

Method of moments for $B^0 \rightarrow K^* \mu\mu$

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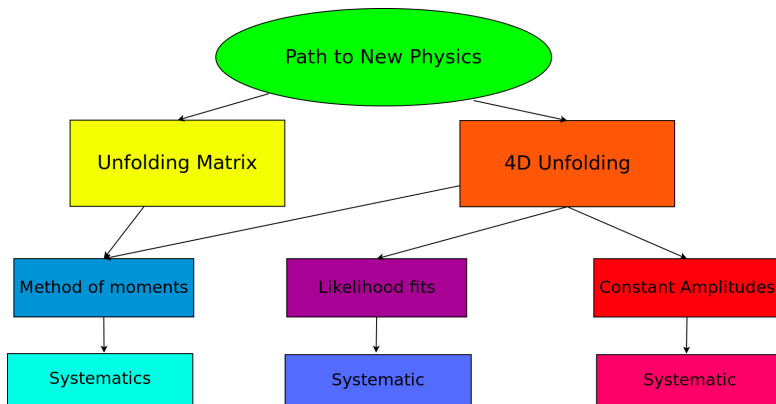
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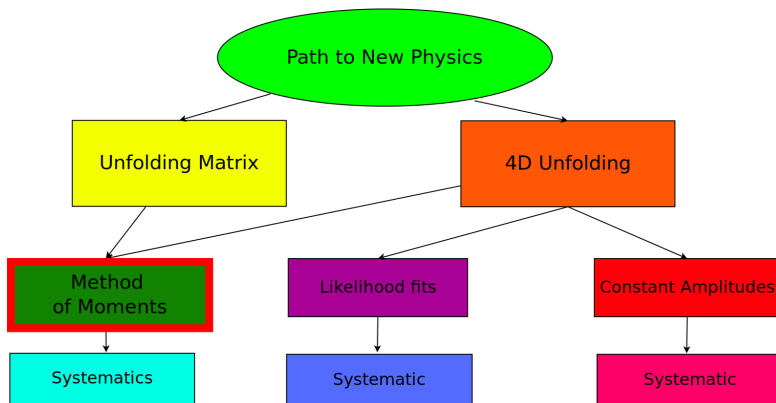
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Quo vadis $B^0 \rightarrow K^* \mu\mu$?



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Why method of moments:

- 1 Complementary approach to LL fits.
- 2 Allows to extract info measuring quantities in event basis depending on the angular distribution.
- 3 Used in $B \rightarrow \rho l \nu$ (SLAC-386 UC-414),
 $J/\psi \rightarrow KK\gamma$ (PRD 71, 032005 (2005)), etc.



Method of moments

Let's assume we have our pdf with k unknown parameters: $PDF(x_i, \alpha)$, $dim(\alpha) = k$. One can calculate k moments, which are the functions of α_j :

$$\mu_i = f(\alpha_1, \dots, \alpha_k) = E[W_i] \quad (1)$$

For n events, we can estimate:

$$\hat{\mu}_i = \frac{1}{n} \sum_{j=0}^{j=n-1} w_j \quad (2)$$

, where $w_j = g(x_i)$

Trivial example

Lets see how this works in practice:

$$f(x) = \frac{x^{a-1}e^{-x/b}}{b^a\Gamma(a)} \quad (3)$$

we measure the moments:

$$m_1 = \frac{X_1 + X_2 + \dots + X_n}{n},$$
$$m_2 = \frac{X_1^2 + X_2^2 + \dots + X_n^2}{n}.$$

and calculate them analytically:

$$m_1 = ab, \quad m_2 = b^2a(a+1)$$

So one just needs to solve this and get the answer:

$$a = \frac{m_1^2}{m_2 - m_1^2}, \quad b = \frac{m_2 - m_1^2}{m_1}$$



The angular terms:

$$\begin{aligned}
 PDF(\cos \theta_k, \cos \theta_l, \phi) = & \frac{9}{32\pi} \left(\frac{3}{4}(1-F_l) \sin^2 \theta_k + F_l \cos^2 \theta_k + \left(\frac{1}{4}(1-F_l) \sin^2 \theta_k \right. \right. \\
 & \left. \left. - F_l \cos^2 \right) \cos 2\theta_l + S_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi + S_4 \sin 2\theta_k \sin \theta_l \cos \phi + \right. \\
 & \left. S_5 \sin 2\theta_k \sin \theta_l \cos \phi + (S_{6s} \sin^2 \theta_k + S_{6c} \cos^2 \theta_k) \cos \theta_l + \right. \\
 & \left. S_7 \sin 2\theta_k \sin \theta_l \sin \phi + S_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right) \quad (4)
 \end{aligned}$$

Since we are fitting a PDF we need to ensure it is normalized:

$$\int_{-\pi}^{\pi} d\phi \int_{-1}^1 d\cos\theta_l \int_{-1}^1 d\cos\theta_k \frac{d^4\Gamma}{dq^2 d\cos\theta_k d\cos\theta_l d\phi} = 1 \quad (5)$$



