Method of moments for $B \to K^{*}\mu\mu$

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Plan

Why method of moments:

1. Complementary approach in performing the fit.
2. Allows to extract info measuring quantities in event basis depending on the angular distribution.
3. Used in $B \to \rho \ell \nu$(SLAC-386 UC-414),
   $J/\psi \to KK\gamma$(PRD 71, 032005 (2005)), etc.

Method of moments

Let's assume we have our pdf with k unknown parameters :$PDF(x_i, \alpha)$, $dim(\alpha) = k$. One can calculate k moments, which are the functions of α_i:

$$\mu_i = f(\alpha_1, ..., \alpha_k) = E[W_i] \tag{1}$$

If we have n events in our q^2 bin, we can estimate:

$$\widehat{\mu}_i = \frac{1}{n} \sum_{j=0}^{j=n-1} w_j \tag{2}$$

, where $w_j = g(x_i)$

Trivial example

Lets see how this works in practice:

$$f(x) = \frac{x^{a-1}e^{-x/b}}{b^a\Gamma(a)} \quad (3)$$

we measure the moments:

$$m_1 = \frac{X_1 + X_2 + ... + X_n}{n},$$
$$m_2 = \frac{X_1^2 + X_2^2 + ... + X_n^2}{n}.$$

and calculate them analytically:

$$m_1 = ab,\ m_2 = b^2a(a+1)$$

So one just needs to solve this and get the answer:

$$a = \frac{m_1^2}{m_2 - m_1^2},\ b = \frac{m_2 - m_1^2}{m_1}$$

Our PDF

The angular terms:

$$\frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = \frac{9}{32\pi}(\frac{3}{4}(1-F_l)\sin^2\theta_k + F_l\cos^2\theta_k + (\frac{1}{4}(1-F_l)\sin^2\theta_k - F_l\cos^2)\cos 2\theta_l + J_3\sin^2\theta_k\sin^2\theta_l cos2\phi + J_4\sin 2\theta_k\sin\theta_l\cos\phi + J_5\sin 2\theta_k\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_k + J_{6c}\cos^2\theta_k)\cos\theta_l + J_7\sin 2\theta_k\sin\theta_l\sin\phi + J_8\sin 2\theta_k\sin 2\theta_l\sin\phi + J_9\sin^2\theta_k\sin^2\theta_l\sin 2\phi) \quad (4)$$

Since we are fitting a PDF we need to ensure it is normalized:

$$\int_{-\pi}^{\pi} d\phi \int_{-1}^{1} dcos\theta_l \int_{-1}^{1} dcos\theta_k \frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = 1 \quad (5)$$

Measuring Js

From equation 2 we have the following:

$$\frac{1}{4}(3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}) = 1 \tag{6}$$

For now we will consider the following PDF:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi}(cos\theta_k, cos\theta_l, \phi) \tag{7}$$

Becouse our PDF is not normalized and we are measuring $\Gamma + \overline{\Gamma}$ we are effectively fitting the S_i (aka $J_i \rightarrow S_i$)

Moments for B → K*$\mu\mu$

Let's calculate the moments for Ss:

$$\frac{d^3\Gamma}{\Gamma d cos\theta_k d cos\theta_l d\phi} sin^2\theta_k sin^2\theta_l cos2\phi = \frac{8S_3}{25} \quad (8)$$

$$\frac{d^3\Gamma}{\Gamma d cos\theta_k d cos\theta_l d\phi} sin2\theta_k sin2\theta_k cos\phi = \frac{8S_4}{25} \quad (9)$$

$$\frac{d^3\Gamma}{\Gamma d cos\theta_k d cos\theta_l d\phi} sin2\theta_k sin\theta_l cos\phi = \frac{2S_5}{5} \quad (10)$$

$$\frac{d^3\Gamma}{\Gamma d cos\theta_k d cos\theta_l d\phi} sin2\theta_k sin\theta_l sin\phi = \frac{2S_7}{5} \quad (11)$$

$$\frac{d^3\Gamma}{\Gamma d cos\theta_k d cos\theta_l d\phi} sin2\theta_k sin2\theta_l sin\phi = \frac{8S_8}{25} \quad (12)$$

Moments for B $\rightarrow$ K$^*\mu\mu$

- The simplest solution one could imagine.
- We are abusing the fact that the basis is orthogonal.
- Each of the J doesn't know about other.
- Only S_{1s}, S_{2s}, S_{1c}, S_{2c} and S_{6s}, S_{6c} are not orthogonal, but to get the answer you just need to solve a linear equation system so it's not a tragedy.

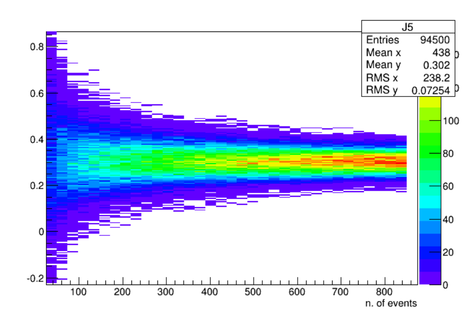
$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_k cos\theta_l = 0.1(S_6c + 4S_6s) \tag{13}$$

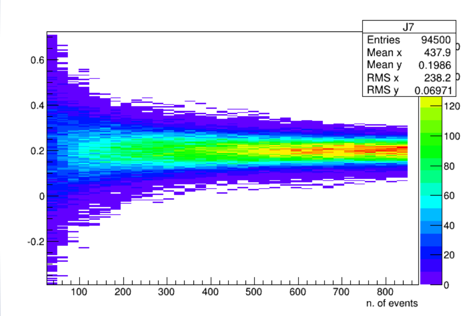
$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} cos\theta_l = 0.25(S_6c + 2S_{6s}) \tag{14}$$

solution: $S_{6c} = 2(4M_{S_{6c}} - 5M_{S_{6s}})$, $S_{6s} = -2M_{S_{6c}} + 5M_{S_{6s}}$

Moments for $B \to K^*\mu\mu$

Lets see if this method actually works. Let's take some random parameters for the PDF and make a toy.





- let's take 300 signal events as a working case.

Moments for $B \to K^*\mu\mu$

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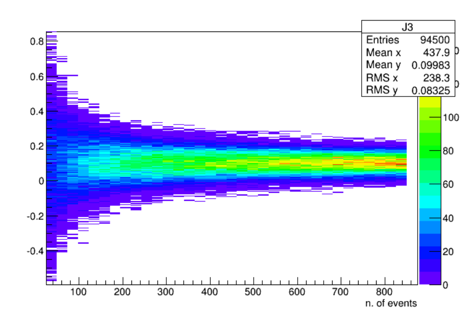
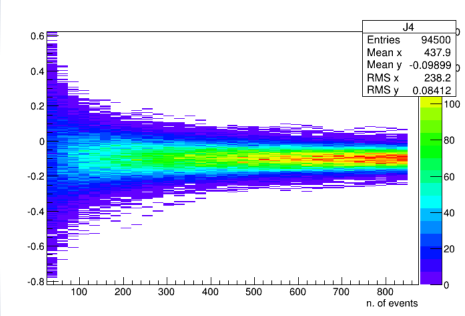
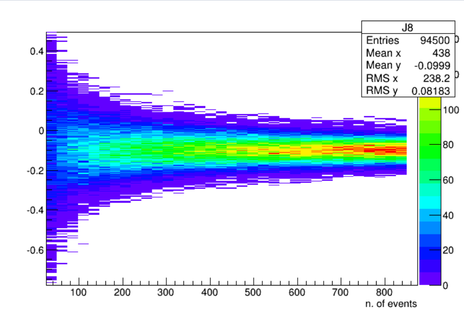


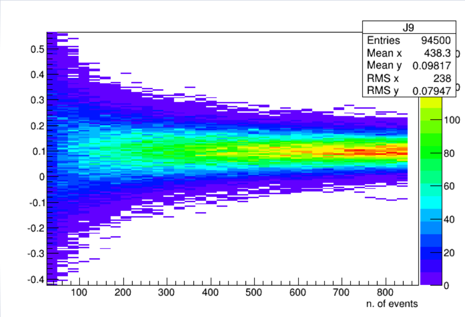


- let's take 300 signal events as a working case... we might still change the binning

Moments for $B \to K^*\mu\mu$

Lets see if this method actually works. Let's take some random parameters for the PDF and make a toy.

