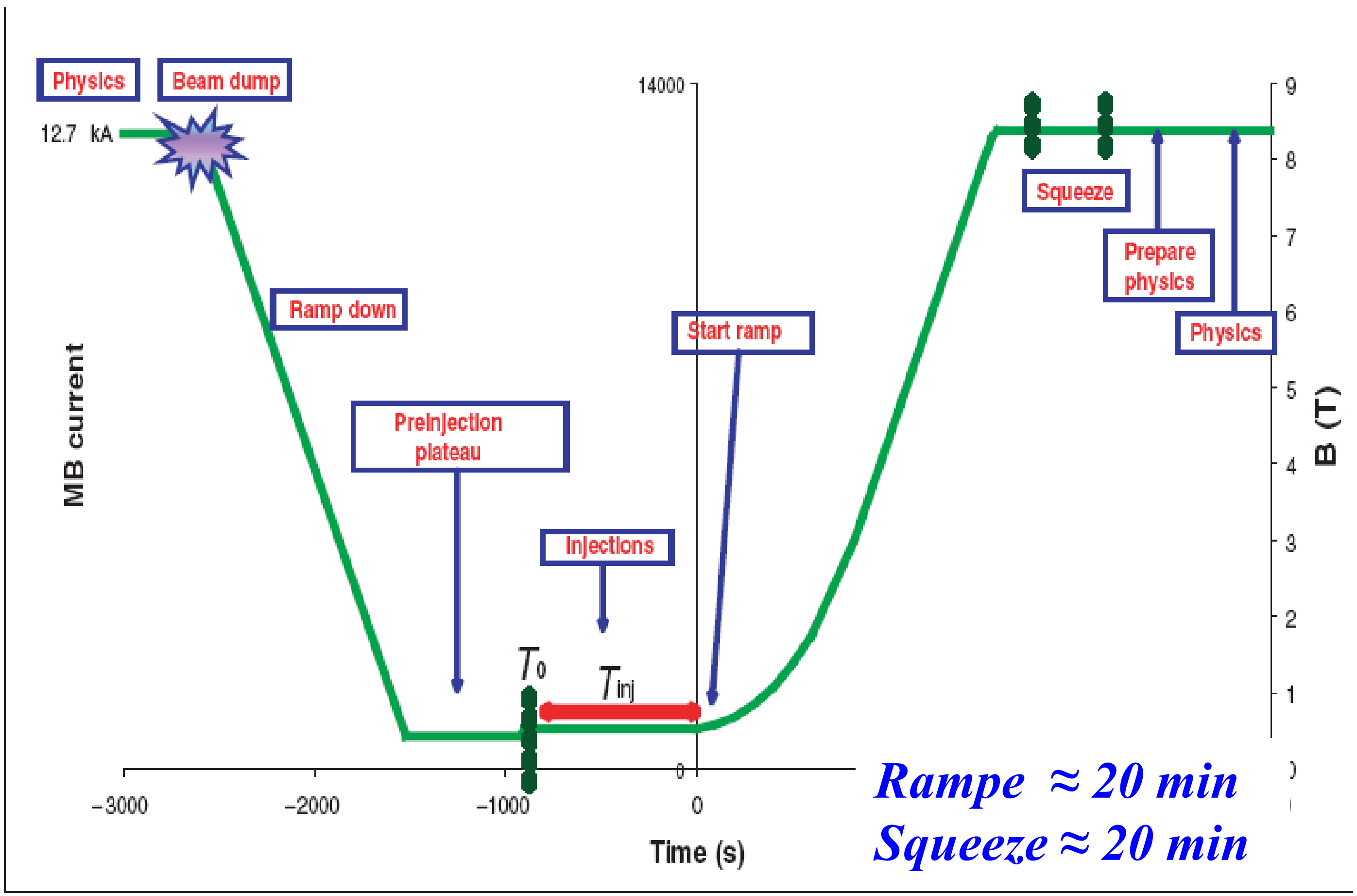
Nummer	Gruppe	Name	aktiv	Sollwerte File1 [A]	Sollwer[te] [A]		
1	HPDIPOL	BPA1	True	4138.993	5646		
2	HPMAINW	QZ51 WL	True	235.462	326.		
3	HPMAINW	QR52 WR	True	258.724	377.		
4	HPMAINW	QC53 WL	True	237.933	327.		
5	HPMAINW	QB28 WL	True	625.429	849.		
6	HPMAINW	QR54 WR	True	291.486	405.		
7	HPMAINW	QR24 WR	True	139.139	185.		
8	HPMAINW	QR50 WL	True	305.348	419.		
9	HPMAINW	QC22 WR	True	75.816	302.046	226.230	35300
10	HPMAINW	QR57 WL	True	260.769	354.833	94.064	12329
11	HPMAINW	QR56 WR	True	190.123	263.722	73.599	11484
12	HPMAINW	QC20 WR	True	91.056	-13.587	-104.643	-16328
13	HPMAINW	QP58 WR	True	-5.517	19.		
14	HPMAINW	QP59 WL	True	-10.401	-11.		
15	HPMAINW	QP60 WR	True	73.600	98.		
16	HPMAINW	QP61 WL	True	69.504	90.		
17	HPMAINW	QP62 WR	True	40.163	58.		
18	HPMAINW	QP63 WL	True	47.489	63.		
19	HPMAINW	QP64 WR	True	-47.780	-71.		

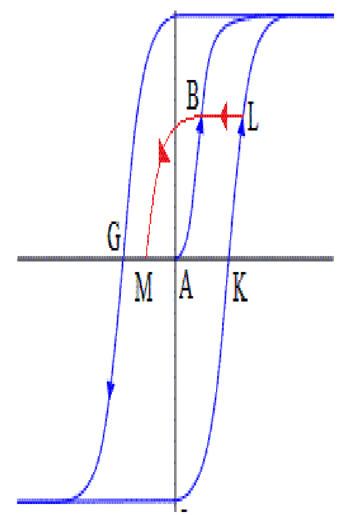


remember: $\Delta B/B \approx 10^{-4}$

LHC Operation: Magnet Preparation Cycle & Ramp

***8 independent sectors**, hysteresis effects, **saturation & remanence in nc and sc magnets**, synchronisation of the power converters, **magnet model** to describe the transfer functions of every element*



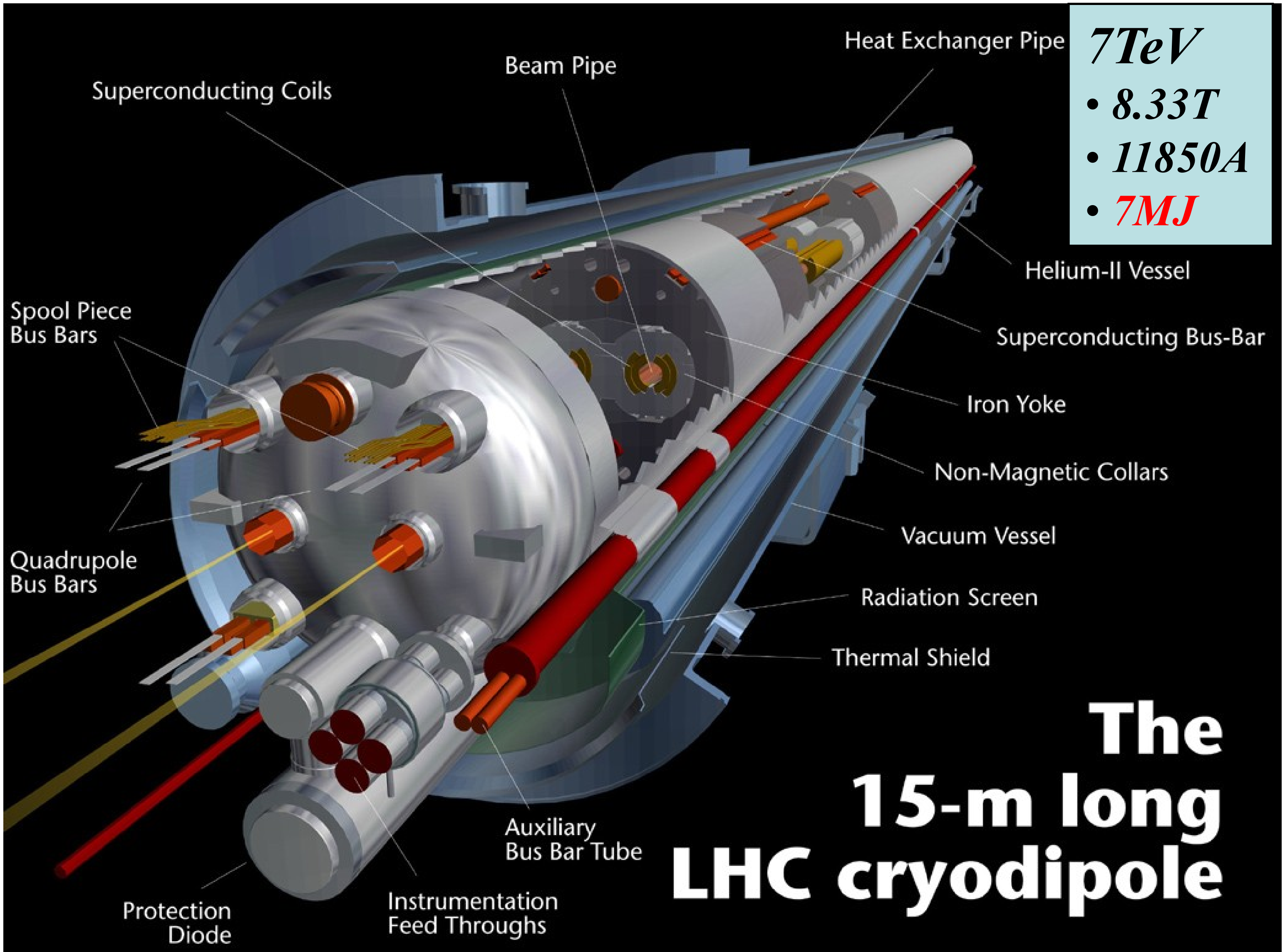


magnet hysteresis

LHC dipoles (1232 of them)

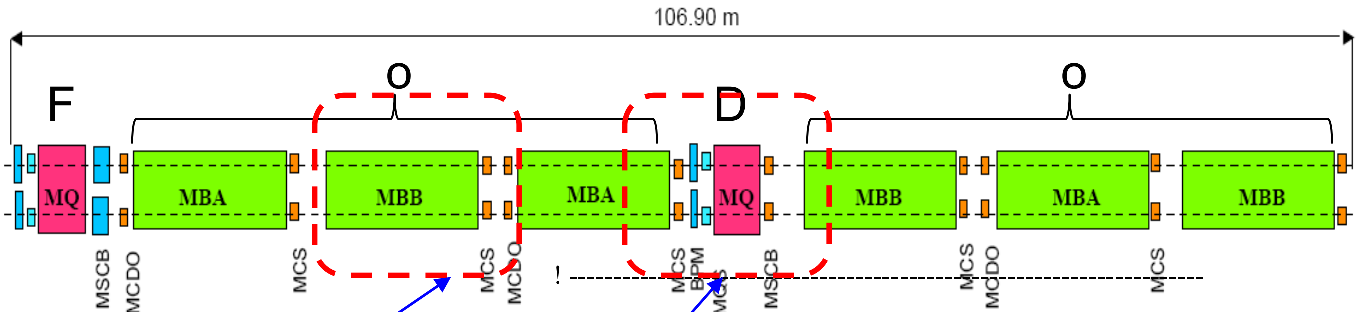
7TeV

- *8.33T*
- *11850A*
- *7MJ*



LHC: Basic Layout of the Machine
multipole corrector magnets

2, 6, 8, 10, 12 pol
skew & trim quad, chroma 6pol
landau 8 pole



! ---

! ********Magnet type : MQXC/MQXD (new Inner Triplet Quad)***************

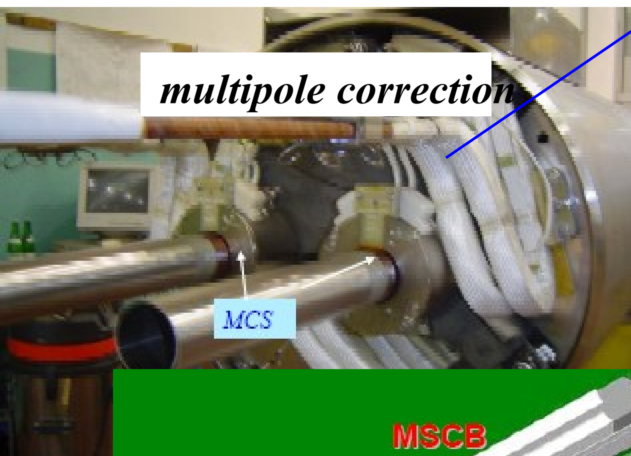
! ---

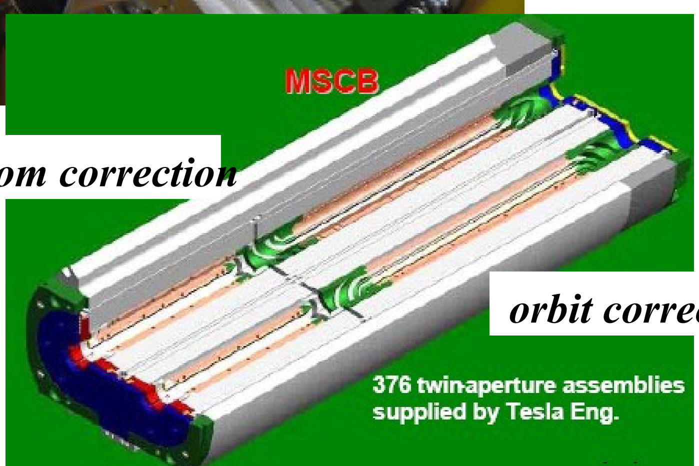
LHC TDR

bn in collision

b1M_MQXCD_col := 0.0000 ; b1U_MQXCD_col := 0.0000 ; b1R_MQXCD_col := 0.0000 ;
b2M_MQXCD_col := 0.0000 ; b2U_MQXCD_col := 0.0000 ; b2R_MQXCD_col := 0.0000 ;
b3M_MQXC ... 0.8900 ;
b4M_MQXC ... 0.6400 ;
b5M_MQXC ... 0.4600 ;
b6M_MQXC ... 1.2800 ;
b7M_MQXC ... 0.2100 ;
b8M_MQXC ... 0.1600 ;
b9M_MQXC ... 0.0800 ;
b10M_MQX ... 0.0600 ;
...M_MQX ... 0.0300 ;
...M_MQX ... 0.0200 ;
b13M_MQX ... 0.0100 ;
b14M_MQX... 0.0100 ;
b15M_MQXCD_col := 0.0000 ; b15U_MQXCD_col := 0.0000 ; b15R_MQXCD_col := 0.0000 ;