The angular terms:

$$\begin{aligned}
 PDF(\cos \theta_k, \cos \theta_l, \phi) = & \frac{9}{32\pi} \left(\frac{3}{4}(1-F_l) \sin^2 \theta_k + F_l \cos^2 \theta_k + \left(\frac{1}{4}(1-F_l) \sin^2 \theta_k \right. \right. \\
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 \end{aligned}$$

For further use let's introduce a notation:

$$\begin{aligned}
 PDF(\cos \theta_k, \cos \theta_l, \phi) = & \frac{9}{32\pi} \left(\frac{3}{4}(1 - F_l) \sin^2 \theta_k + F_l \cos^2 \theta_k + \right. \\
 & \left. \left(\frac{1}{4}(1 - F_l) \sin^2 \theta_k - F_l \cos^2 \right) \cos 2\theta_l + \sum_{x=3}^9 S_x f_x(\cos \theta_k, \cos \theta_l, \phi) \right) \quad (5)
 \end{aligned}$$

Moments for $B \rightarrow K^* \mu \mu$ 1/2

Let's calculate the moments (means of the given distribution):

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF(\cos \theta_k, \cos \theta_l, \phi) \sin^2 \theta_k = \frac{2}{5}(2 - F_l) \quad (6)$$

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF(\cos \theta_k, \cos \theta_l, \phi) \cos^2 \theta_k = \frac{1}{5}(2F_l + 1) \quad (7)$$

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF(\cos \theta_k, \cos \theta_l, \phi) \cos^2 \theta_k \cos 2\theta_l = -\frac{2}{25}(2 + F_l) \quad (8)$$

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF(\cos \theta_k, \cos \theta_l, \phi) \sin^2 \theta_k \cos 2\theta_l = -\frac{1}{25}(1 + 8F_l) \quad (9)$$



Moments for $B \rightarrow K^* \mu \mu$ 2/2

Let's calculate the moments (means of the given distribution):

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi \text{PDF}(\cos \theta_k, \cos \theta_l, \phi) f_{S_x} = \frac{8}{25} S_x, \quad (10)$$

for $x = 3, 4, 8, 9$, and:

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi \text{PDF}(\cos \theta_k, \cos \theta_l, \phi) f_{S_x} = \frac{2}{5} S_x, \quad (11)$$

for $x = 5, 6, 7$.

New physics apparently as we like orthogonal world:

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi (f_{S_x} \times f_{S_y}) = \alpha_{xy} \delta_x y \quad (12)$$



Moments for $B \rightarrow K^* \mu \mu$

- We are abusing the fact that the basis is orthogonal and moments do not mix.
- Makes live easier and reduces the systematics.
- Each of the S does not know about other.
- In case of full PDF, S_{1s} , S_{2s} , S_{1c} , S_{2c} , S_{6s} , S_{6c} are not orthogonal.
- Still we can get them solving equation system:

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin^2\theta_k \cos\theta_l = 0.1(S_{6c} + 4S_{6s}) \quad (13)$$

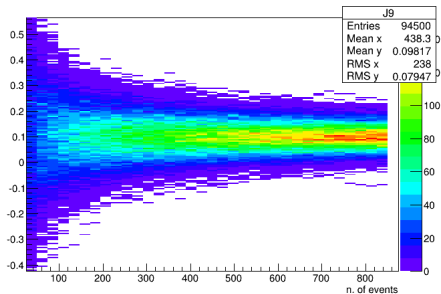
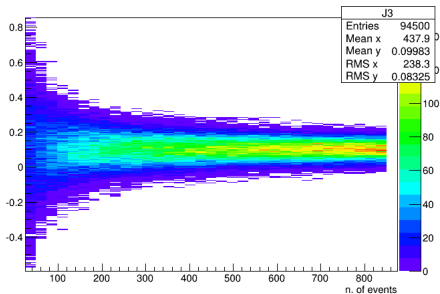
$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \cos\theta_l = 0.25(S_{6c} + 2S_{6s}) \quad (14)$$

solution: $S_{6c} = 2(4M_{S_{6c}} - 5M_{S_{6s}})$, $S_{6s} = -2M_{S_{6c}} + 5M_{S_{6s}}$



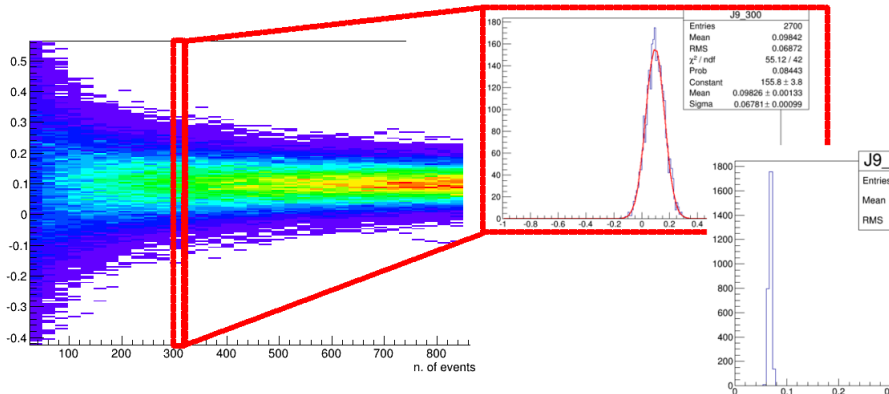
Moments for $B \rightarrow K^* \mu\mu$

Lets see if this method actually works. Let's take some random parameters for the PDF and make a toy.