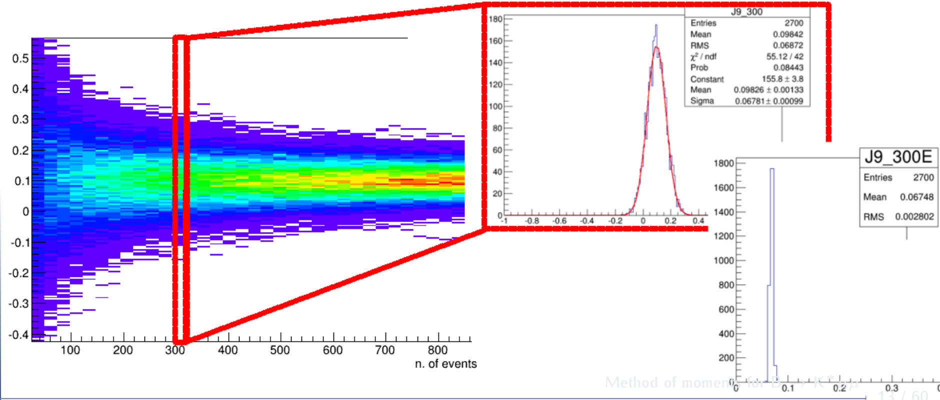




- let's take 300 signal events as a working case... we might still change the binning

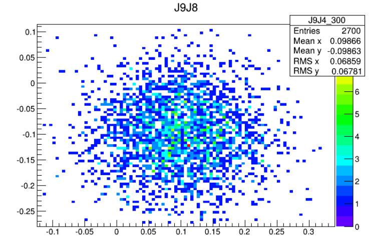
Error Estimation

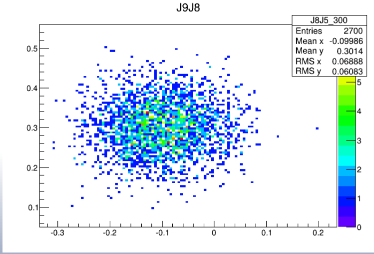
- Since moment is the mean of a given distribution the error can be estimated as *mean/RMS*
- use TOY MC to check this assumption



Correlation check

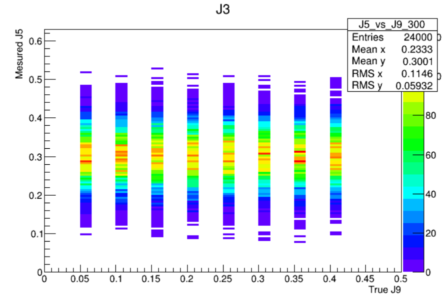
- In theory S_i shouldn't be correlated to S_j in the moment calculation.
- Lets put this to a test.

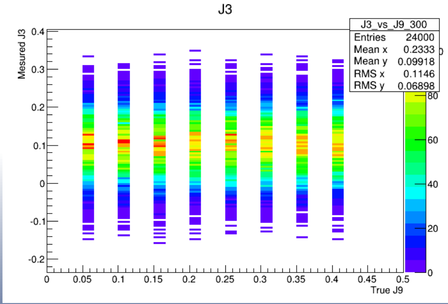




Correlation check 2

- Let's now FIX J_x and simulate different J_y
- Again theory would suggest that one J shouldn't know about the other, so J_x shouldn't change with scanning J_y parameter





What will happen to our problem with an S-wave?

Reminder:

$$\frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = \frac{9}{32\pi}(J_{1s}sin^2\theta_k + J_{1c}cos^2\theta_k + (J_{2s}sin^2\theta_k + J_{2c}cos^2)cos2\theta_l + J_3 sin^2\theta_k sin^2\theta_l cos2\phi + J_4 sin2\theta_k sin\theta_l cos\phi + J_5 sin2\theta_k sin\theta_l cos\phi + (J_{6s}sin^2\theta_k + J_{6c}cos^2\theta_k)cos\theta_l + J_7 sin2\theta_k sin\theta_l sin\phi + J_8 sin2\theta_k sin2\theta_l sinphi + J_9 sin^2\theta_k sin^2\theta_l sin2\phi) \quad (15)$$

Let's add a very discussing things that keeps us awake at night:

$$W_s = \frac{2}{3}F_s \sin^2\theta_l + \frac{4}{3}A_s \sin^2\theta_l \cos\theta_k + I_4 sin\theta_k sin2\theta_l cos\phi + I_5 sin\theta_k sin\theta_l cos\phi + I_7 sin\theta_k sin\theta_l + sin\phi + I_8 sin\theta_k sin2\theta_l sin\phi) \quad (16)$$

What will happen to our problem with an S-wave?

So now our PDF is sum of eq. 15 and 16. Of coz we need to require normalization:

$$\frac{1}{12}(32I_{1a} + 9J_{1c} + 18J_{1s} - 3J_{2c} - 6J_{2s}) = 1 \quad (17)$$

No surprises here. If we have a S-wave it has to enter in Γ. To build up the preasure, what will happen to our Ss?

NOTHING!!!!!!!!

We are completely insensitive to S-wave:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_k sin^2\theta_l cos2\phi = \frac{8S_3}{25} \quad (18)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_k cos\phi = \frac{8S_4}{25} \quad (19)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l cos\phi = \frac{2S_5}{5} \quad (20)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l sin\phi = \frac{2S_7}{5} \quad (21)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_l sin\phi = \frac{8S_8}{25} \quad (22)$$

Thins get better :)

We can even measure directly the S-wave:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_l cos\theta_k = \frac{32 I_{1b}}{45} \tag{23}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l cos\phi = \frac{16 I_4}{45} \tag{24}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin\theta_l cos\phi = \frac{4 I_5}{9} \tag{25}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l sin\phi = \frac{4 I_7}{9} \tag{26}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l sin\phi = \frac{16 S_8}{45} \tag{27}$$

Conclusions on S-wave

- S-wave components are transparent to method of moments.
- If they are orthogonal to others all they toy studies holds for them as well(will reapat for robustness but can bet my house that there is nothing going on there).
-

Conclusions

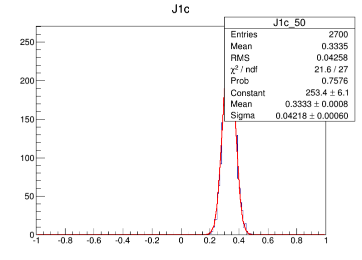
- Implemented moments method for the K*mm and start testing with toy MC
- The method converge fast and works for the "simple case", i.e. signal only.
- Method completely insensitive to S-wave component, thanks to orthogonality.
- Complementary one can measure in-depended S-wave component.