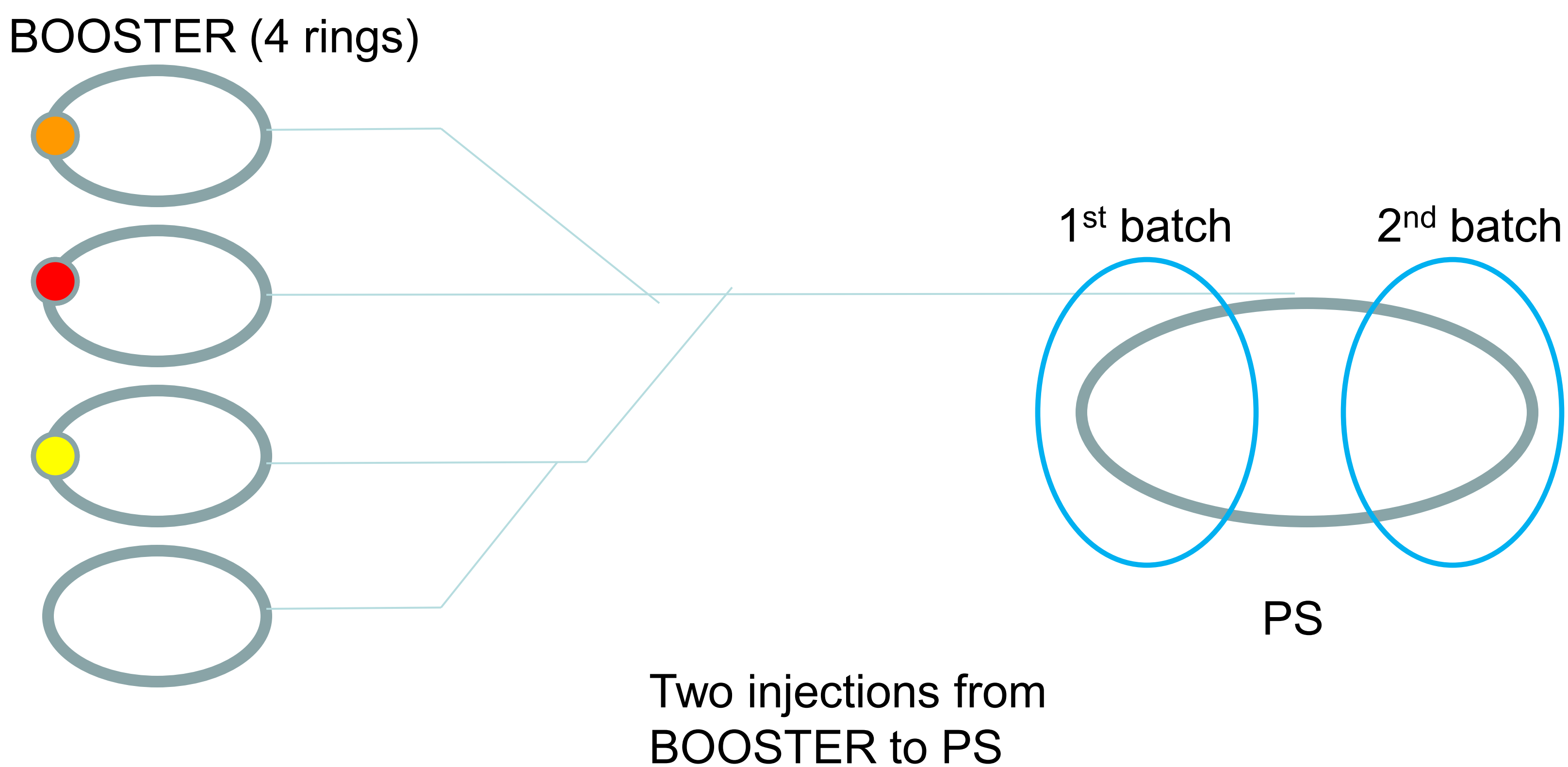
MB: main dipole
MQ: main quadrupole
MQT: Trim quadrupole
MQS: Skew trim quadrupole
MO: Lattice octupole (Landau damping)
MSCB: Normal & Skew sextupole
Orbit corrector dipoles
MCS: Spool piece sextupole
MCDO: Spool piece 8 / 10 pole
BPM: Beam position monitor + diagnostics





LHC Operation: Pre-Accelerators and Injection

BOOSTER (1.4 GeV) ➔ PS (26 GeV) ➔ SPS (450 GeV) ➔ LHC

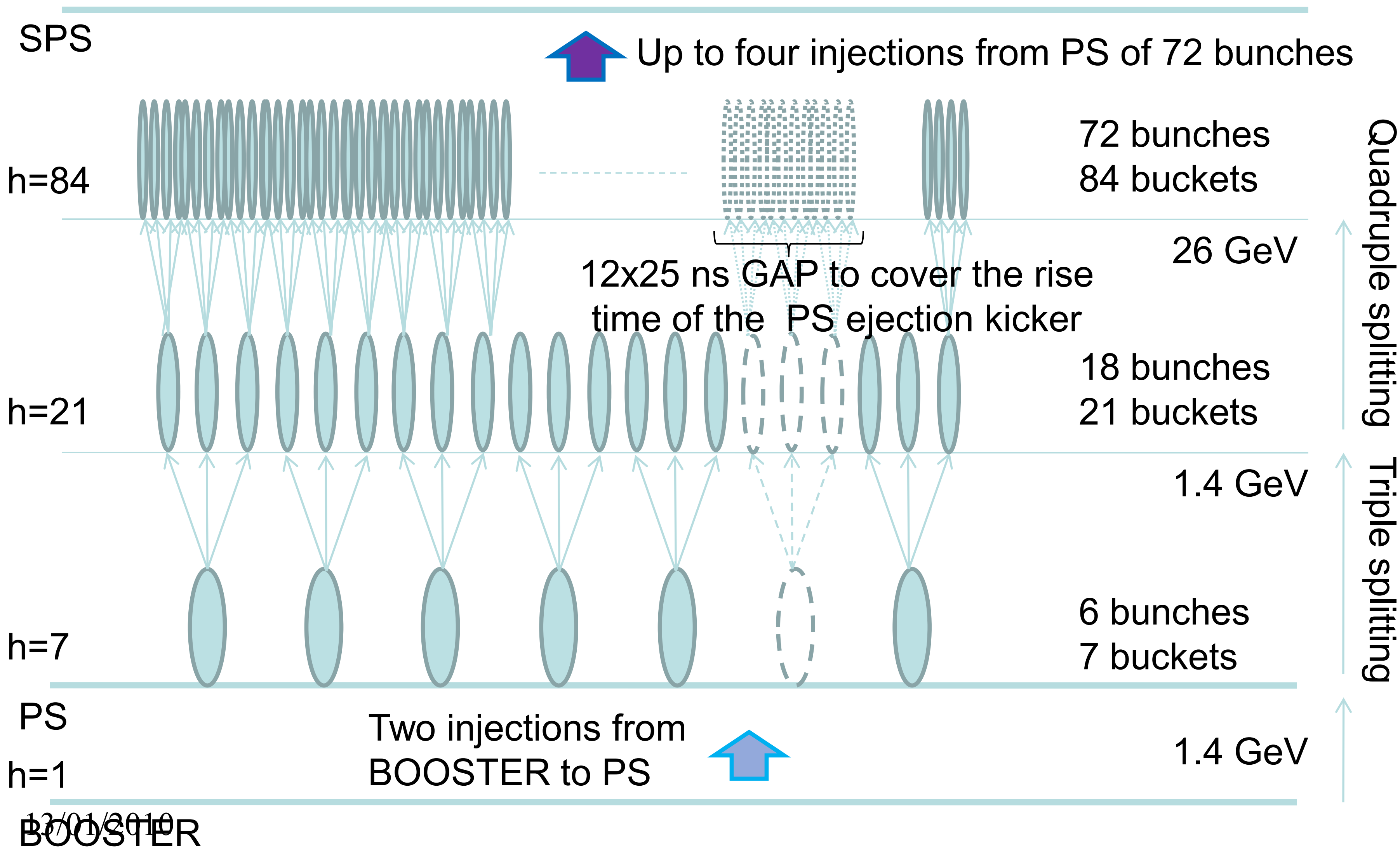


h=1

h=7 (6 buckets filled + 1 empty)

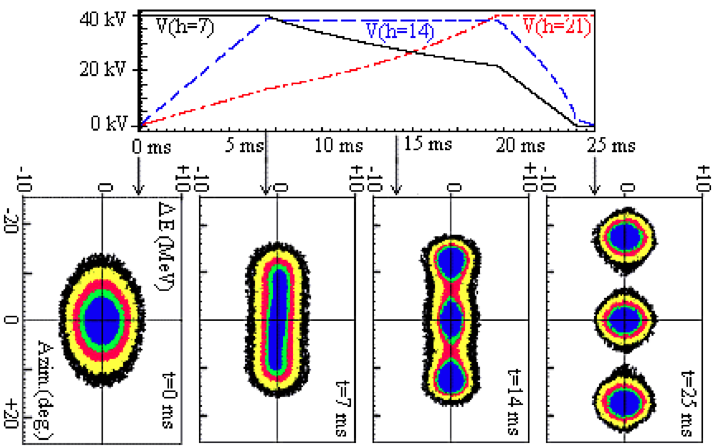
court. R. Alemany

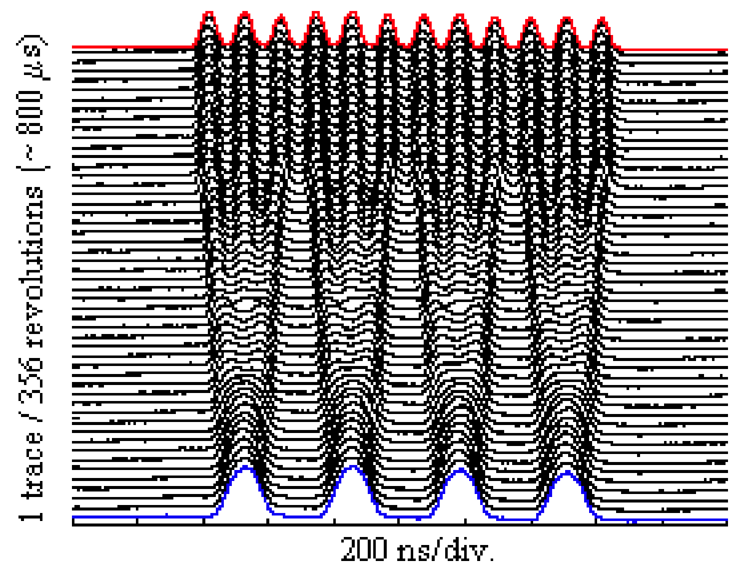
LHC Injection: Preparing the Bunch Trains



Beam Injection

Bunch Splitting in the PS





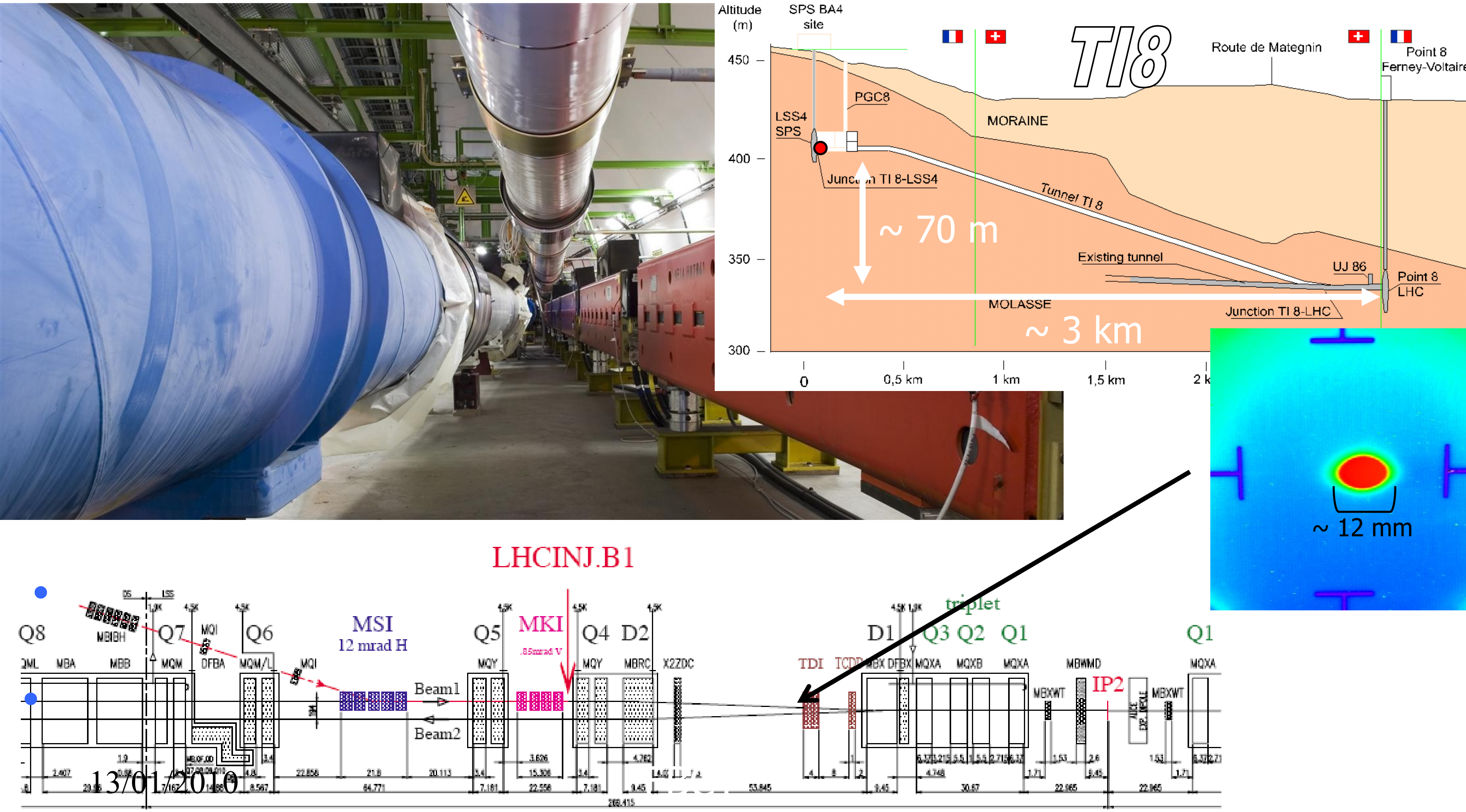
CERN: Linac 2 injection into PSB

$N_p \approx 1.5*10^{13}$ protons per bunch, ***$E_{inj} = 50$ MeV***

$\beta = 0.31$

$\gamma = 1.05$

Injection mechanism: the transfer lines



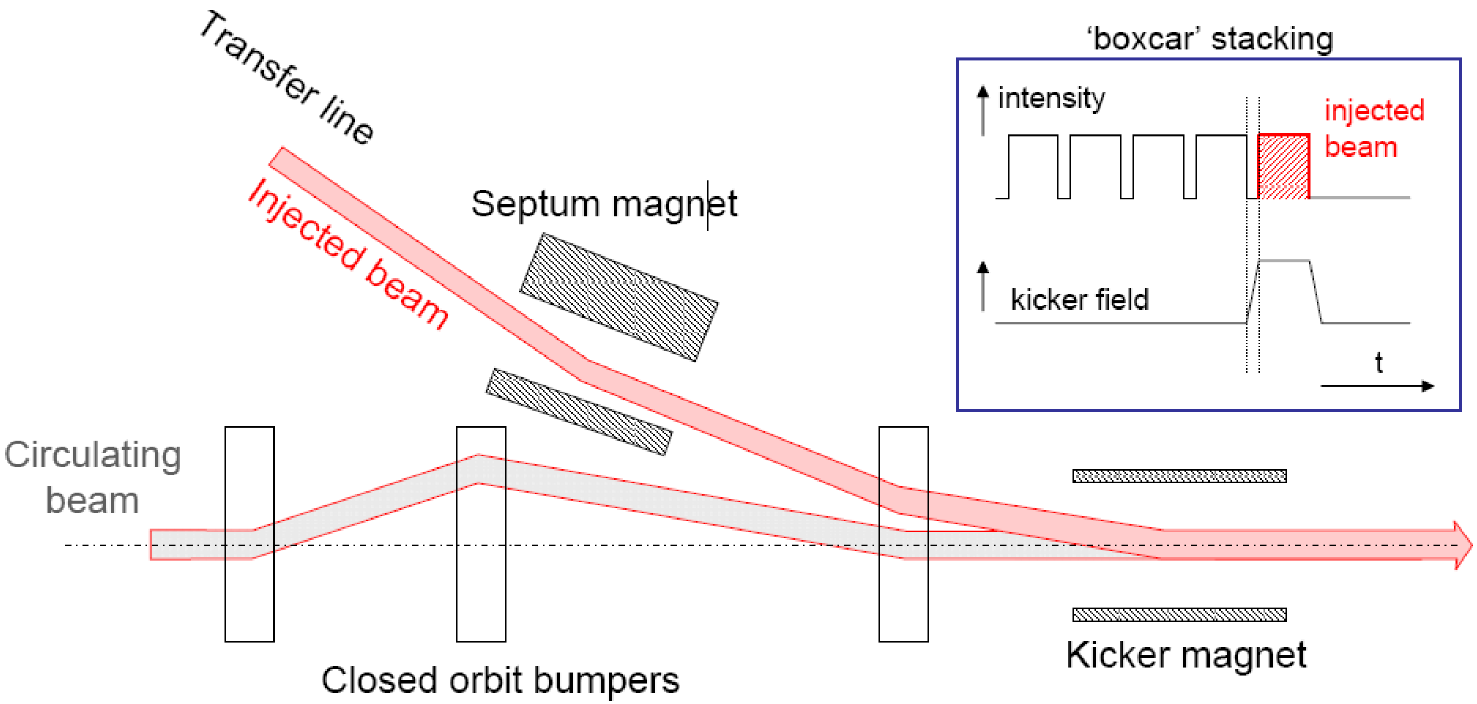
court. R. Alemany

Injection schemes:

Standard Proton Beam ... single turn Injection
Electron Beam .... "off axis" Injection
Ion Beam .... "multi turn" injection

Single Turn Injection

Example: LHC, HERA-P



Transferlines & Injection: Errors & Tolerances

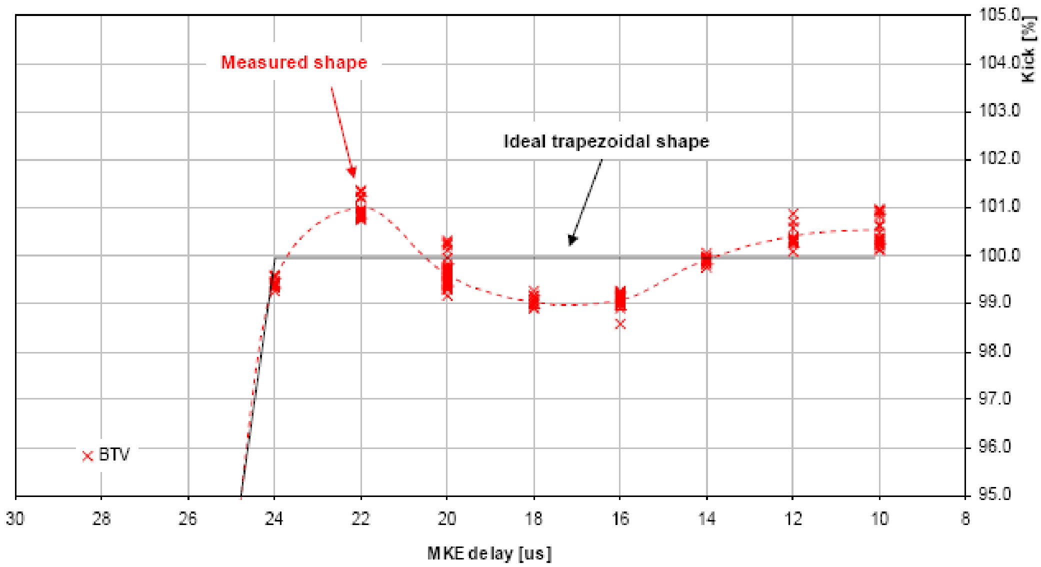
** quadrupole strengths --> "beta beat" $\Delta\beta / \beta$*
** alignment of magnets --> orbit distortion in transferline & storage ring*
** septum & kicker pulses --> orbit distortion & emittance dilution in storage ring*

Example: Error in position Δa:

$$\varepsilon_{new} = \varepsilon_0 * (1 + \frac{\Delta a^2}{2})$$

Δa =0.5 σ

$$\rightarrow \quad \varepsilon_{new} = 1.125 * \varepsilon_0$$



Kicker "plateau" at the end of the PS - SPS transferline measured via injection - oscillations

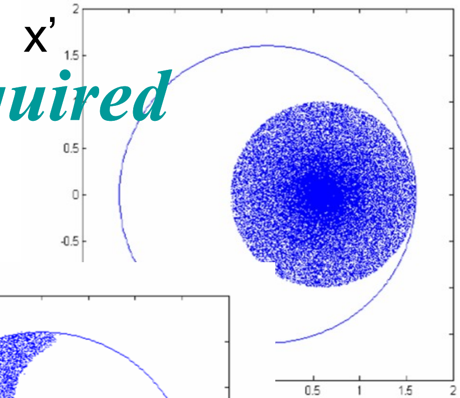
LHC Injection: Again ... high accuracy required

Filamentation

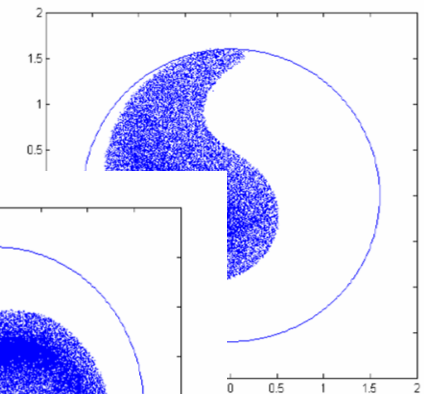
Injection errors (position or angle) dilute the beam emittance

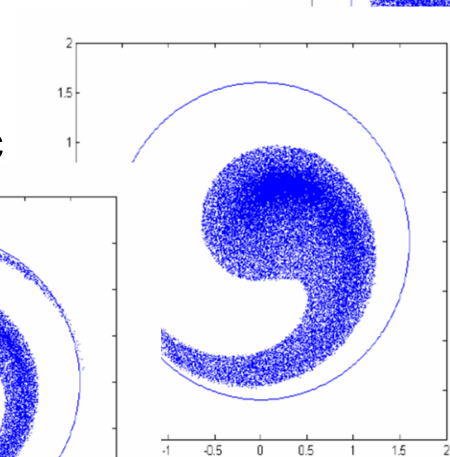
Non-linear effects (e.g. magnetic field multipoles) introduce distort the harmonic oscillation and lead to ampl dependent effects into parti

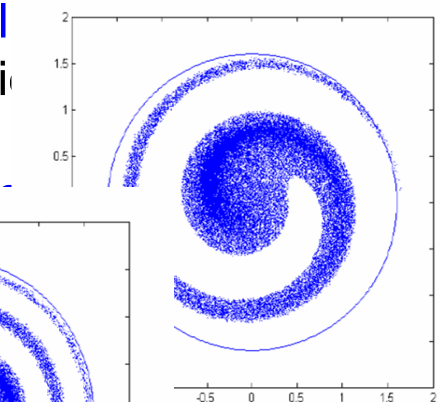
Over many tu
oscillation is t
increase.

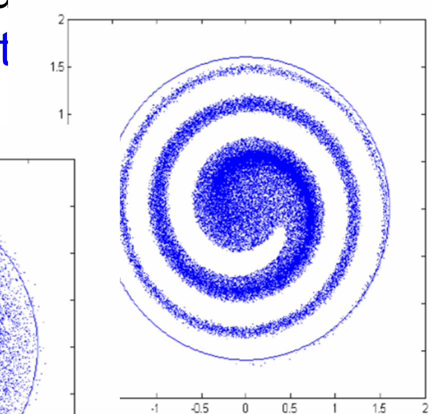


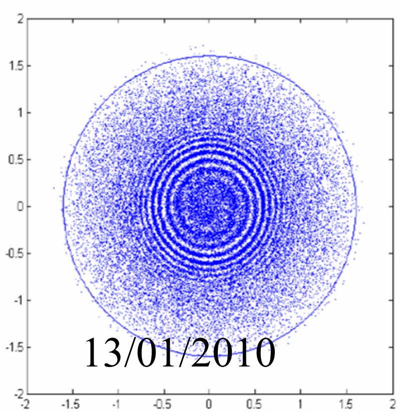
verse
space





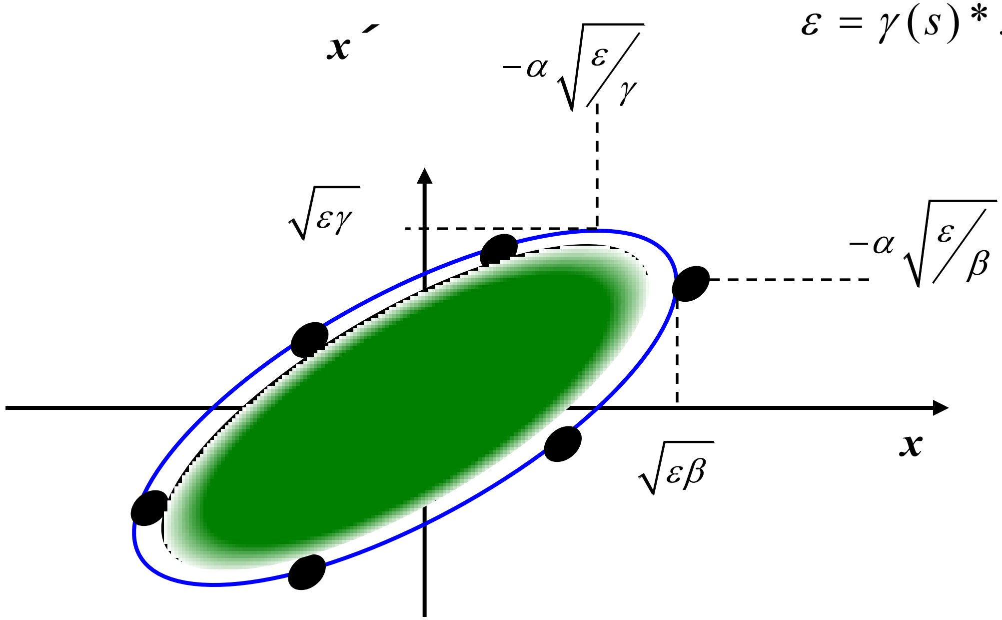




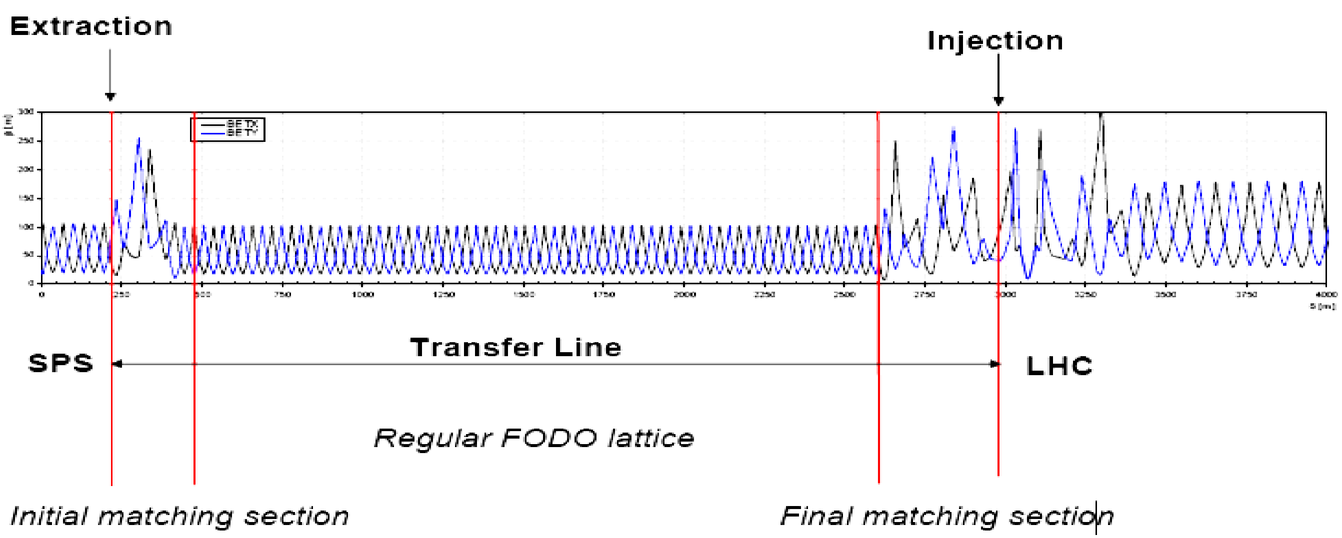


LHC Injection: remember the phase space

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



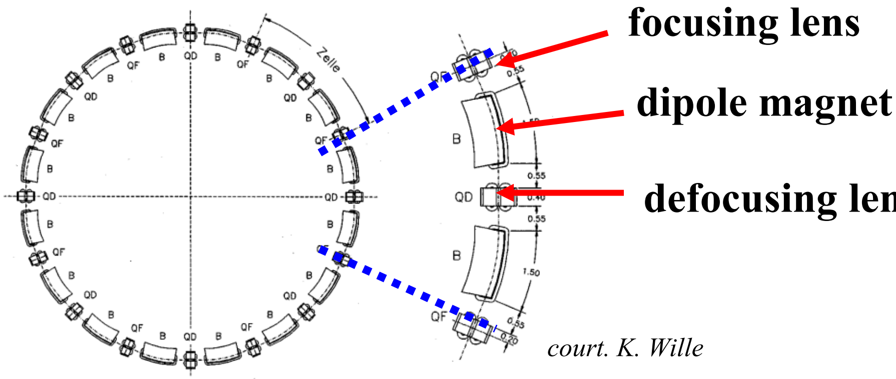
Injected Beam has to be matched to the optics of the storage ring



LHC First Turn Steering

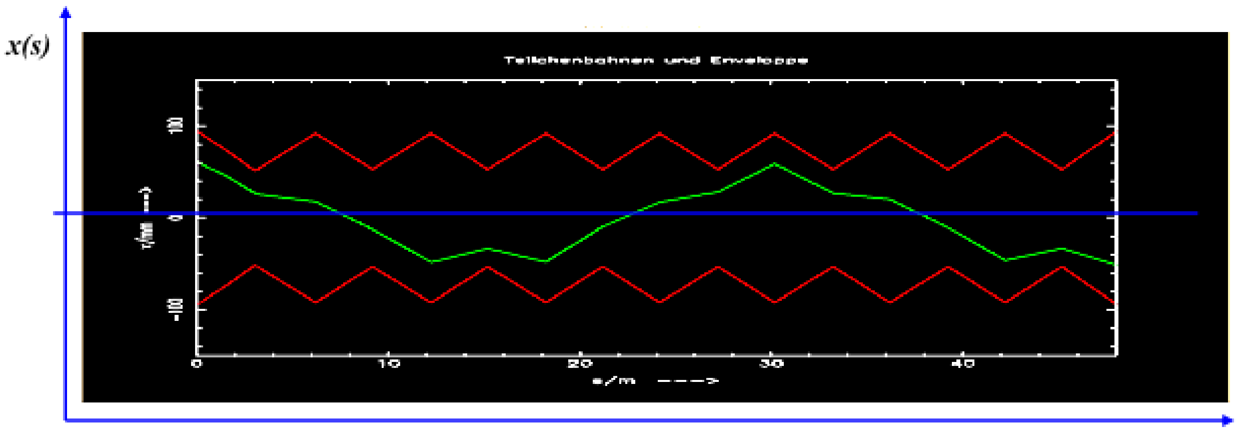
$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