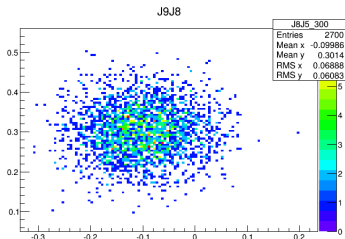
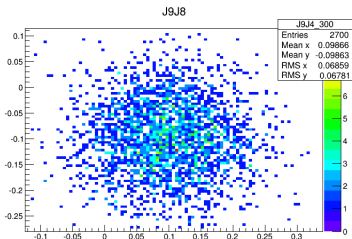
Error Estimation

- Since moment is the mean of a given distribution the error can be estimated as $mean/RMS$
- use TOY MC to check this assumption
- Do not worry, detail description an numbers will come in other presentation.



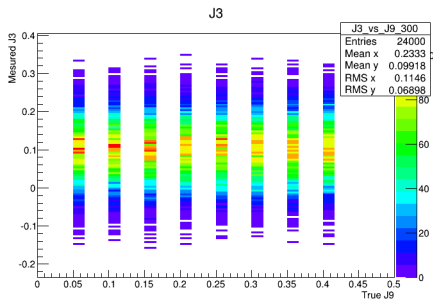
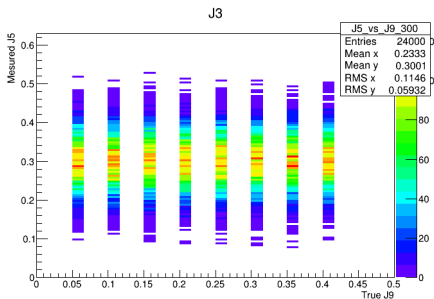
Correlation check

- In theory S_i shouldn't be correlated to S_j in the moment calculation.
- Lets put this to a test.



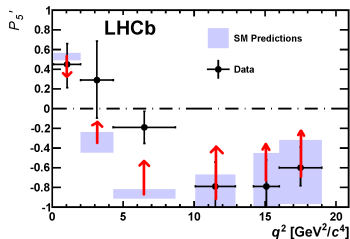
Correlation check 2

- Let's now FIX J_x and simulate different J_y
- Again theory would suggest that one J shouldn't know about the other, so J_x shouldn't change with scanning J_y parameter



S-wave pollution

- Unfortunately in our perfect orthogonal world lives an imposter.
- This imposter is $B^0 \rightarrow (K\pi)_{S\text{-wave}} \mu\mu$
- This "ghost" dilutes our NP! Like dark matter the universe.
- We need something to bust this ghost away



S-wave hunting

Our PDF with the S-wave will look as follows:

$$\begin{aligned} PDF_{full}(\cos \theta_k, \cos \theta_l, \phi) = & \frac{9}{32\pi} ((1 - F_s) \left(\frac{3}{4}(1 - F_l) \sin^2 \theta_k + F_l \cos^2 \theta_k + \right. \\ & \left. \left(\frac{1}{4}(1 - F_l) \sin^2 \theta_k - F_l \cos^2 \theta_k \right) \cos 2\theta_l + S_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi + \right. \\ & S_4 \sin 2\theta_k \sin \theta_l \cos \phi + S_5 \sin 2\theta_k \sin \theta_l \cos \phi + \\ & (S_{6s} \sin^2 \theta_k + S_{6c} \cos^2 \theta_k) \cos \theta_l + S_7 \sin 2\theta_k \sin \theta_l \sin \phi + \\ & \left. S_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right) + \\ & \frac{2}{3} F_s \sin^2 \theta_l + \frac{4}{3} A_s \sin^2 \theta_l \cos \theta_k + I_4 \sin \theta_k \sin 2\theta_l \cos \phi \\ & + I_5 \sin \theta_k \sin \theta_l \cos \phi + I_7 \sin \theta_k \sin \theta_l + \sin \phi + I_8 \sin \theta_k \sin 2\theta_l \sin \phi \end{aligned} \quad (15)$$

In this form we ensure normalization.

How does the dilution work? 1/2

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi \text{PDF}_{full}(\cos \theta_k, \cos \theta_l, \phi) f_{S_x} = \frac{8}{25} S_x (1 - F_s), \quad (16)$$

for $x = 3, 4, 8, 9$, and:

$$\int_{-1}^1 d \cos \theta_l \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi \text{PDF}_{full}(\cos \theta_k, \cos \theta_l, \phi) f_{S_x} = \frac{2}{5} S_x (1 - F_s), \quad (17)$$

for $x = 5, 6, 7$.

Not much harm and easy to control.