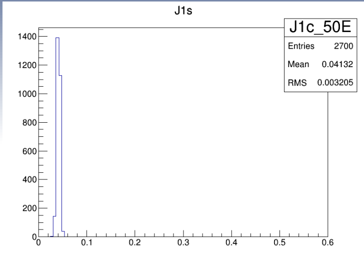
TO DO:

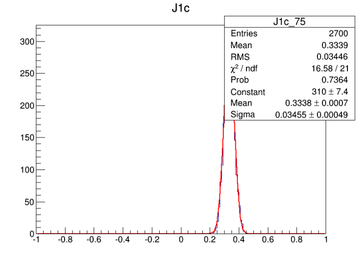
- add realism: backgrounds
- Do the unfolding
- Study binning

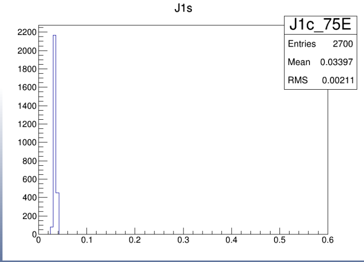
BACKUPS

2 am discovery









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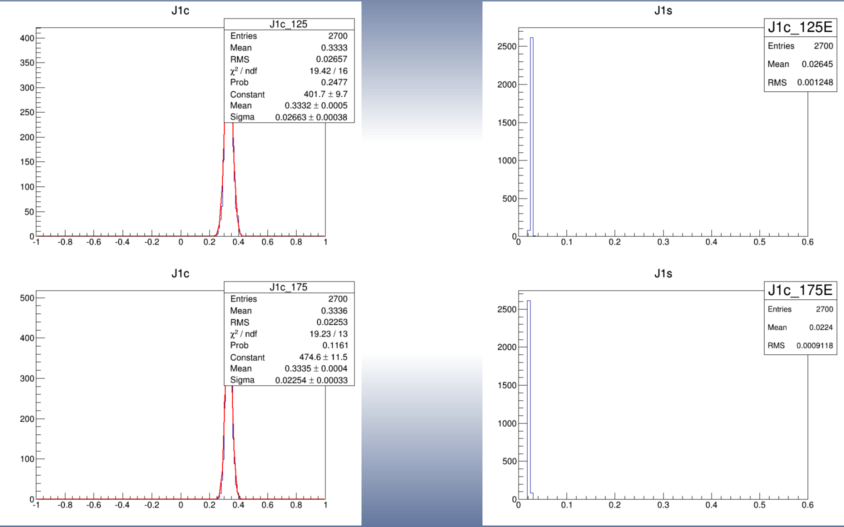

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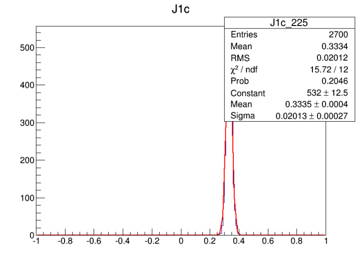

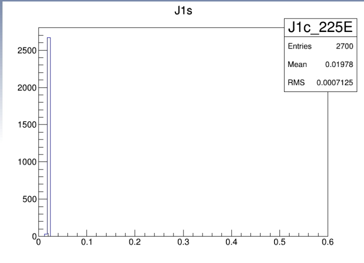

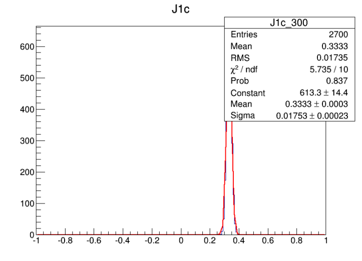

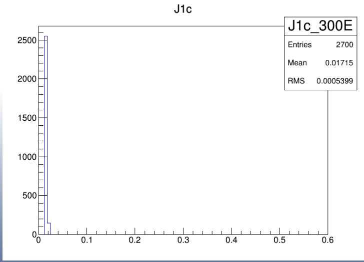

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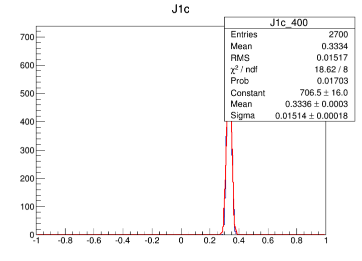

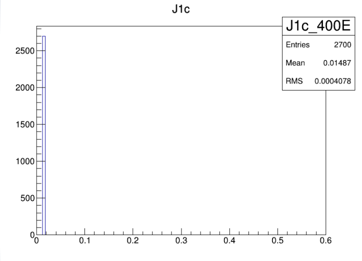

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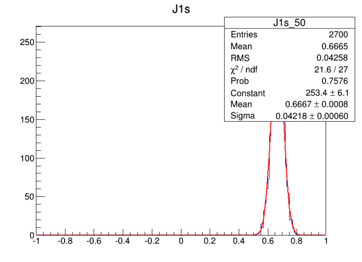

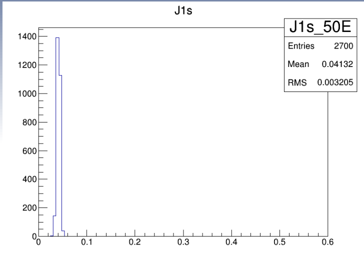

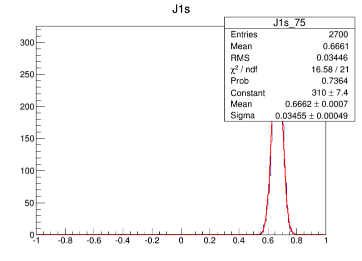

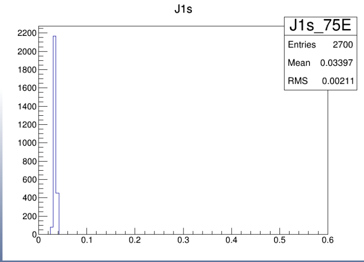

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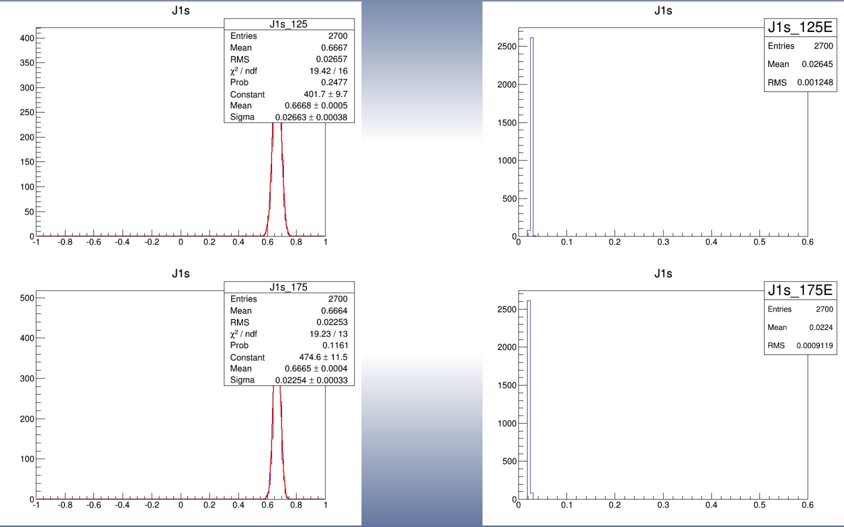

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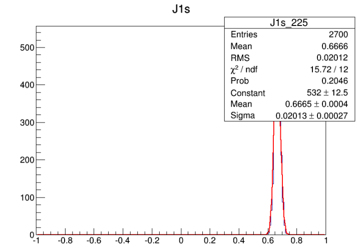

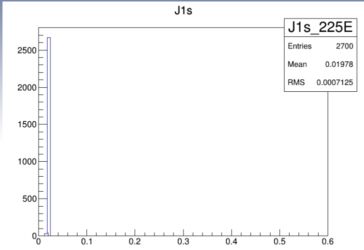

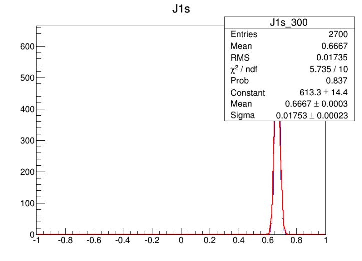

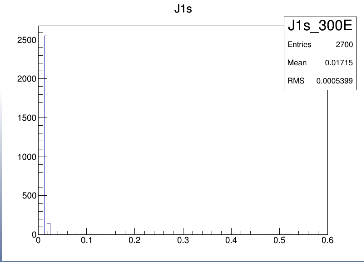

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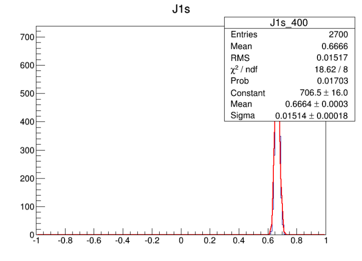