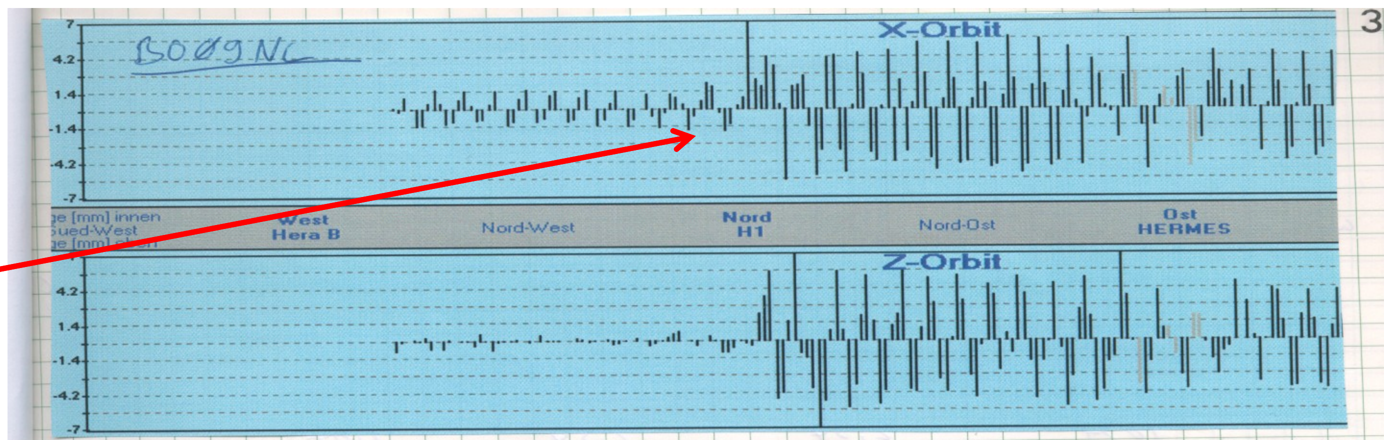
court. K. Wille

in theory
nice harmonic oscillation



in reality:
effect of many localised
orbit distortions

-> correct

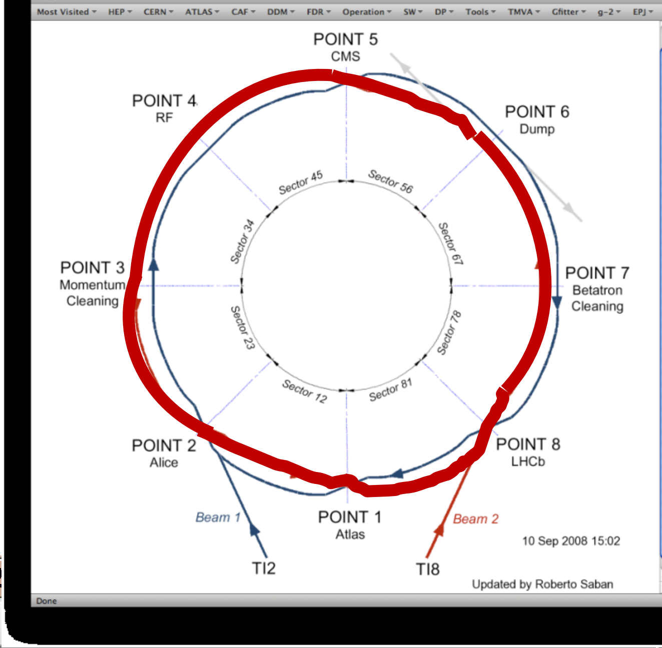


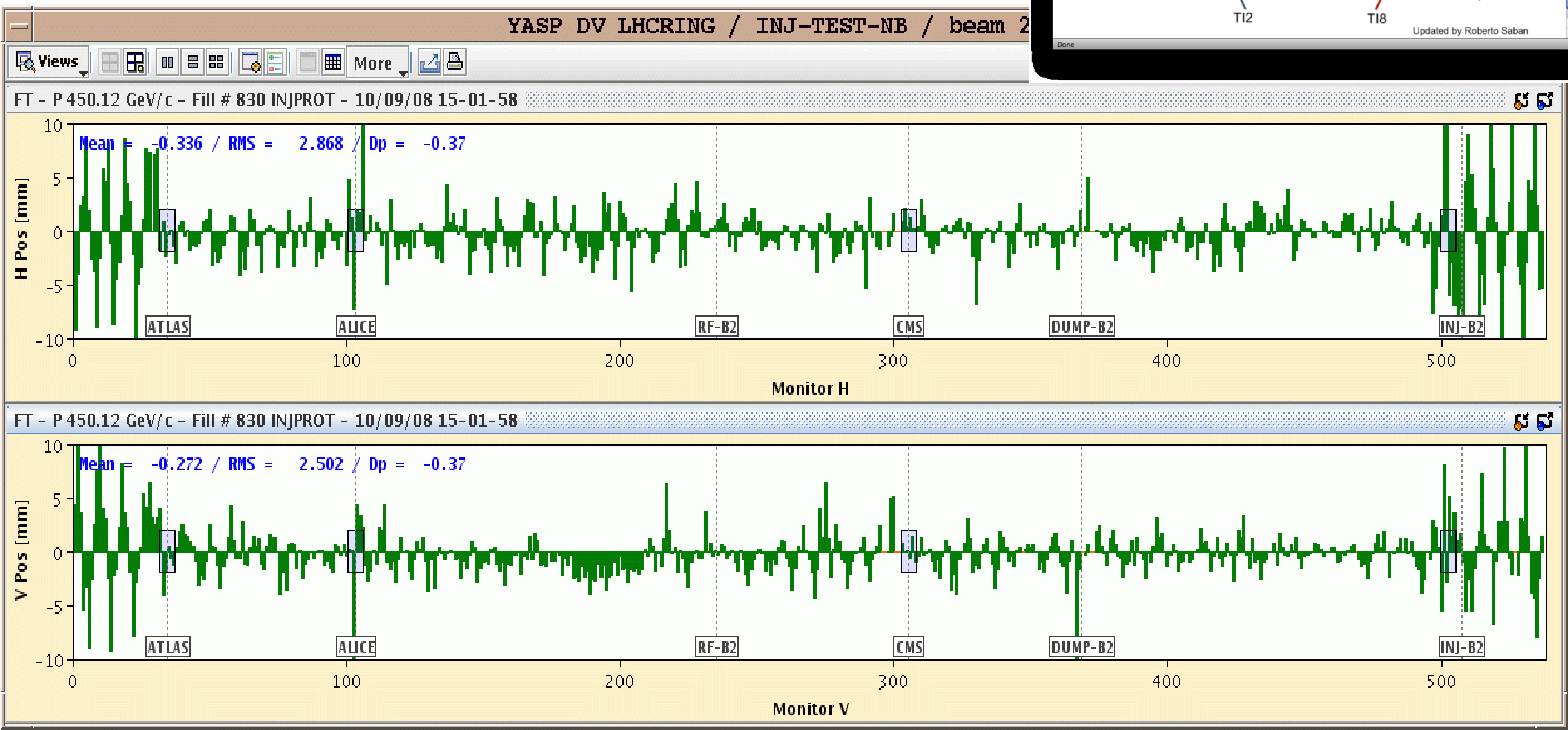
LHC Operation: Beam Commissioning

First turn steering "by sector:"

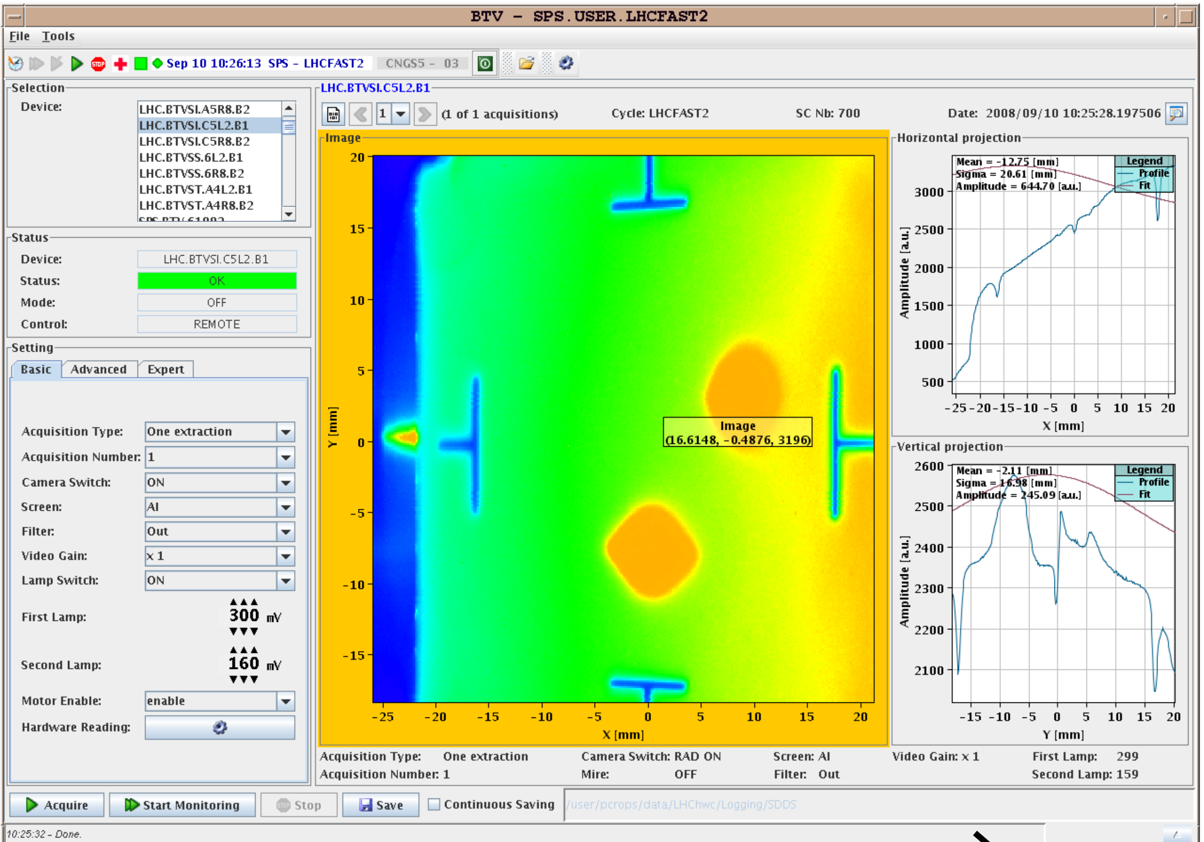
- One beam at the time
- Beam through 1 sector (1/8 ring),

correct trajectory, open collimator and move on.



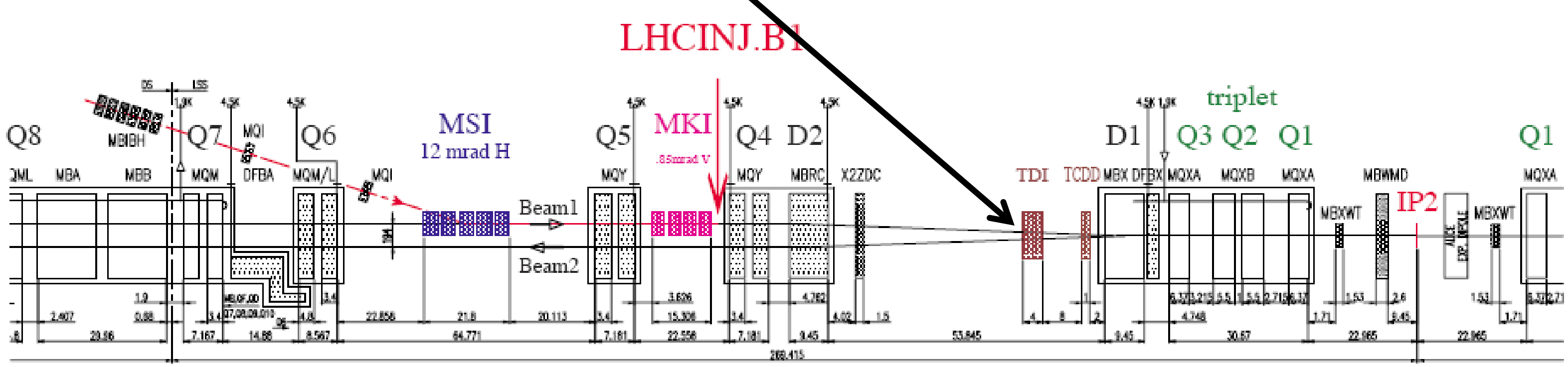


LHC Operation: the First Turn

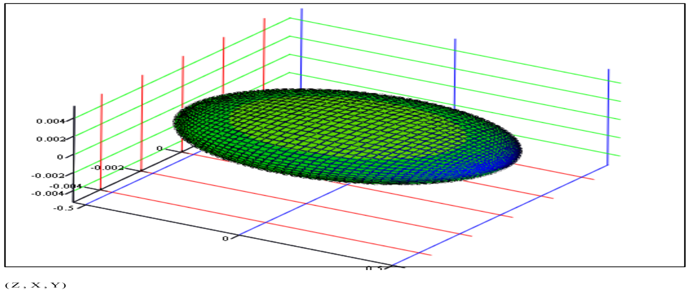


Beam 1 on OTR screen
1st and 2nd turn

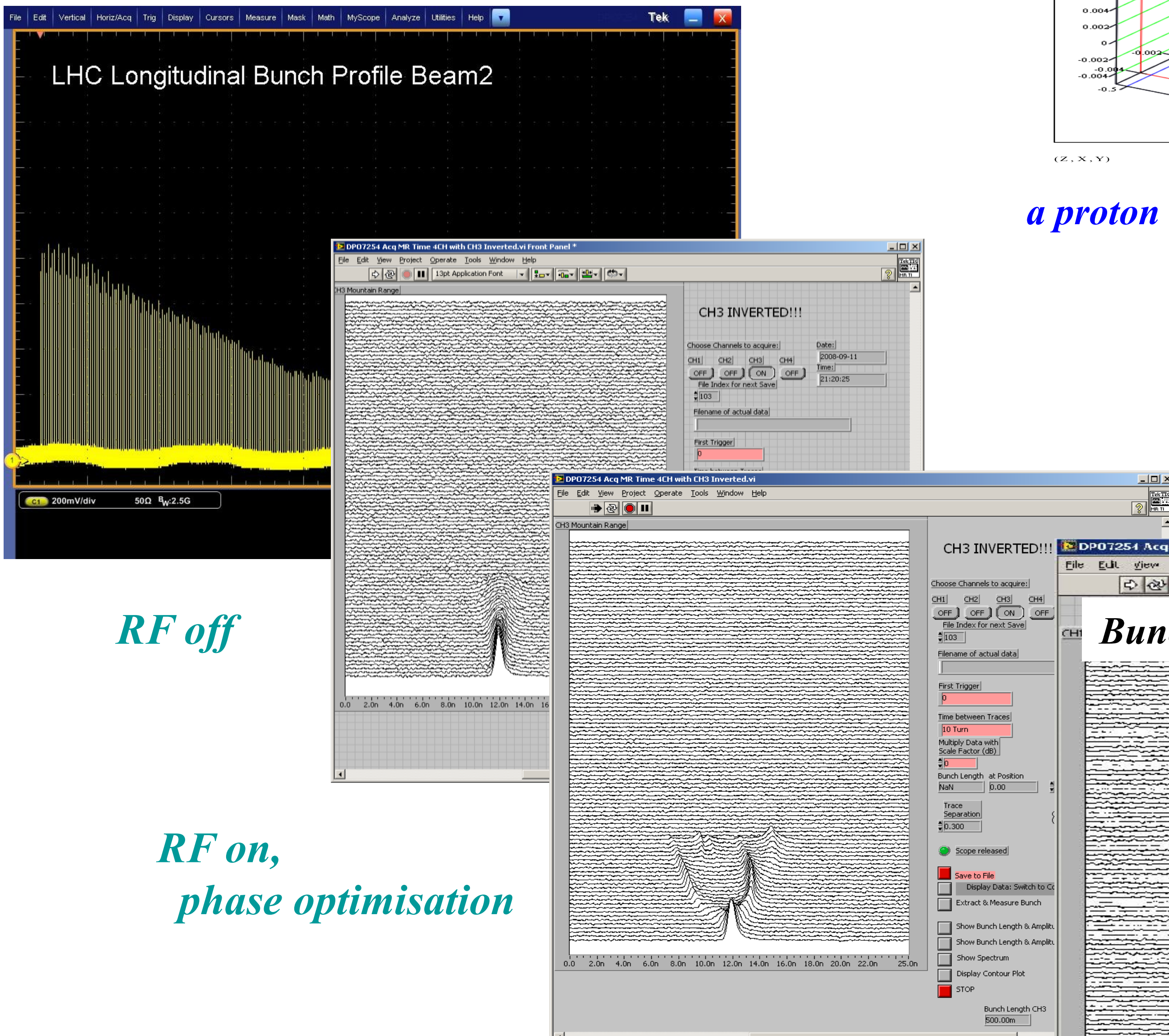
Correct x, x', y, y' to obtain the Closed Orbit