How does the dilution work? 2/2

Unfortunately F_I and F_S will mix with each other:

$$\int_{-1}^1 d \cos \theta_I \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF_{full}(\cos \theta_k, \cos \theta_I, \phi) \sin^2 \theta_k = \frac{2}{15}(6 + 3F_I(F_S - 1) - F_S) = M_{F_I} \quad (18)$$

$$\int_{-1}^1 d \cos \theta_I \int_{-1}^1 d \cos \theta_k \int_{-\pi}^{\pi} d \phi PDF_{full}(\cos \theta_k, \cos \theta_I, \phi) \sin^2 \theta_I = \frac{1}{5}(3 + F_I + F_S - F_I F_S) = M_{F_S} \quad (19)$$

They can solve this system:

$$\begin{cases} F_S = \frac{15}{4}(M_{F_I} + 2M_{F_S}) \\ F_I = \frac{(15M_{F_I} + 10M_{F_S} - 18)}{(15M_{F_I} + 30M_{F_S} - 34)} \end{cases}$$

S-wave moments

We can even measure directly the S-wave:

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin^2\theta_l \cos\theta_k = \frac{32I_{1b}}{45} \quad (20)$$

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin\theta_k \sin 2\theta_l \cos\phi = \frac{16I_4}{45} \quad (21)$$

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin\theta_k \sin\theta_l \cos\phi = \frac{4I_5}{9} \quad (22)$$

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin\theta_k \sin 2\theta_l \sin\phi = \frac{4I_7}{9} \quad (23)$$

$$\frac{d^3\Gamma}{\Gamma d\cos\theta_k d\cos\theta_l d\phi} \sin\theta_k \sin 2\theta_l \sin\phi = \frac{16I_8}{45} \quad (24)$$



Conclusions

- Method of moments very suitable for $B^0 \rightarrow K^* \mu \mu$.
- The method converge fast and works for the "simple case", i.e. signal only.
- Method very insensitive to S-wave component, thanks to orthogonality.
- Complementary one can measure in-dependent S-wave component.
- No problem with boundary problems.

What comes in the next talks(stay tuned):

- Sensitivity will be given tmr.
- This method reduces the error on unfolding.
- Systematics easy accessible.

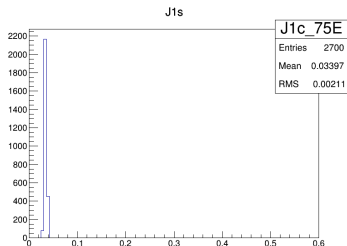
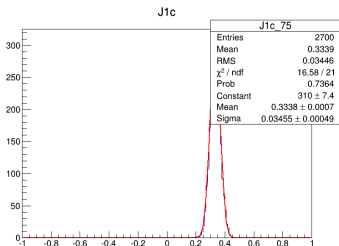
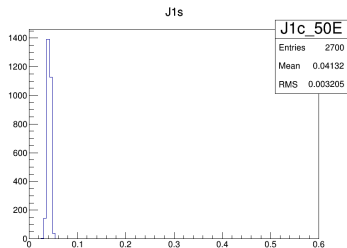
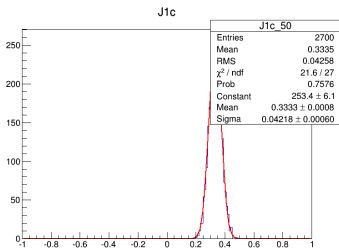


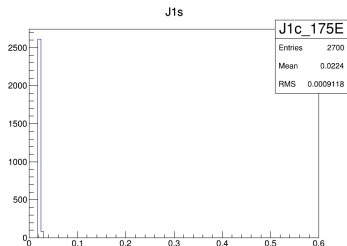
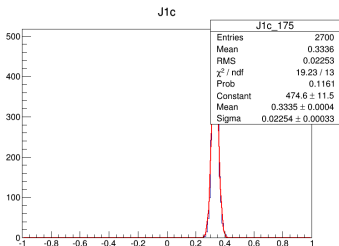
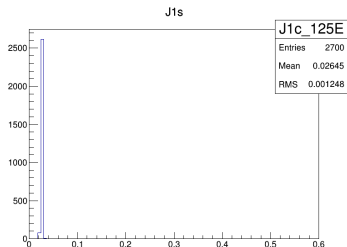
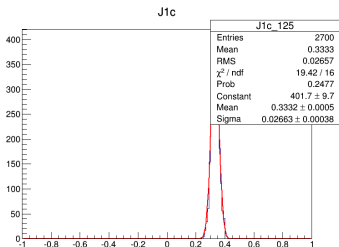
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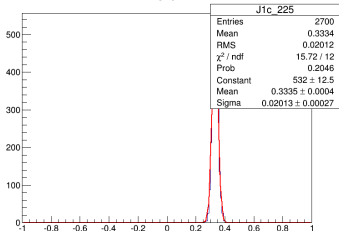
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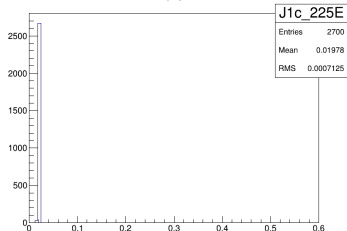