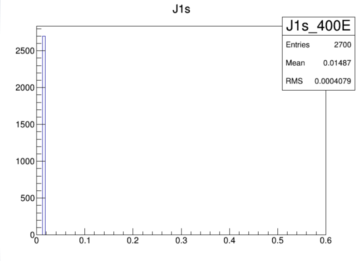

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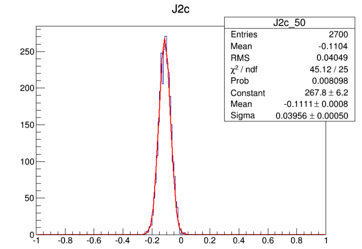

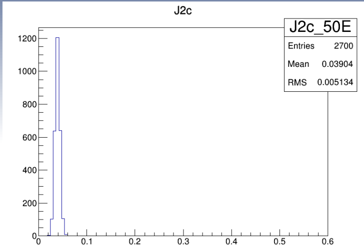

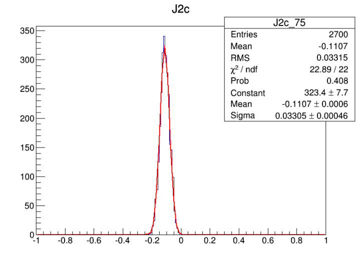

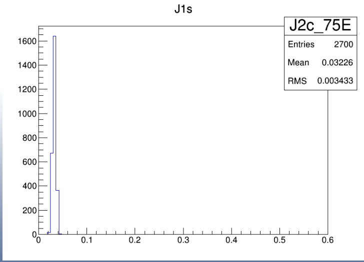

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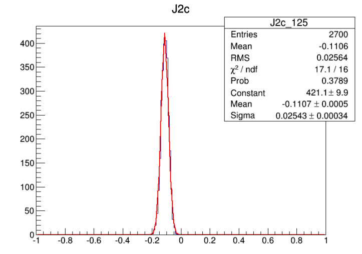

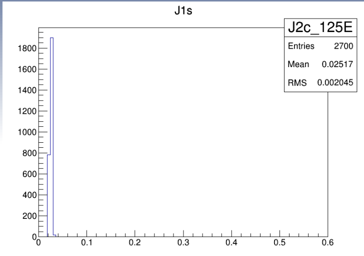

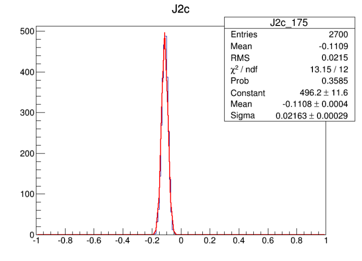

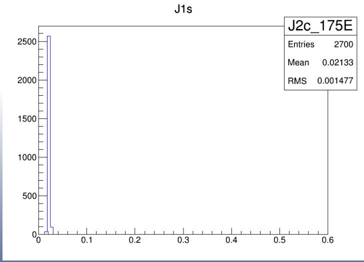

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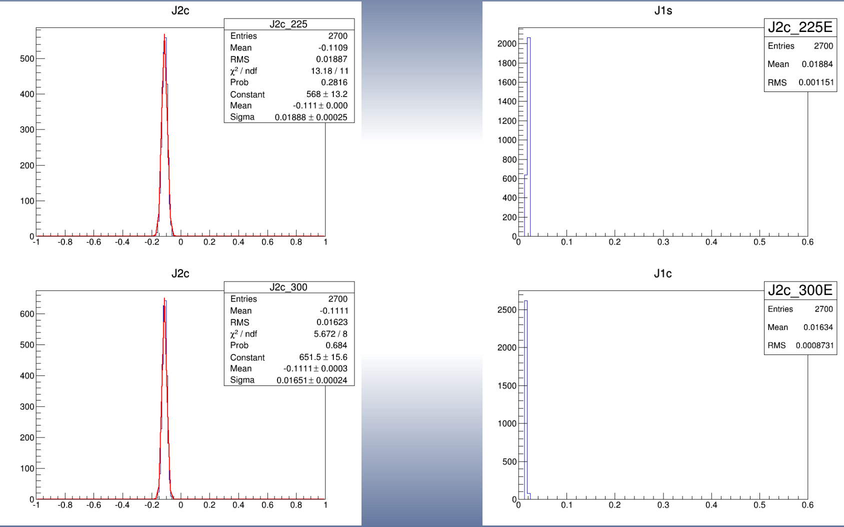

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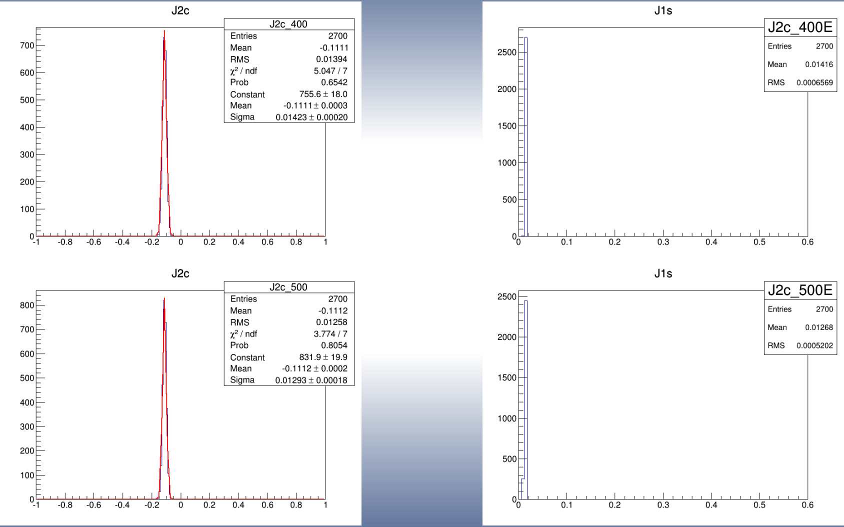

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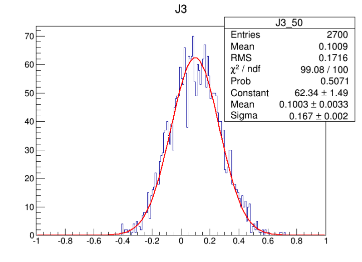

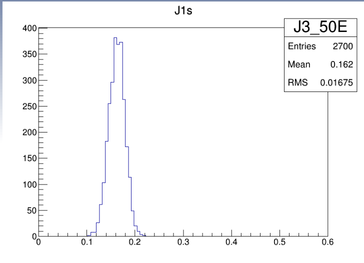

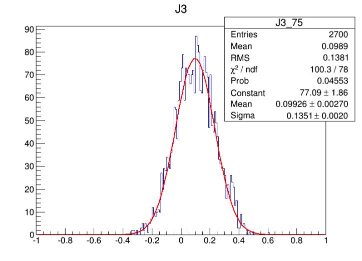

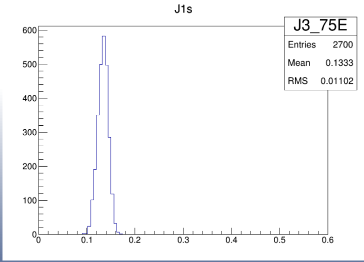

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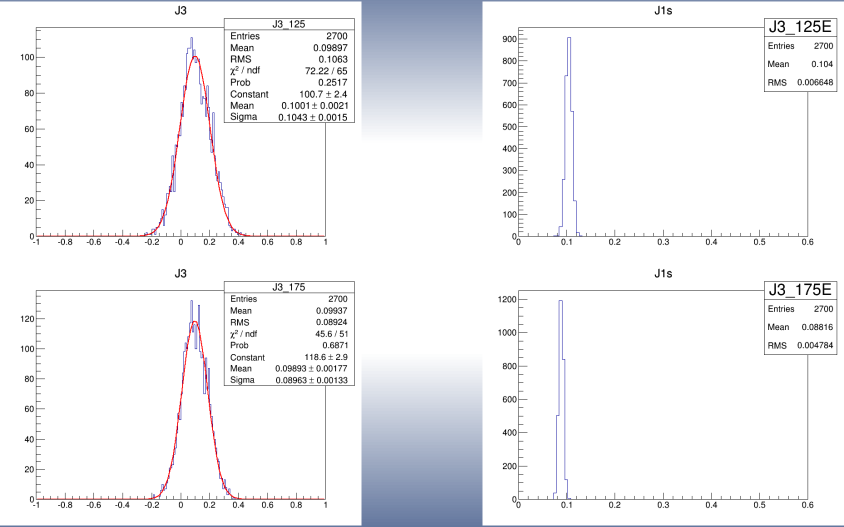