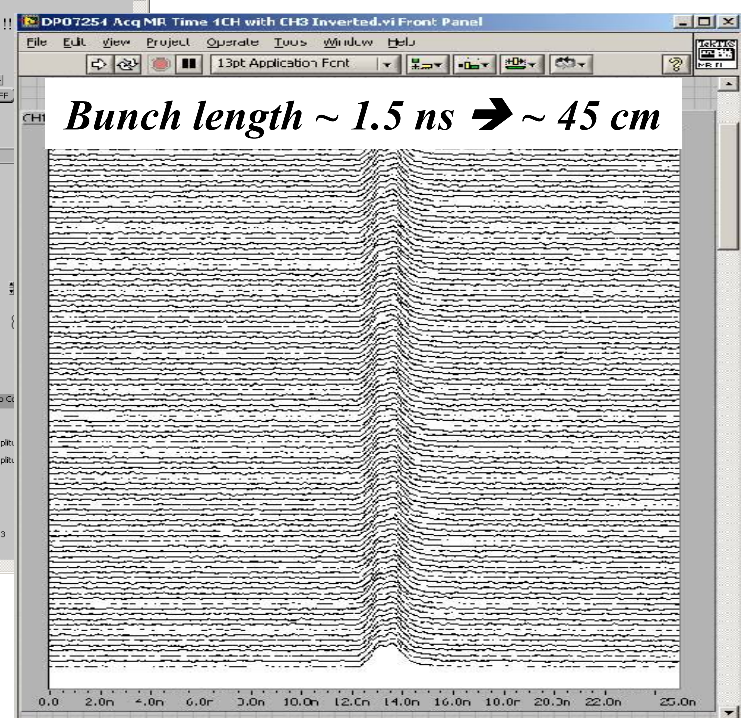


LHC Commissioning: RF



a proton bunch: focused longitudinal by the RF field





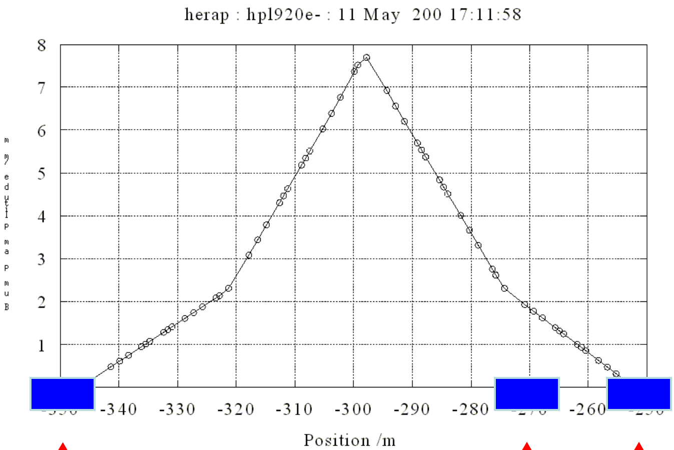
RF off

RF on, phase optimisation

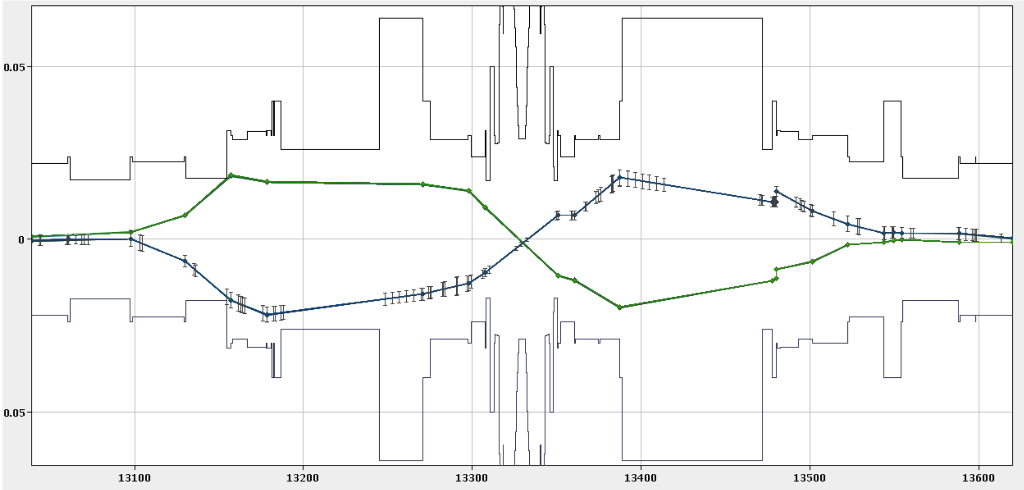
RF on, phase adjusted, beam captured

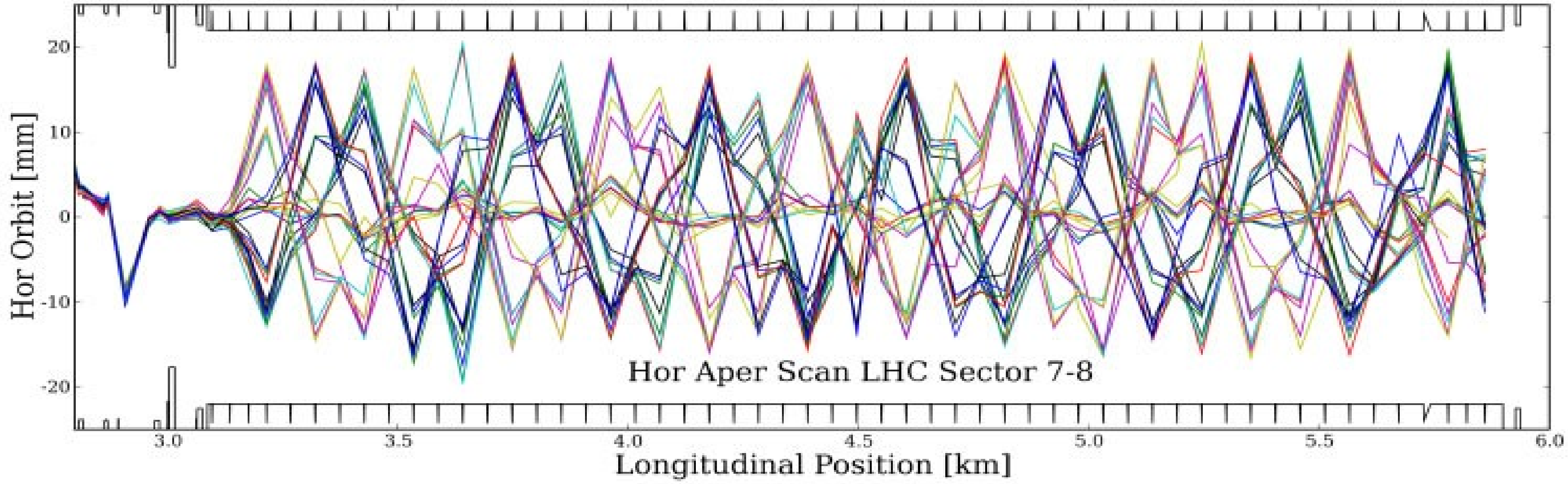
LHC Operation: Aperture Scans

Apply closed orbit bumps until losses indicate the aperture limit
... what about the beam size ?



3 bump to create local orbit distortion



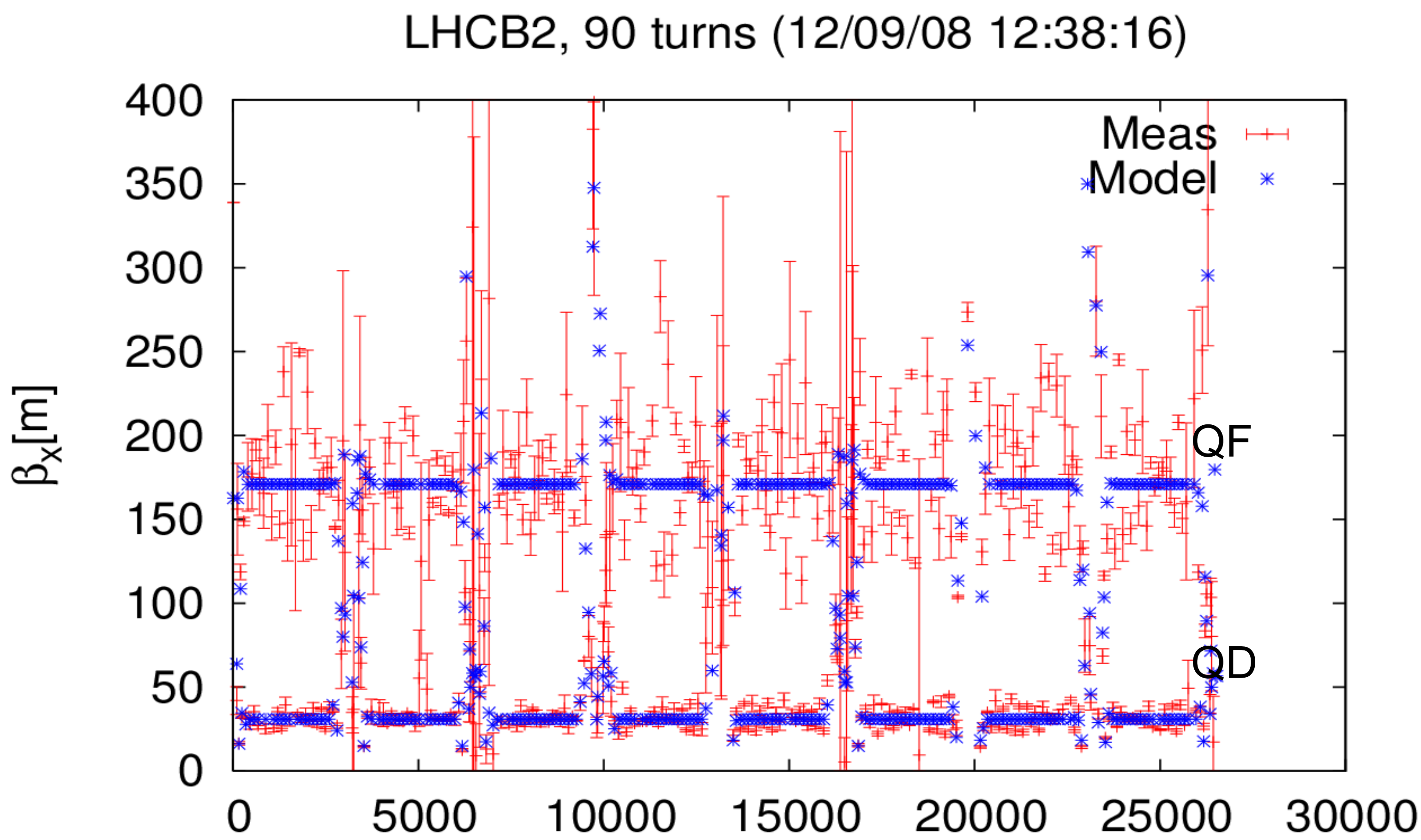


LHC Operation: the First Beam

Measurement of β :

$$\Delta\beta(s_0) = \frac{\beta_0}{2\sin 2\pi Q}\int_{s1}^{s1+l}\beta(s_1)\Delta K\cos\left(2\left|\psi_{s1}-\psi_{s0}\right| - 2\pi Q\right)ds$$

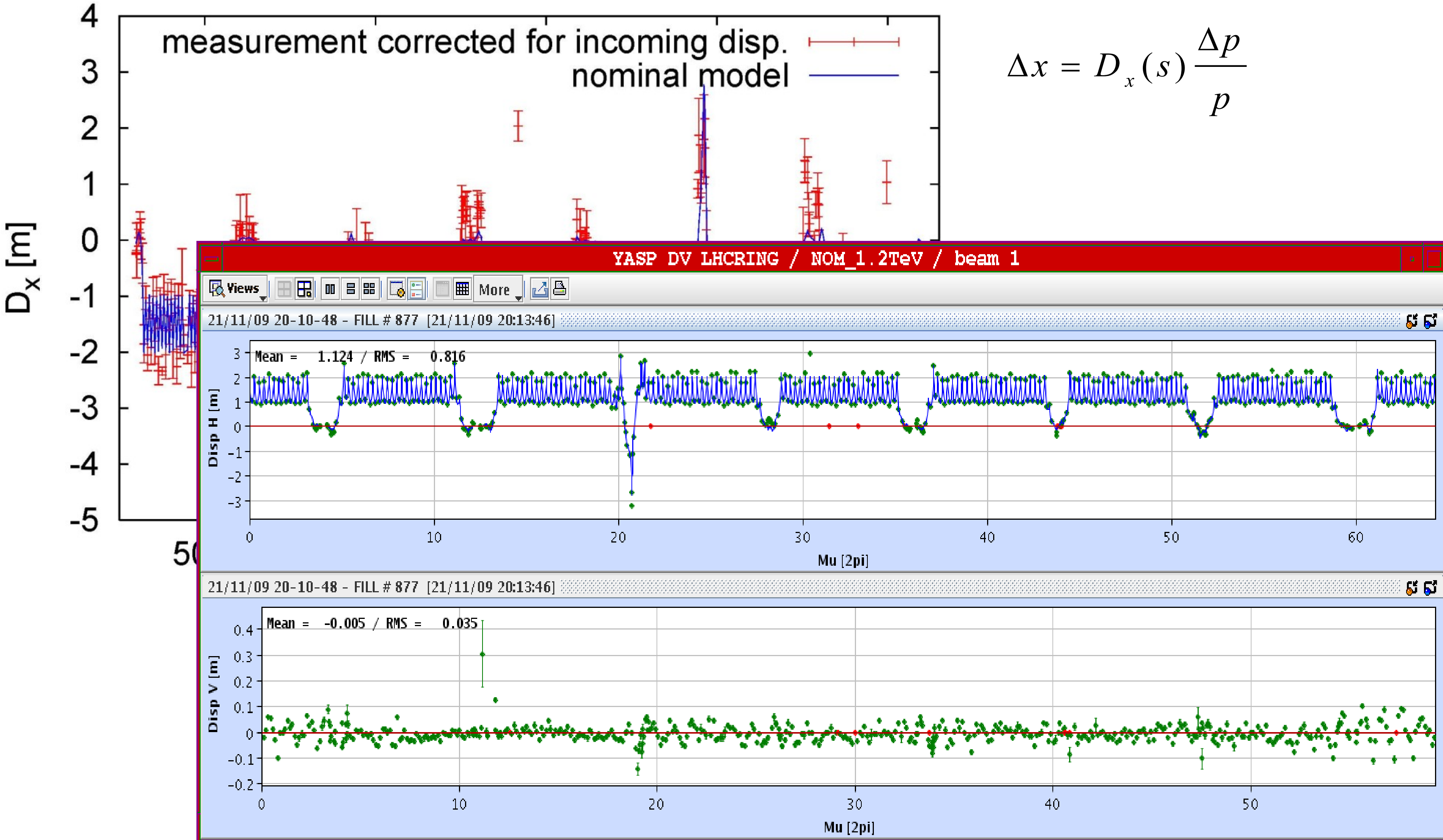
Δβ / β = 50 %



LHC Operation: the First Beam

Dispersion Measurement

$$\Delta x = D_x(s)\frac{\Delta p}{p}$$



Luminosity optimization

$$L = \frac{N_1 N_2 f_{rev} N_b}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} F \cdot W$$

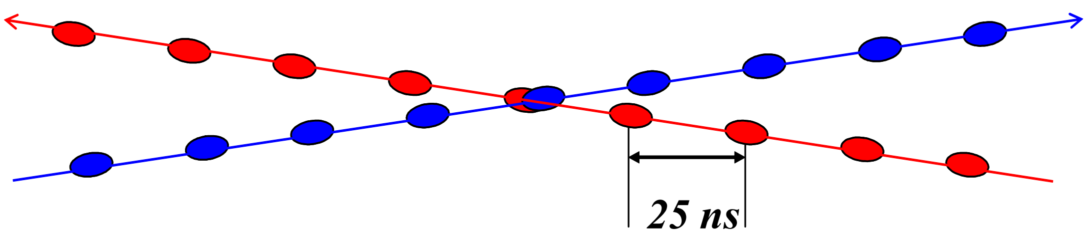
N_i = number of protons/bunch
N_b = number of bunches
f_{rev} = revolution frequency
σ_{ix} = beam size along x for beam i
σ_{iy} = beam size along y for beam i

F is a pure **crossing angle (*Φ*) contribution**:

$$F = \frac{1}{\sqrt{1 + 2\frac{\sigma_s^2}{\sigma_{1x}^2 + \sigma_{2x}^2}\tan^2\frac{\phi}{2}}}$$

F_{LHC} = 0.836

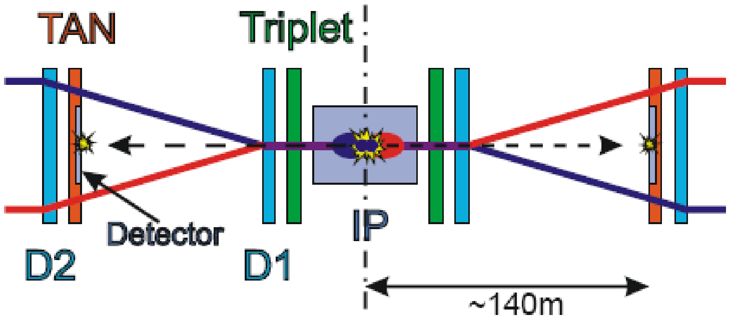
... cannot be avoided



W is a **pure beam offset contribution**.