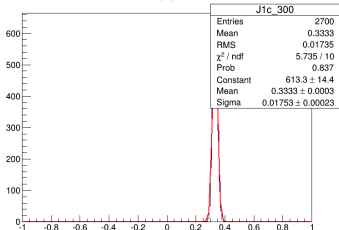
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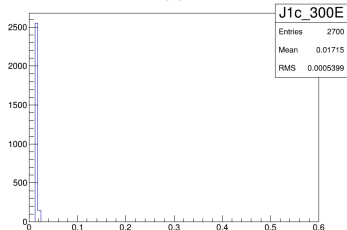
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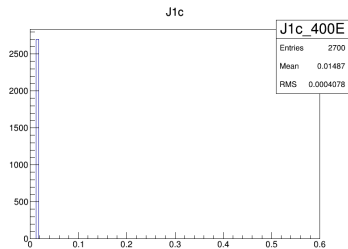
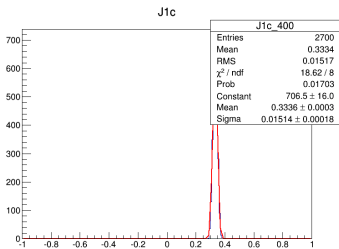


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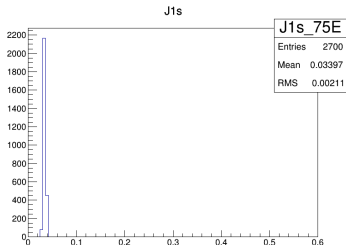
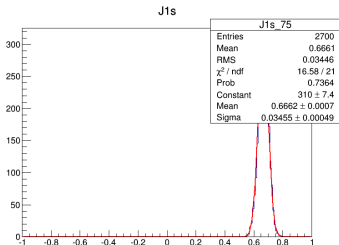
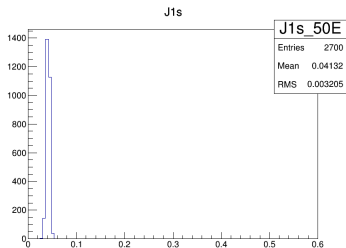
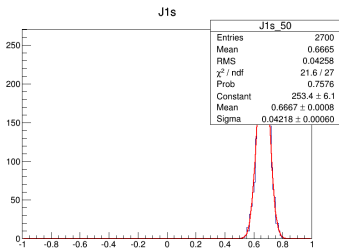
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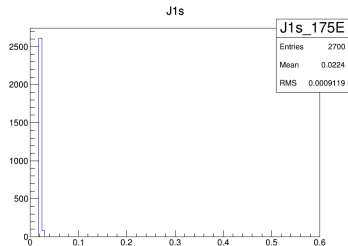
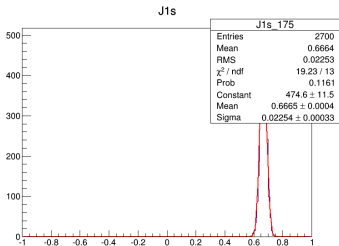
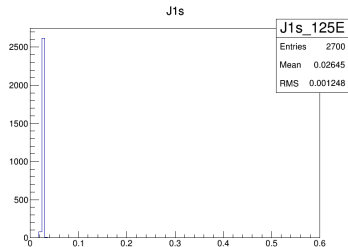
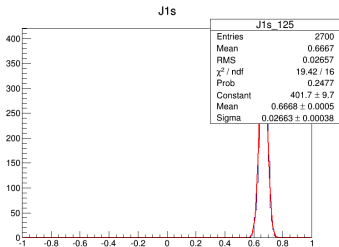




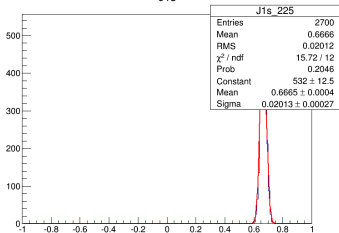
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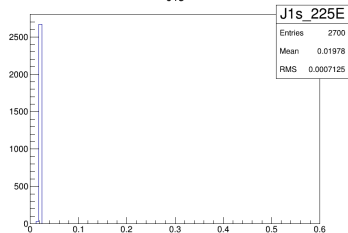




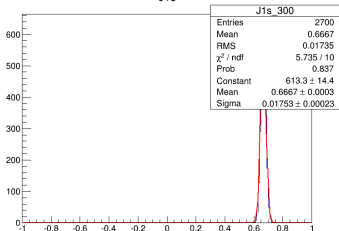
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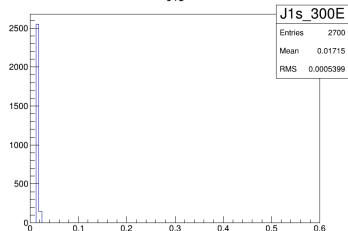
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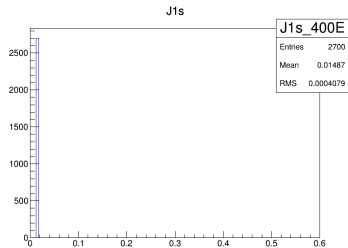
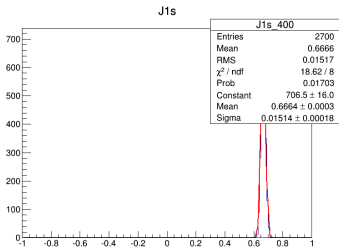