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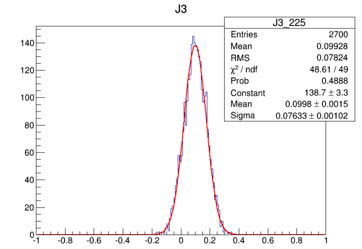

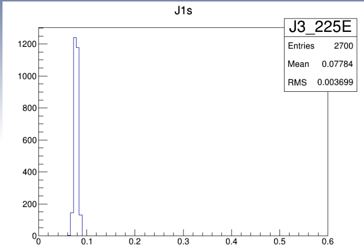

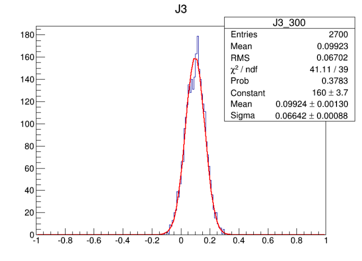

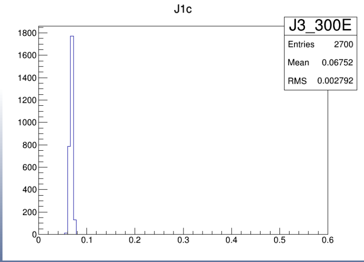

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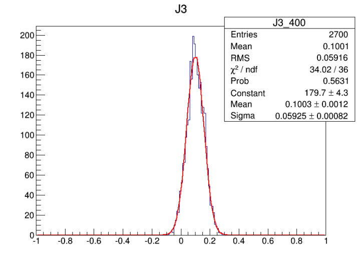

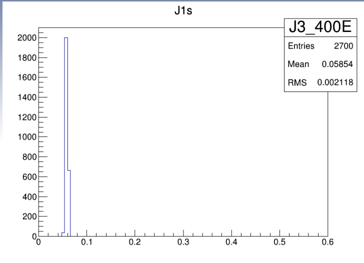

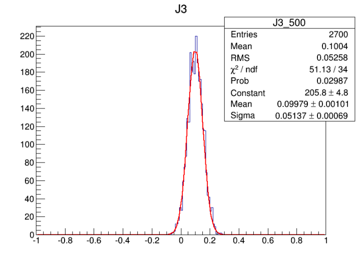

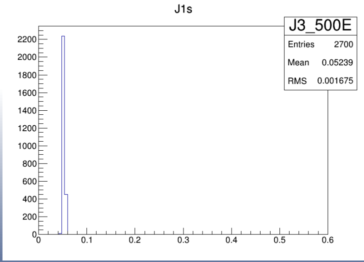

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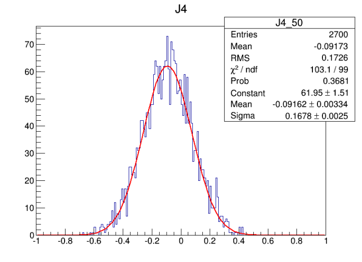

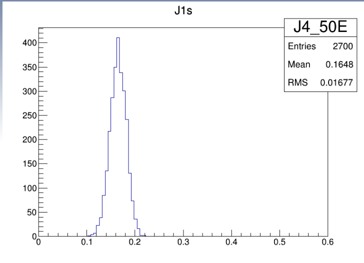

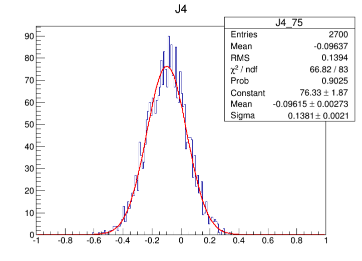

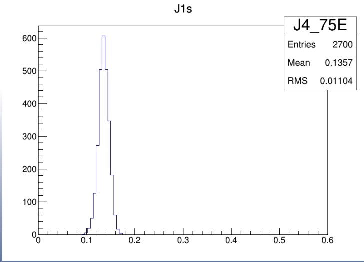

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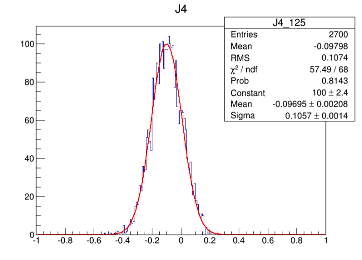

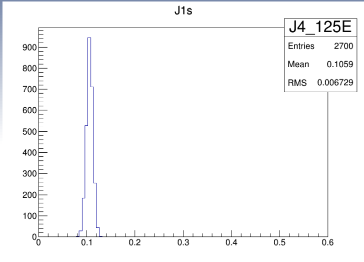

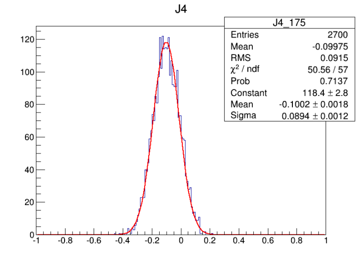

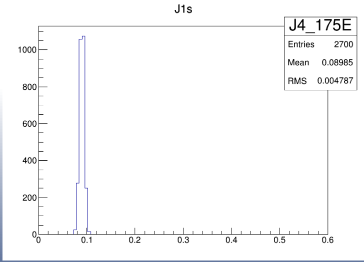

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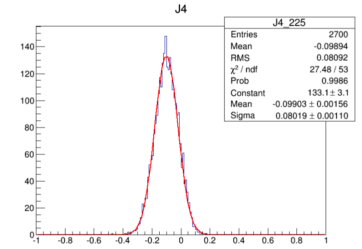

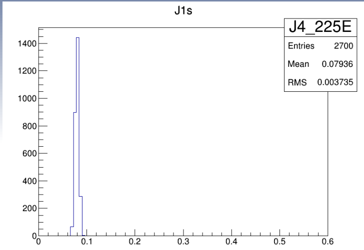

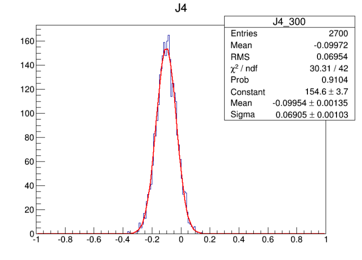

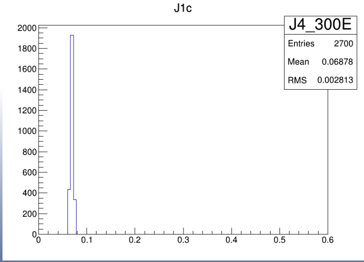

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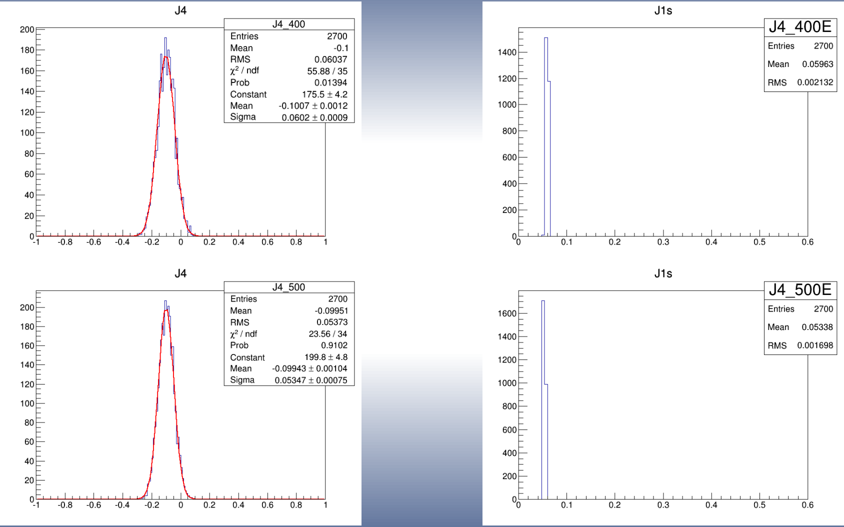