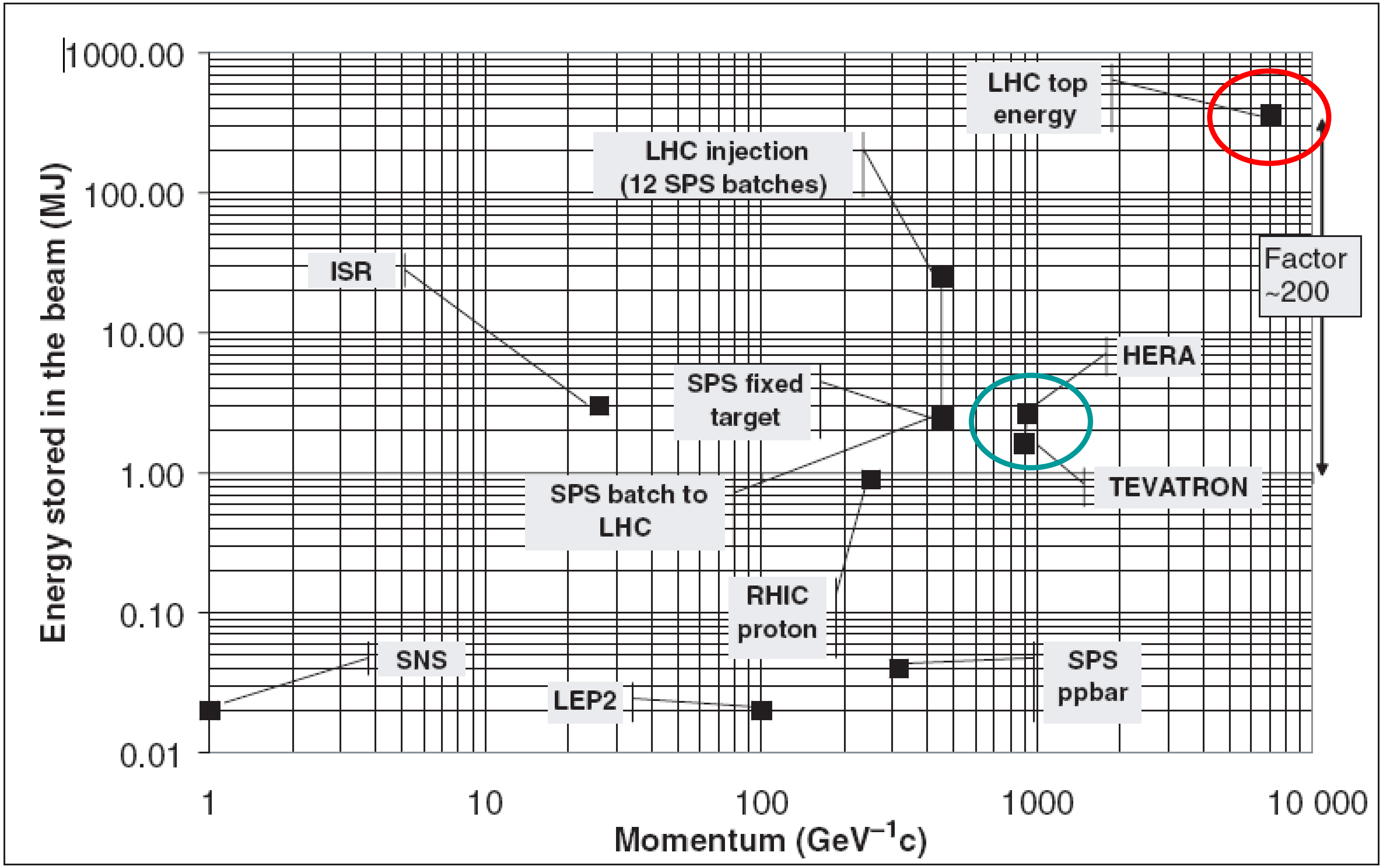
... can be avoided by careful tuning

$$W = e^{-\frac{(d_2 - d_1)^2}{2(\sigma_{x1}^2 + \sigma_{x2}^2)}}$$



LHC Operation:
Machine Protection & Safety

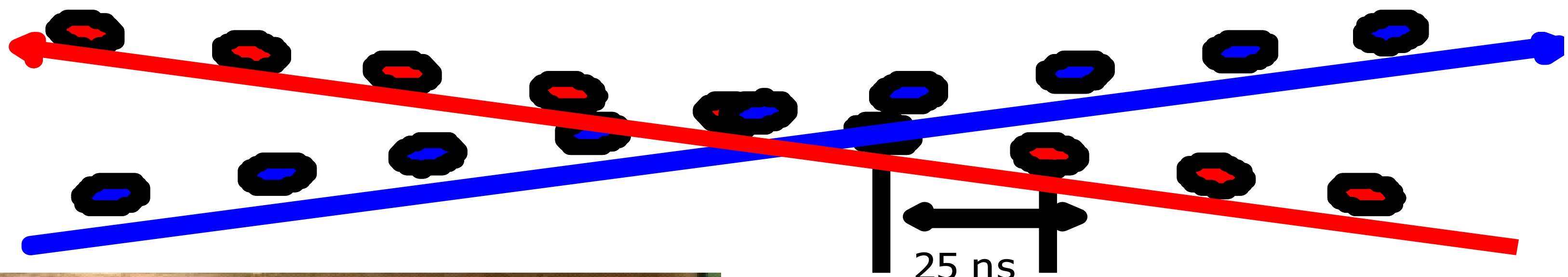
Energy Stored in the Beam of different Storage Rings

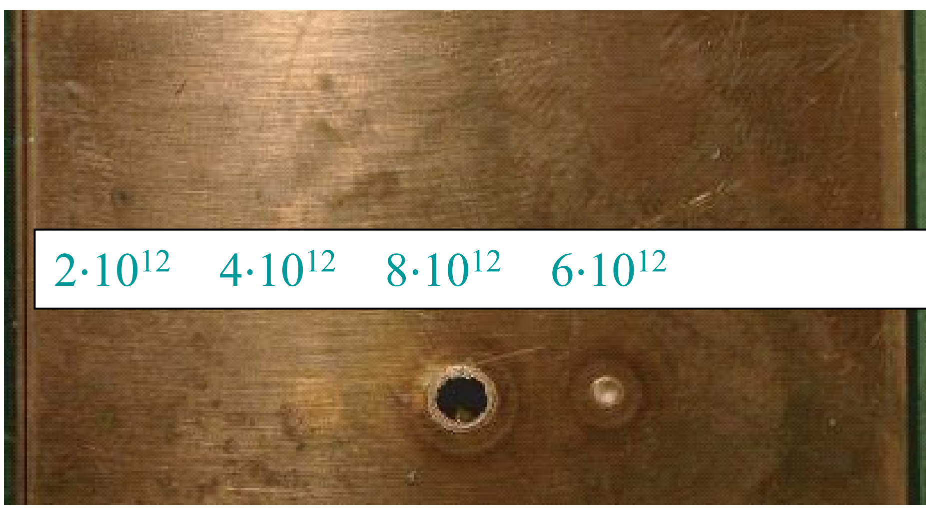


LHC Operation:
Machine Protection & Safety

Energy stored in magnet system	10	GJ
Energy stored in one main dipole circuit	1.1	GJ
Energy stored in one beam	362	MJ

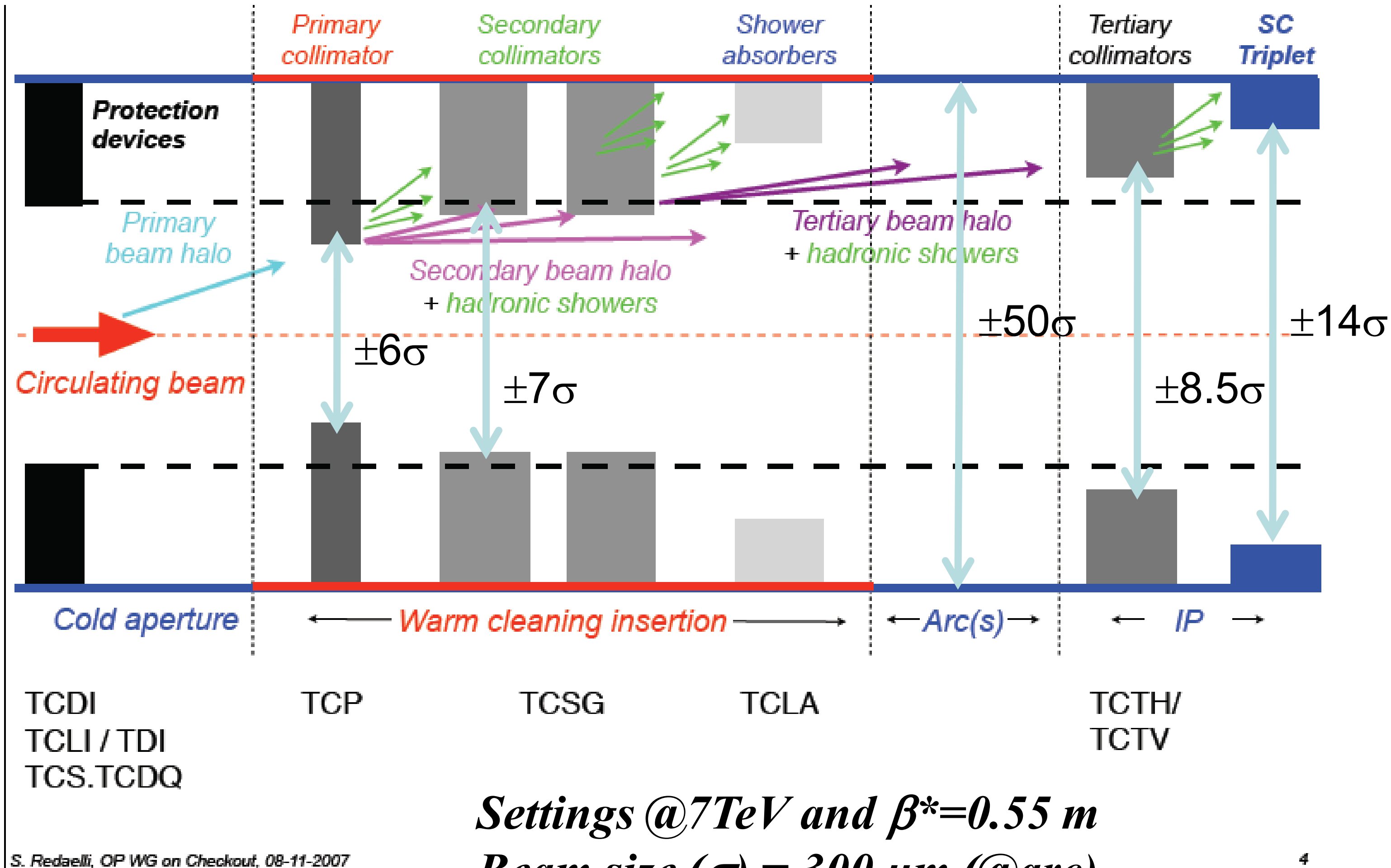
Enough to melt 500 kg of copper





$2\cdot10^{12}$ $4\cdot10^{12}$ $8\cdot10^{12}$ $6\cdot10^{12}$ 450 GeV p Strahl

LHC Aperture and Collimation



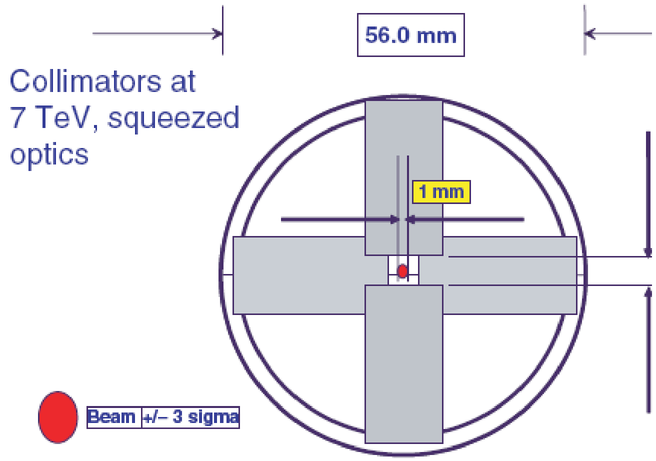
Settings @7TeV and β*=0.55 m

Beam size (σ) = 300 μm (@arc)

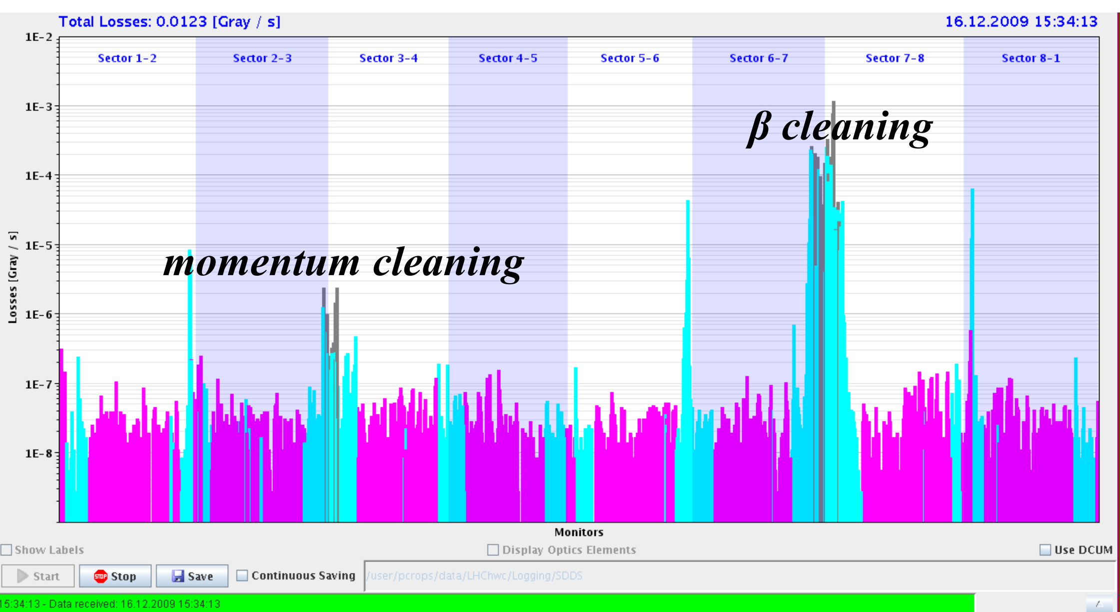
Beam size (σ) = 17 μm (@IR1, IR5)