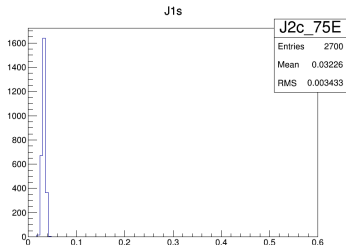
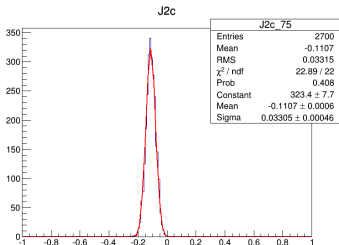
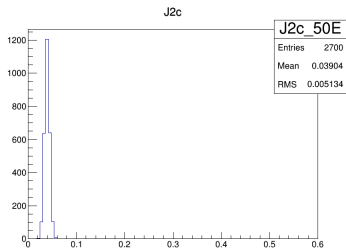
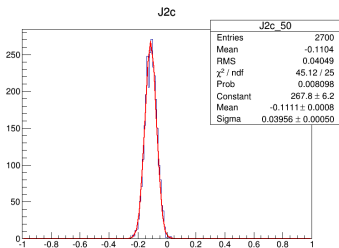
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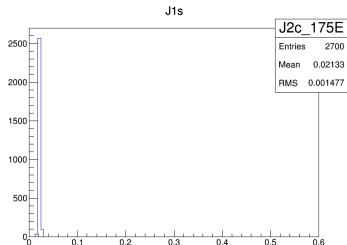
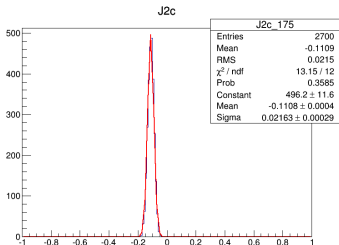
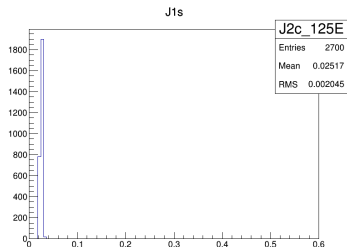
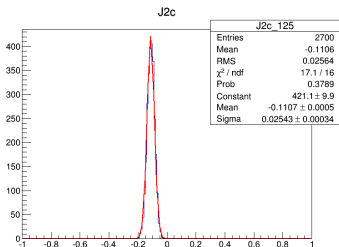


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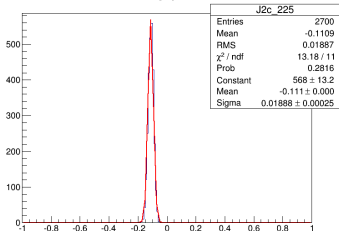




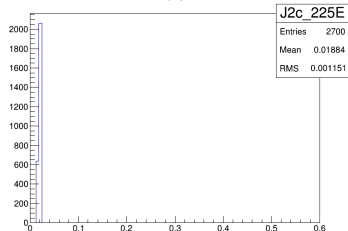




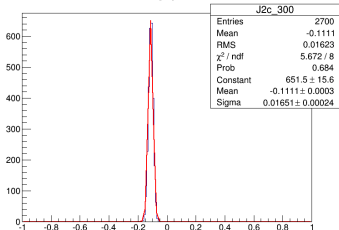
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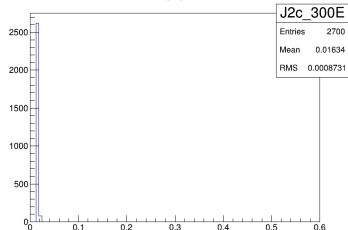
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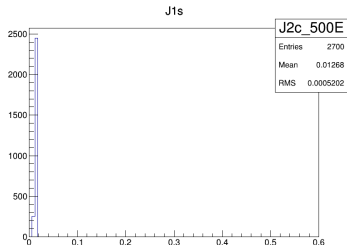
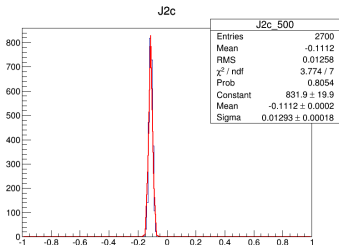
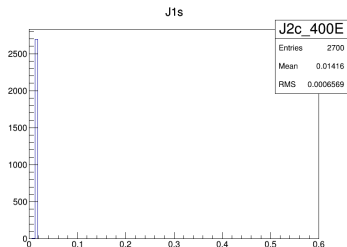
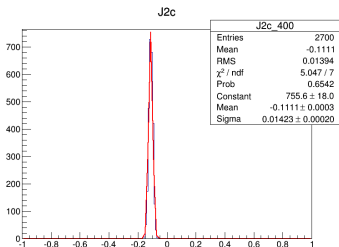