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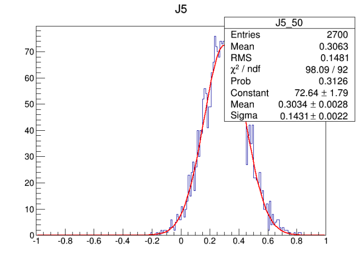

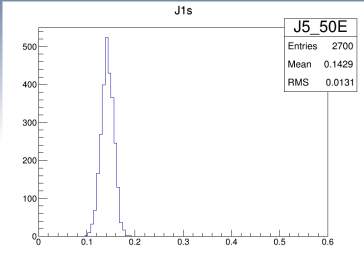

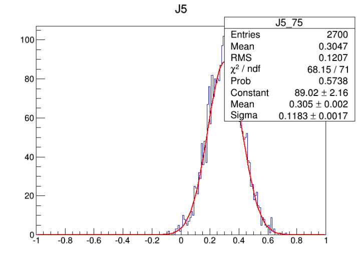

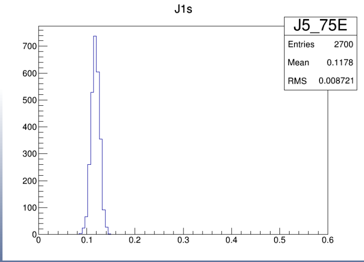

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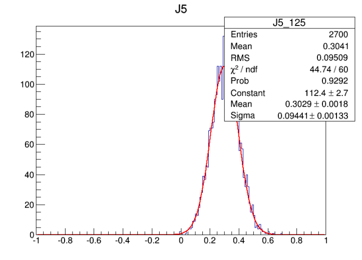

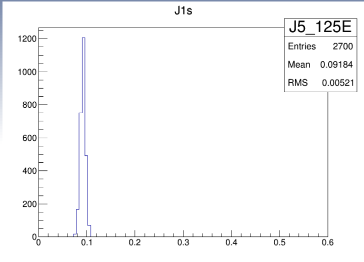

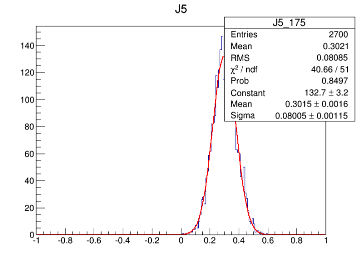

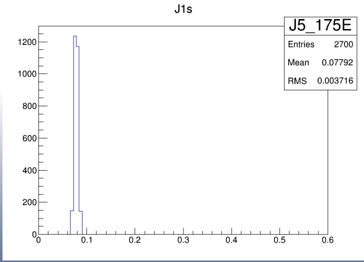

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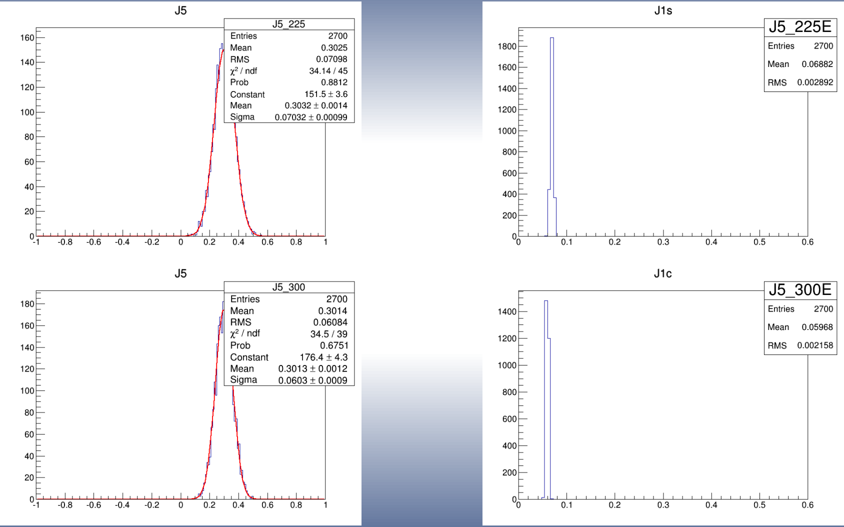

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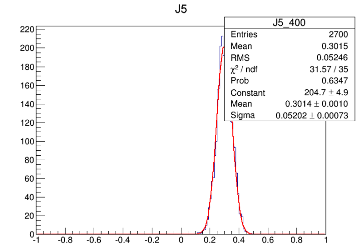

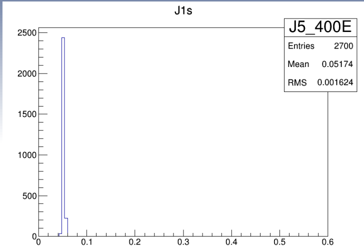

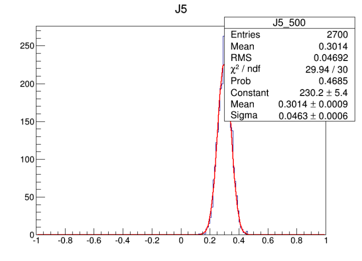

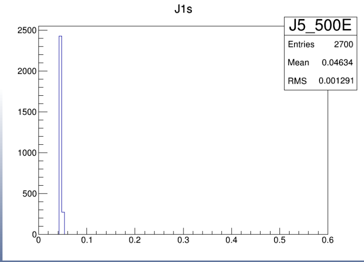

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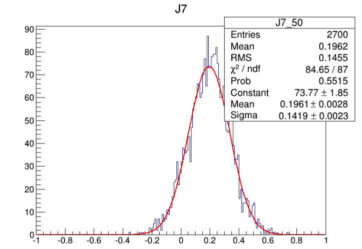

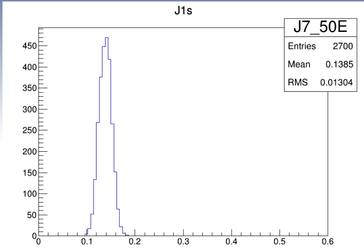

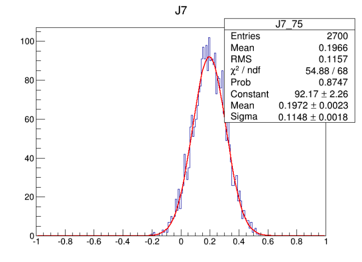

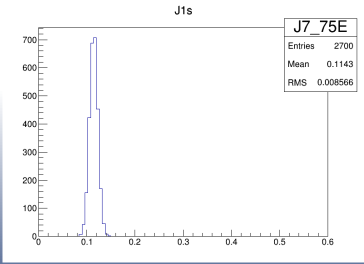

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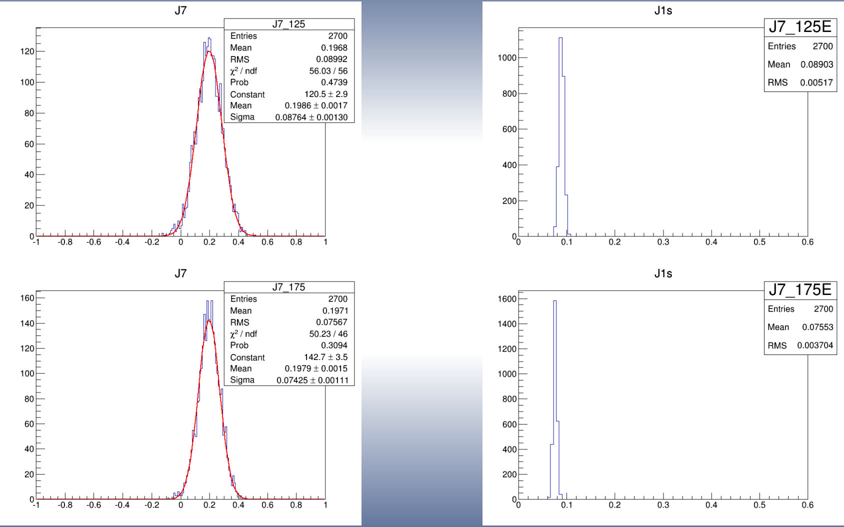