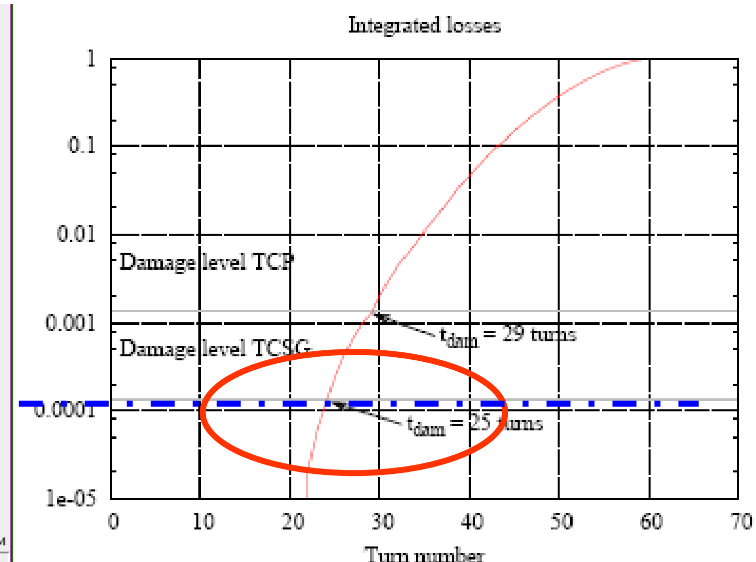
LHC Operation:
Machine Protection & Safety

... Komponenten des Machine Protection Systems :

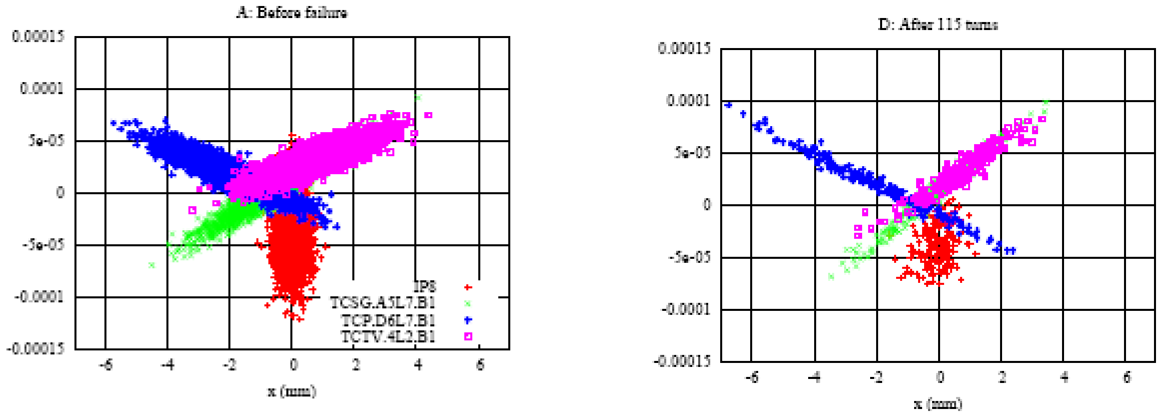


beam loss monitors
QPS
permit server
orbit control
power supply control
collimators
online on beam check of all (?)
hardware components
a fast dump
the gaussian beam profile





LHC Operation: Machine Protection & Safety



What will happen in case of Hardware Failure

Phase space deformation in case of failure of RQ4.LR7 (A. Gómez)

Short Summary of the studies:

quench in sc. arc dipoles: τ_{loss} = 20 - 30 ms

BLM system reacts in time, QPS is not fast enough

quench in sc. arc quadrupoles: τ_{loss} = 200 ms

BLM & QPS react in time

failure of nc. quadrupoles: τ_{det} = 6 ms

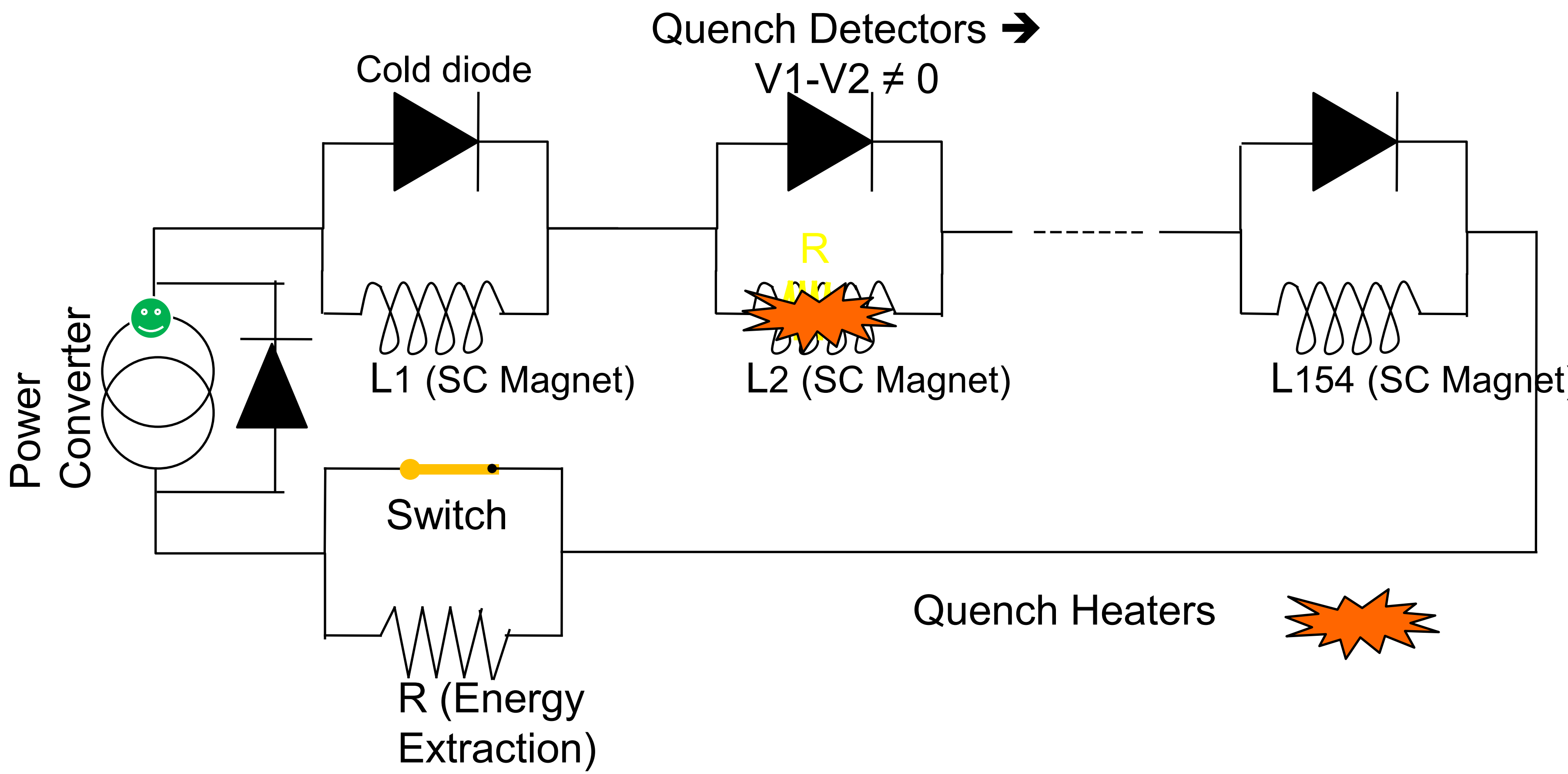
τ_{damage} = 6.4 ms

failure of nc. dipole: τ_{damage} = 2 ms

→ *FMCM installed*

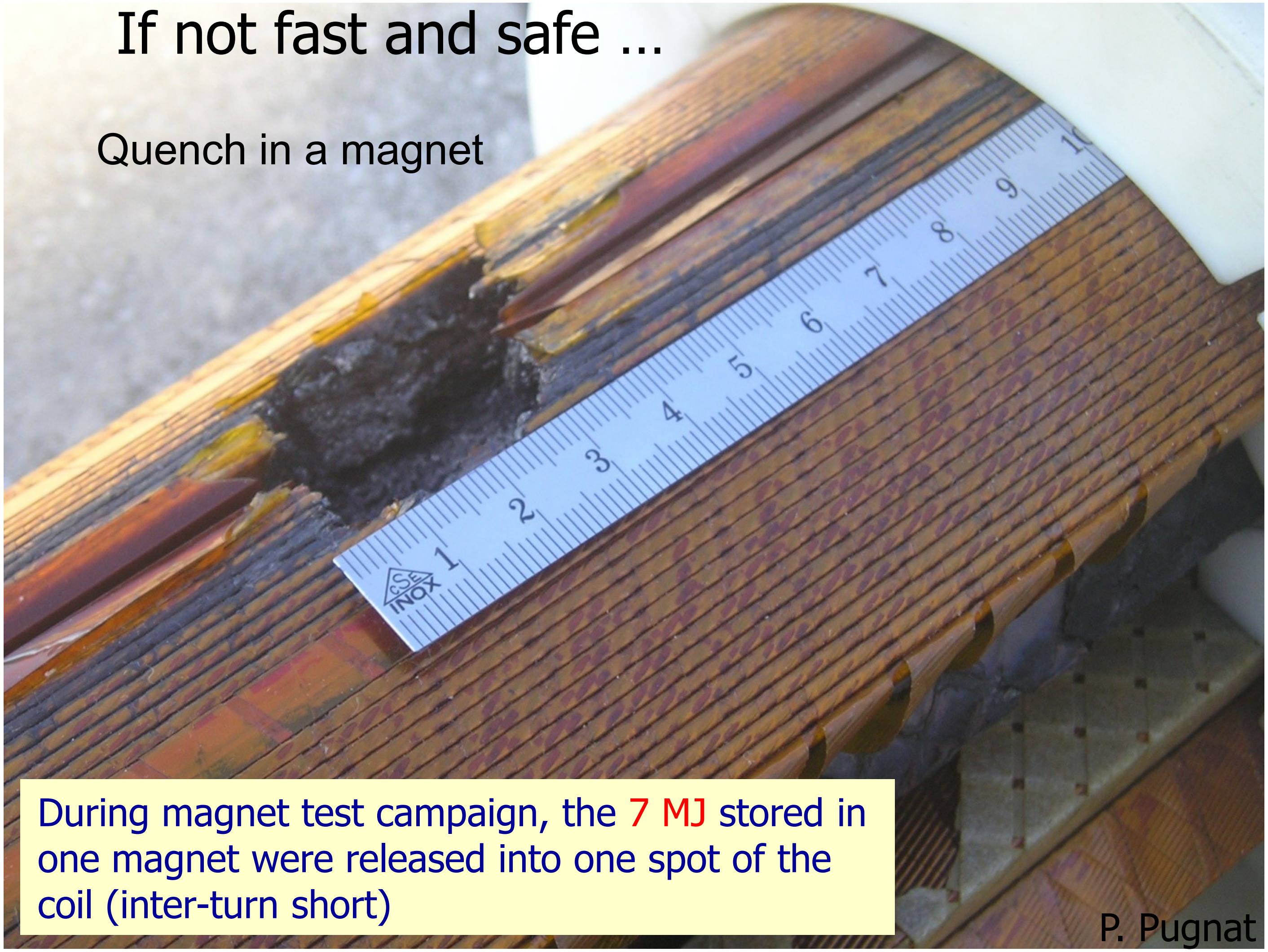
Energy stored in the magnets: 10 GJ
Quench Protection System