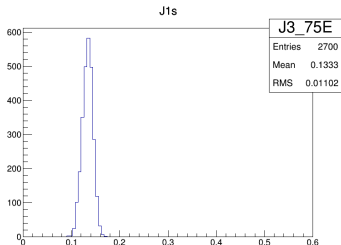
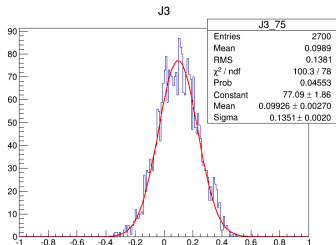
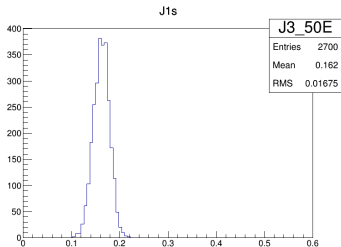
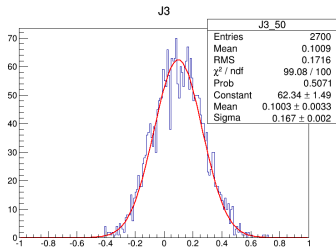
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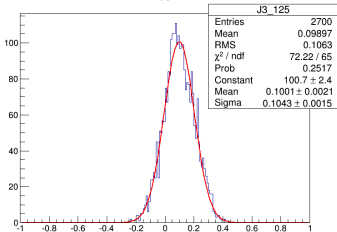
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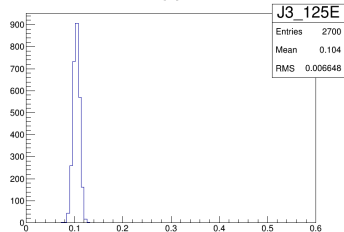




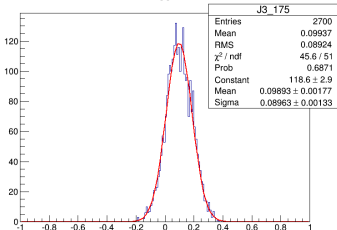
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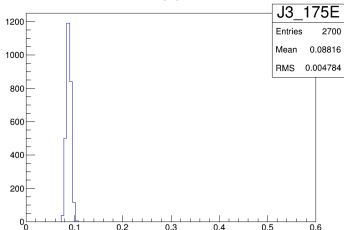
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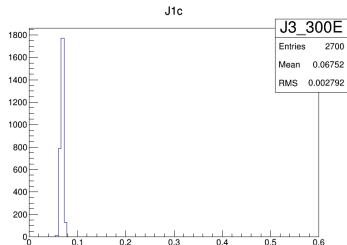
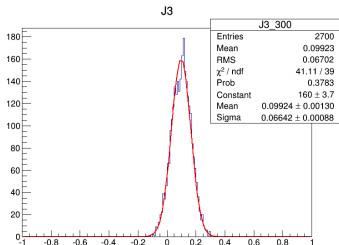
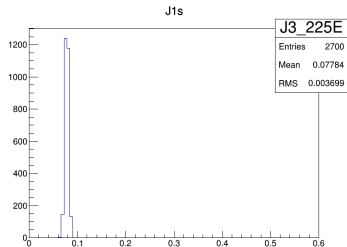
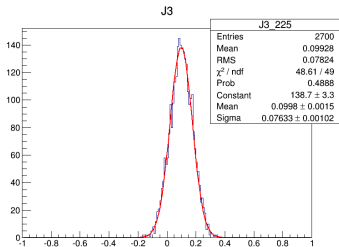


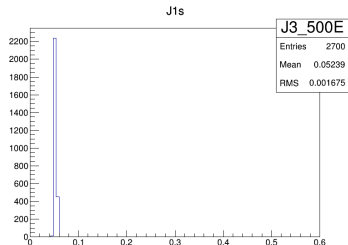
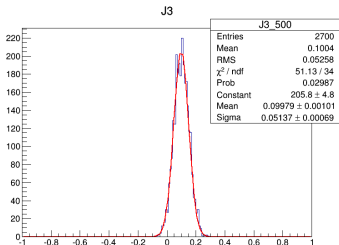
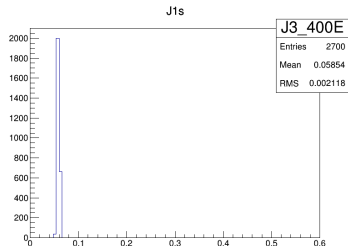
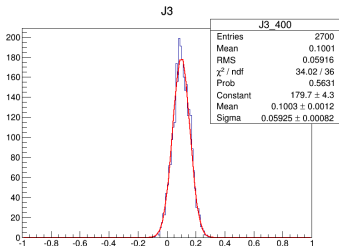
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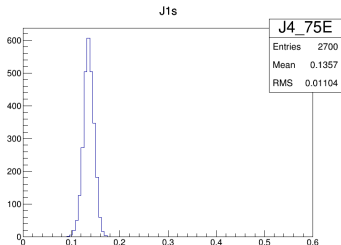
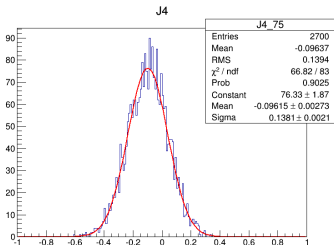
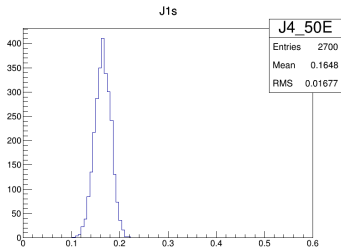
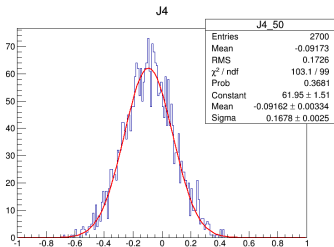