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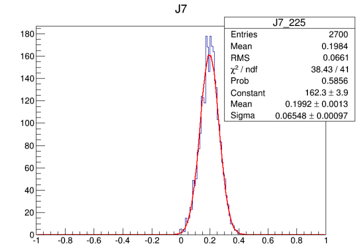

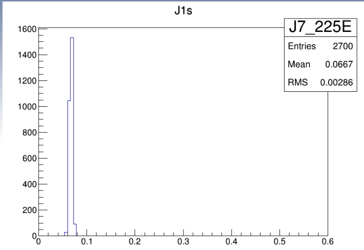

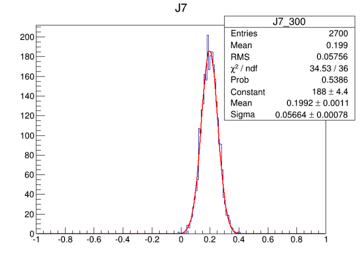

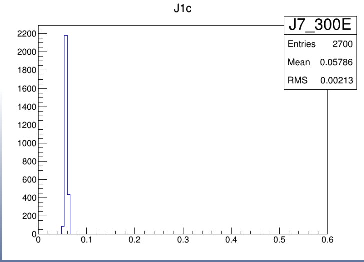

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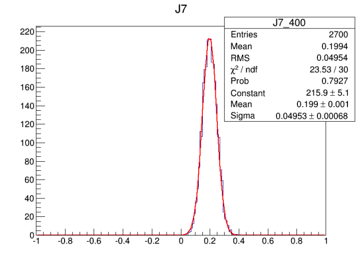

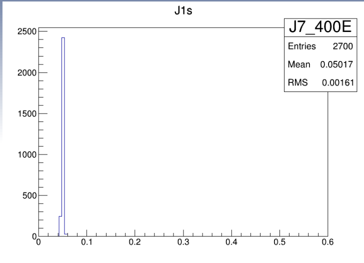

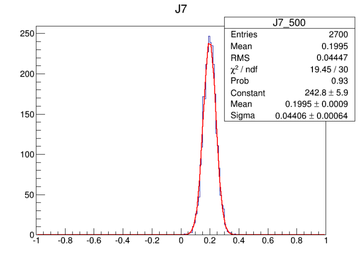

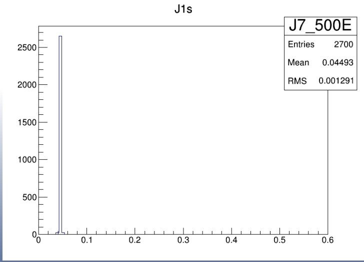

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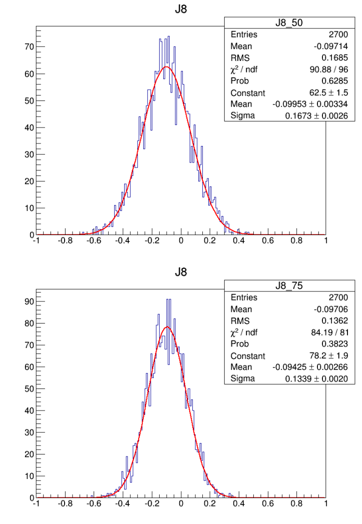

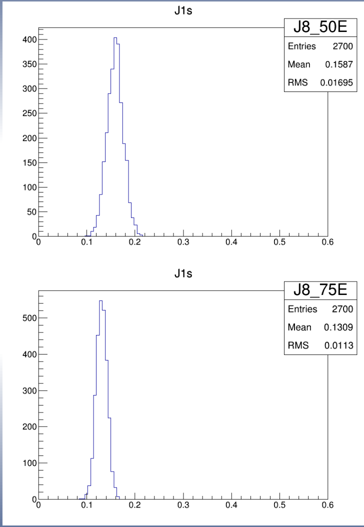

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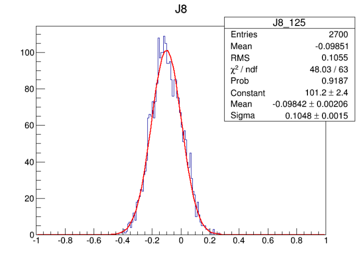

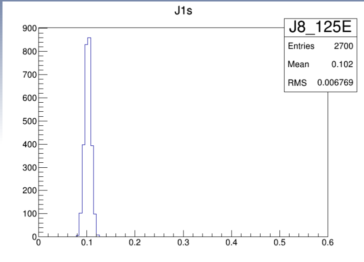

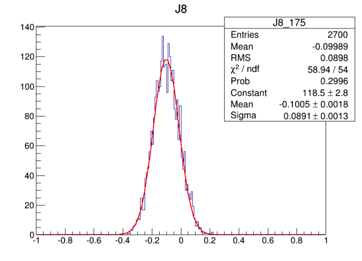

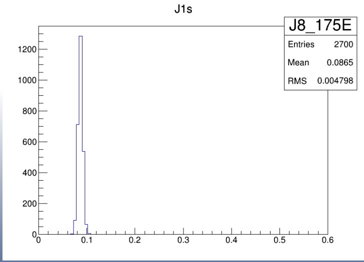

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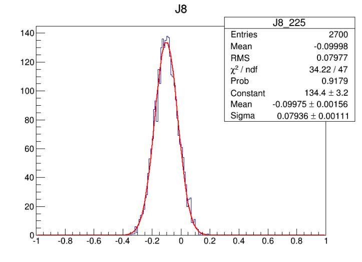

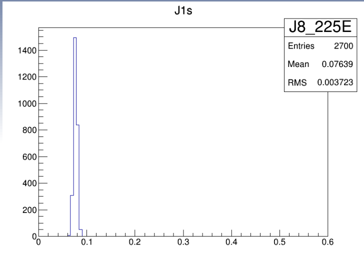

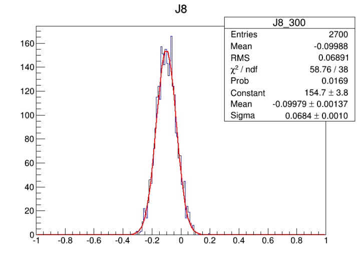

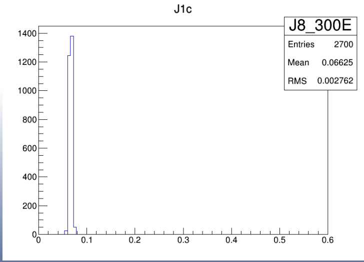

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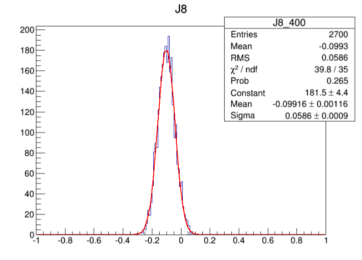