Schematics of the QPS in the main dipoles of a sector

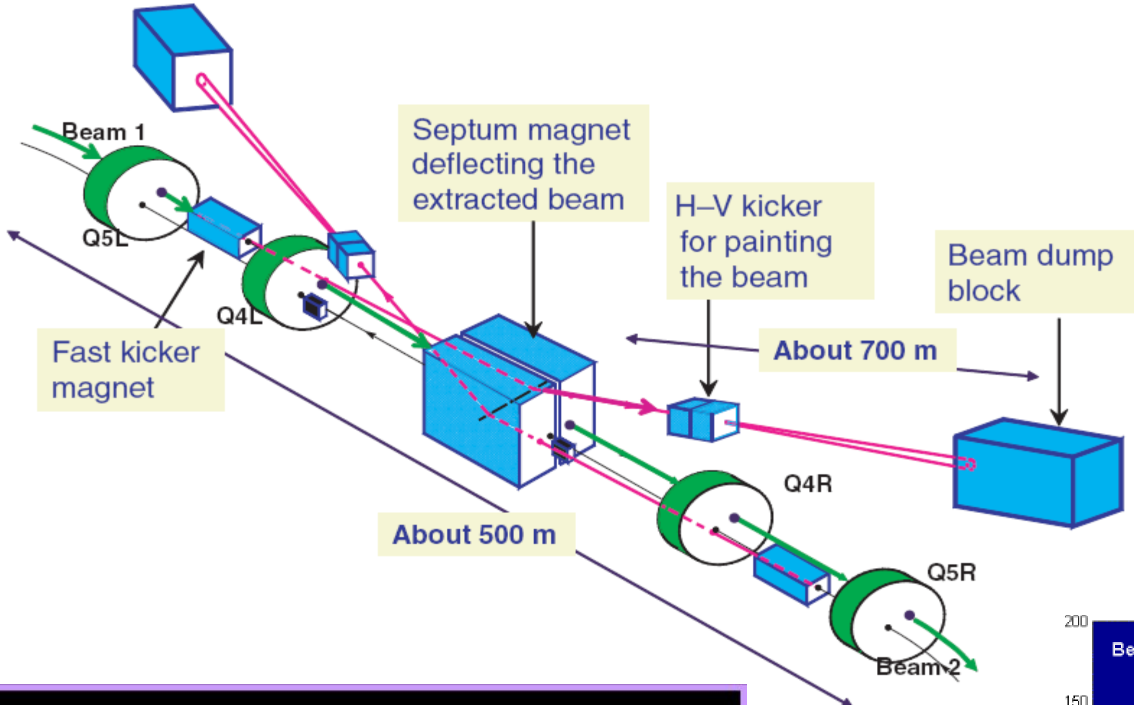


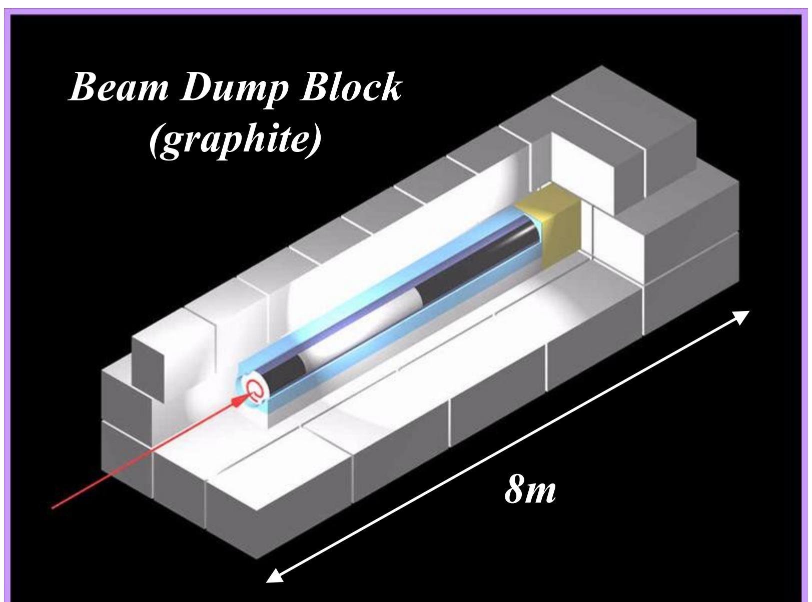
court. R. Alemany

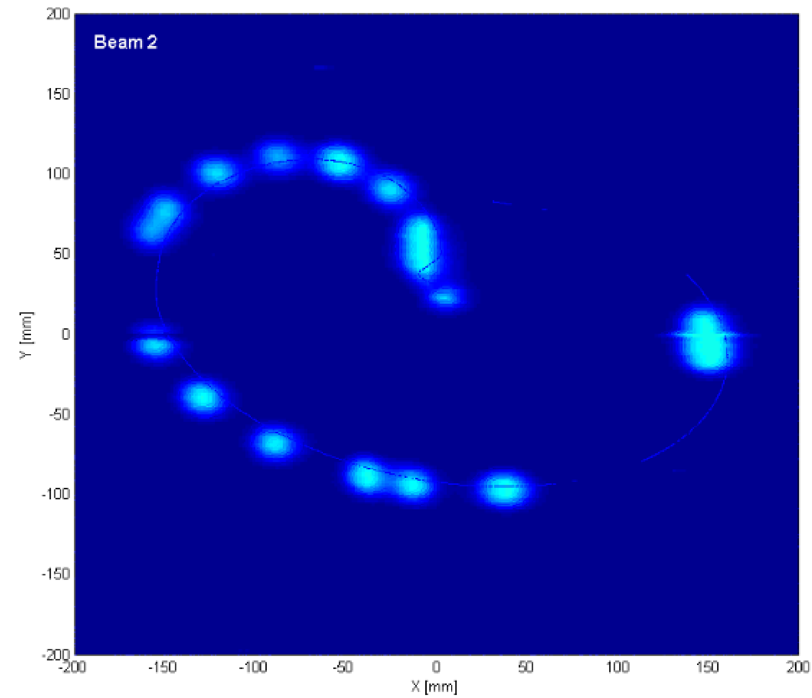
Energy stored in the magnets: quench



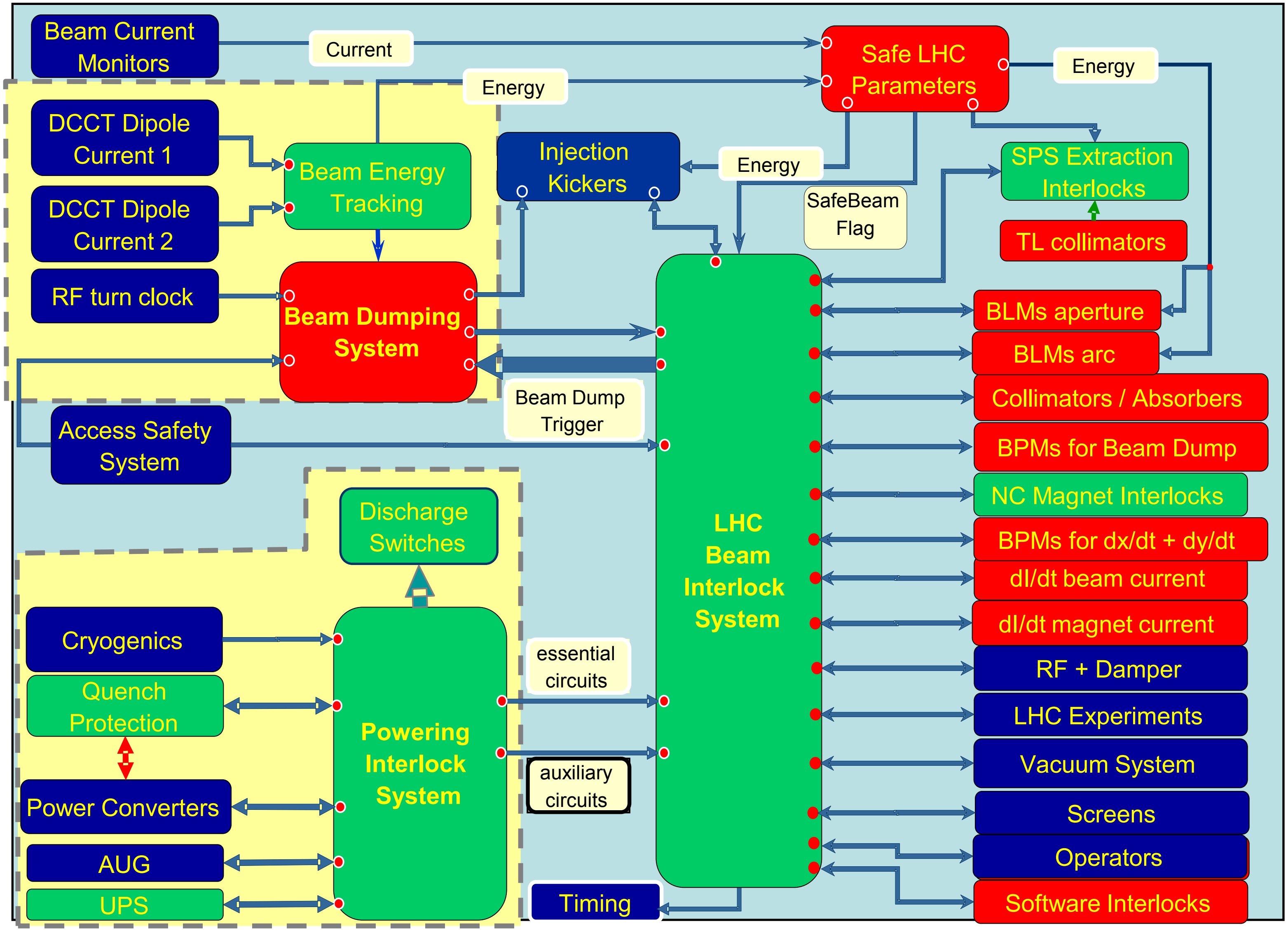
LHC Operation: Dump System





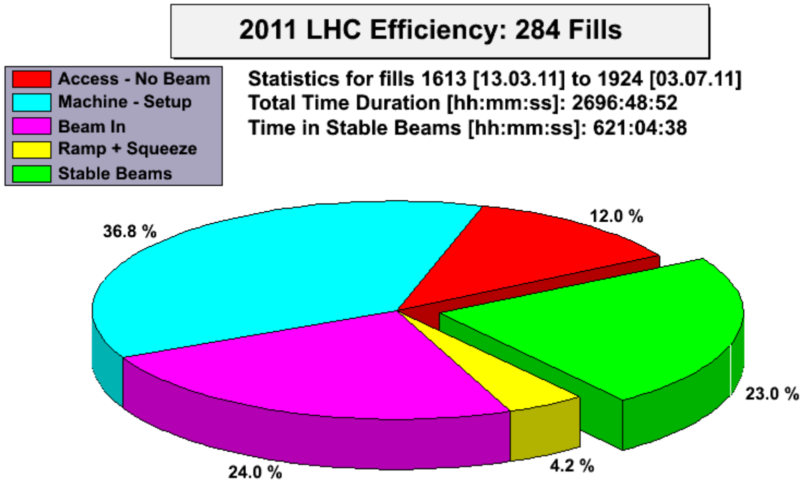


LHC Operation: Machine Protection & Safety

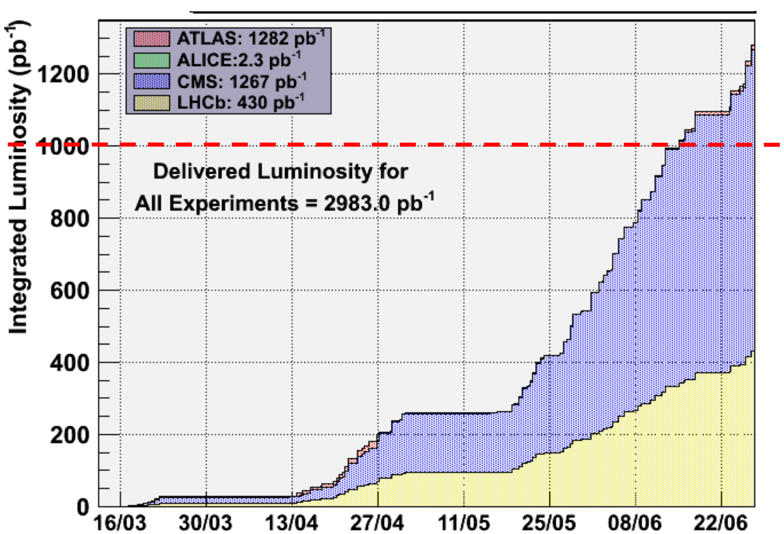


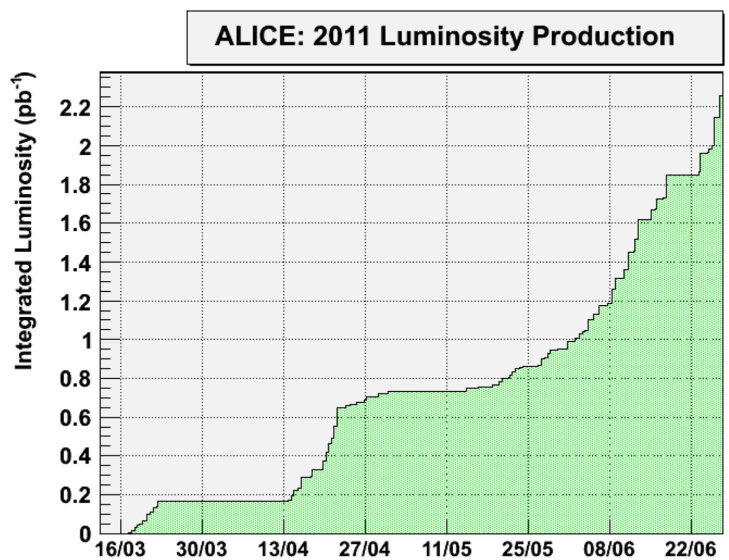
... no comment

LHC Operation where are we ?



Luminosity Efficiency:
time spent in collisions / overall time





LHC Operation where are we ?

	LHC Design	LHC 2011
Momentum at collision	7 TeV /c	3.5 TeV
Dipole field	8.33 T	4.16 T
Protons per bunch	1.15×10^{11}	1.15×10^{11}
Number of bunches/beam	2808	1380
Nominal bunch spacing	25 ns	50 ns
Normalized emittance	3.75 μm	2.2 μm
Absolute Emittance	5×10^{-10}	6.7×10^{-10}
Beta Function	0.5 m	1.5 m
rms beam size (IP)	16 μm	31 μm
Luminosity	1.0×10^{34}	1.3×10^{33}

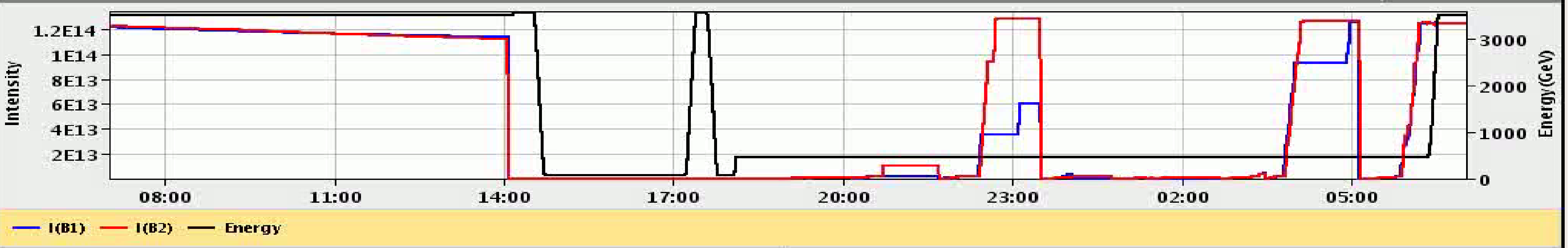
LHC Operation

18-Jun-2011 07:00:48 Fill #: 1880 Energy: 3500 GeV I(B1): 1.25e+14 I(B2): 1.25e+14

	ATLAS	ALICE	CMS	LHCb
Experiment Status	PHYSICS	STANDBY	STANDBY	PHYSICS
Instantaneous Lumi (ub.s)^-1	1064.381	0.413	0.040	19.147
BRAN Luminosity (ub.s)^-1	1152.313	0.657	634.113	17.844
Fill Luminosity (nb)^-1	0.0	0.0	0.0	1.6
BKGD 1	0.263	0.343	1207047.896	0.263
BKGD 2	20.001	0.543	0.002	0.124
BKGD 3	8.543	2.265	2.732	0.161

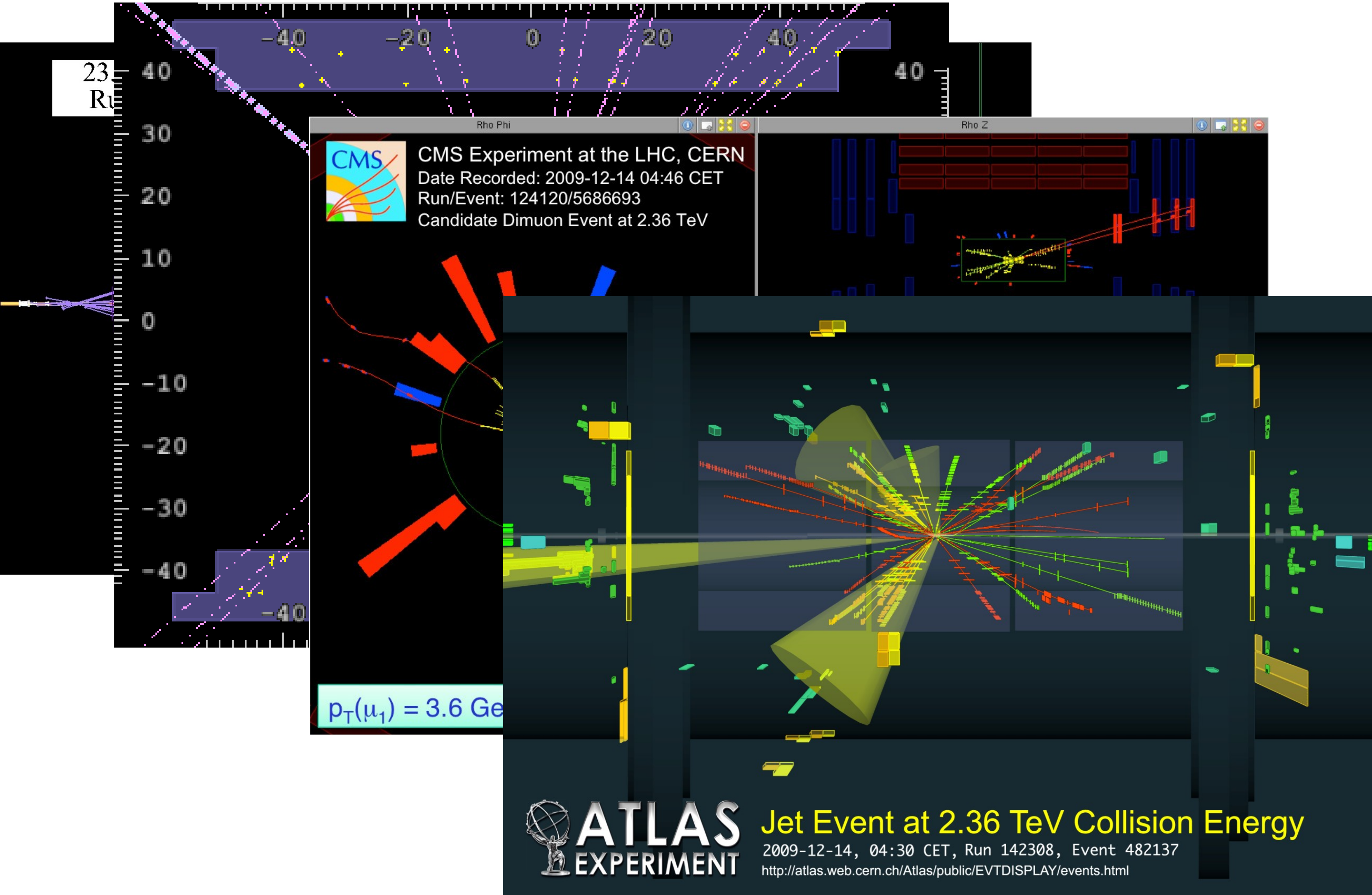
LHCb VELO Position OUT Gap: -0.0 mm STABLE BEAMS TOTEM: STANDBY

Performance over the last 24 Hrs Updated: 07:00:46



... so sorry but this is world record in hadron collisions

LHC Operation: Collisions at 3.5 TeV per beam



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