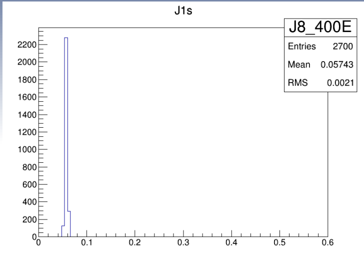

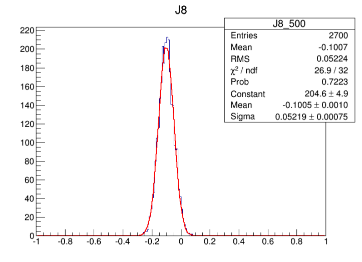

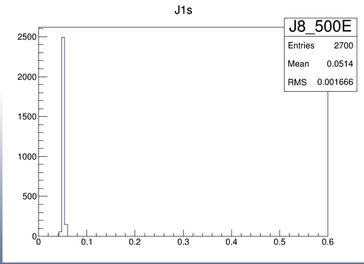

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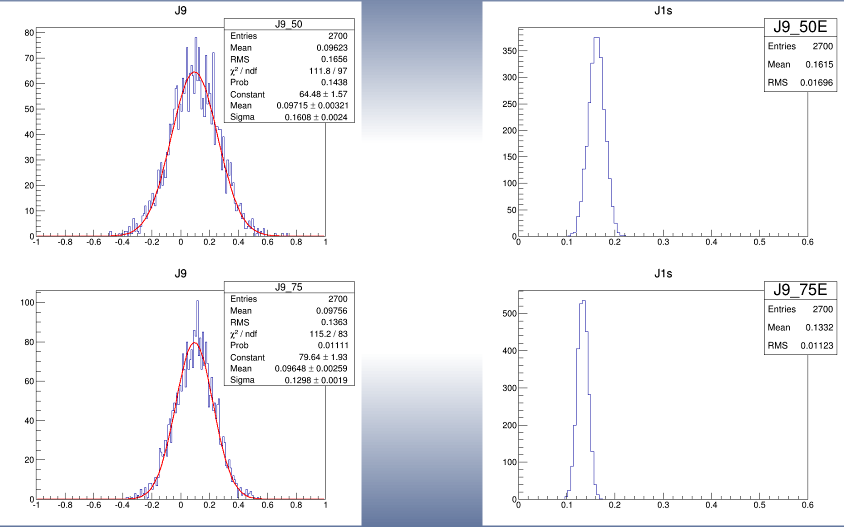
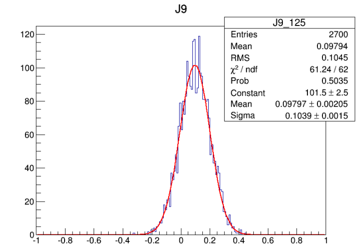
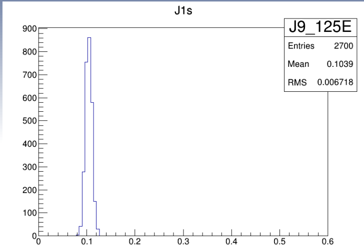



J9
J9_125
Entries 2700
Mean 0.09794
RMS 0.1045
χ² / ndf 61.24 / 62
Prob 0.5035
Constant 101.5 ± 2.5
Mean 0.09797 ± 0.00205
Sigma 0.1039 ± 0.0015
0 20 40 60 80 100 120
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1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J9_125E
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Mean 0.1039
RMS 0.006718
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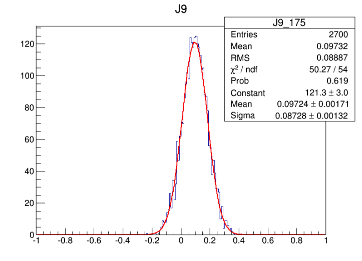
0 0.1 0.2 0.3 0.4 0.5 0.6


J9
J9_175
Entries 2700
Mean 0.09732
RMS 0.08887
χ² / ndf 50.27 / 54
Prob 0.619
Constant 121.3 ± 3.0
Mean 0.09724 ± 0.00171
Sigma 0.08728 ± 0.00132
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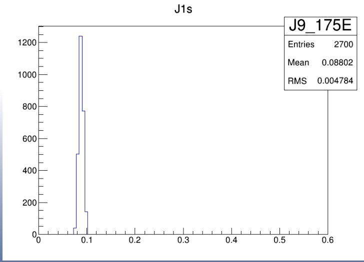
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J9_175E
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RMS 0.004784
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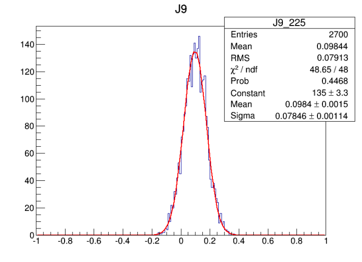
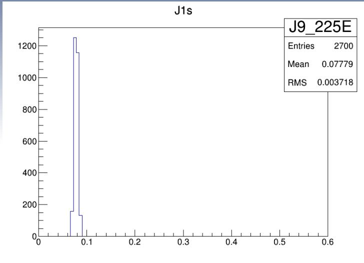
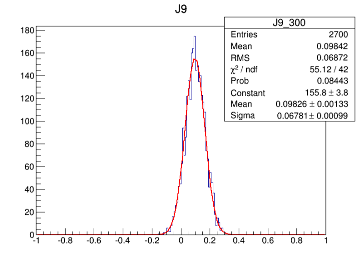
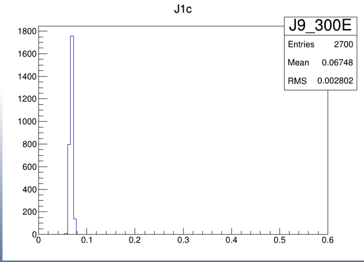
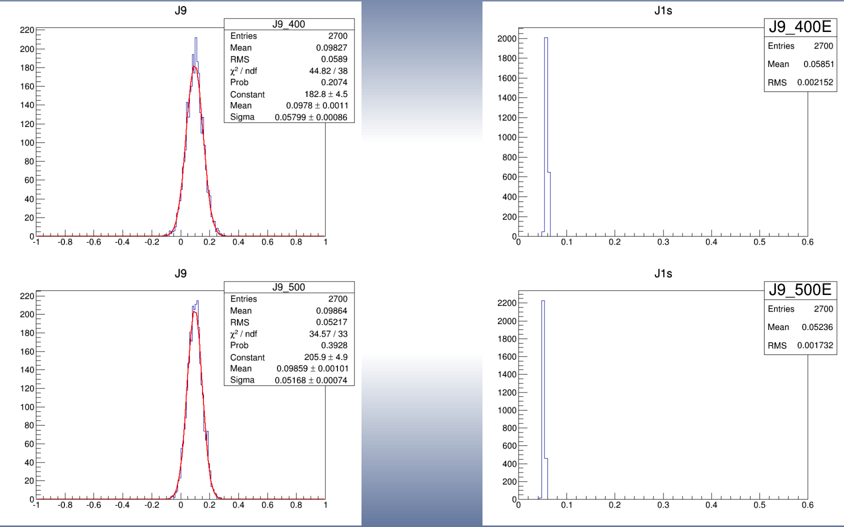


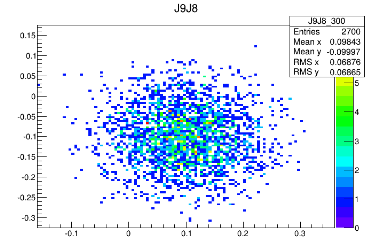






2 am discovery

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J9J8_300
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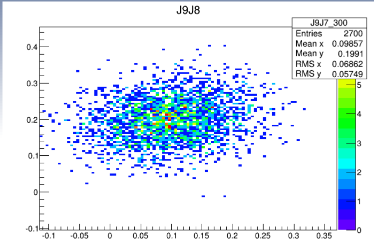
Mean y -0.09997
RMS x 0.06876
RMS y 0.06865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RMS x 0.06862
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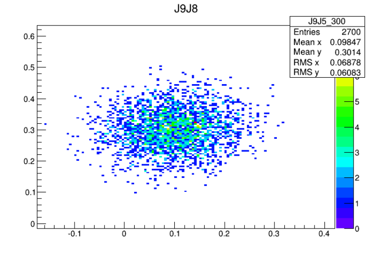
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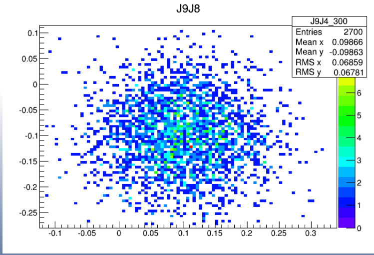
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2 am discovery