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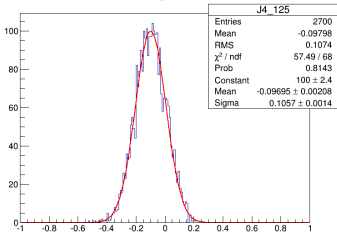




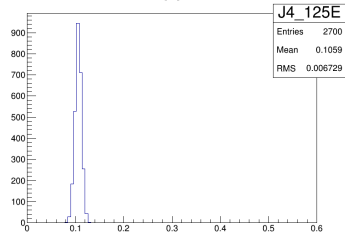




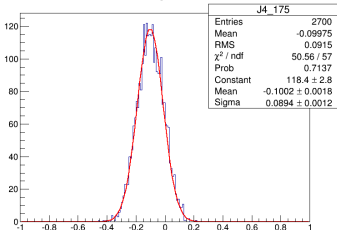
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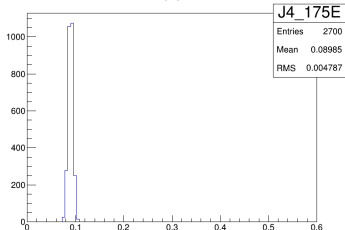
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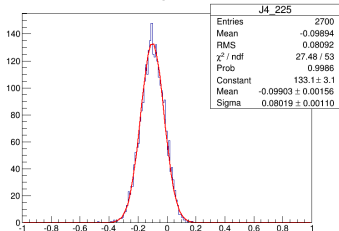
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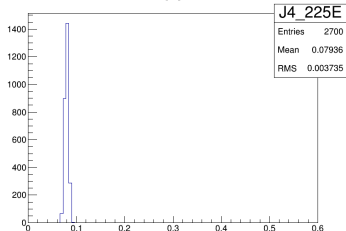
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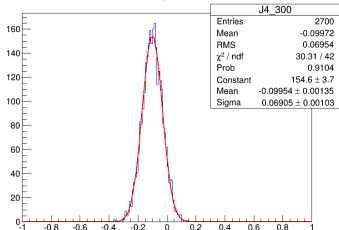
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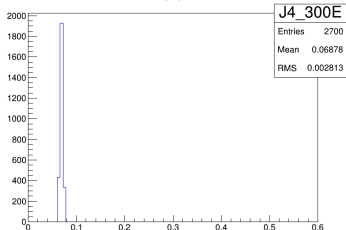
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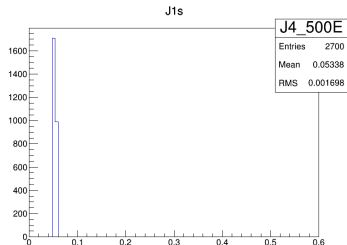
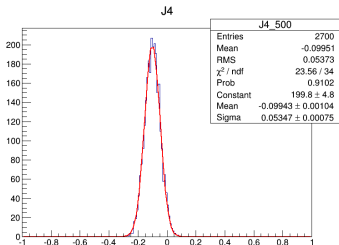
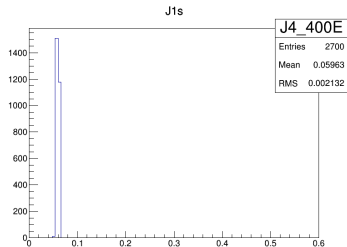
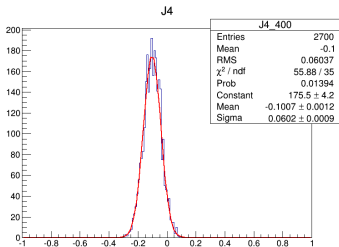


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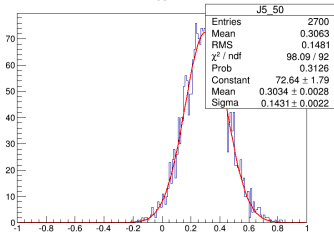


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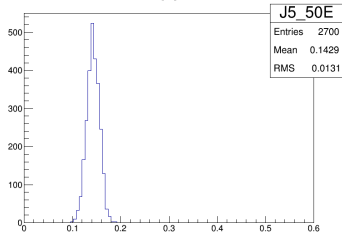




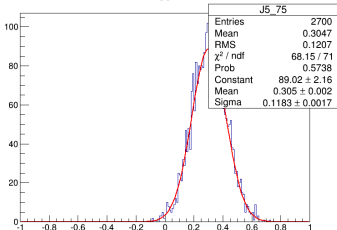
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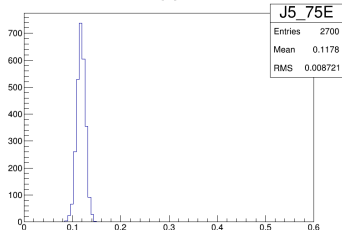
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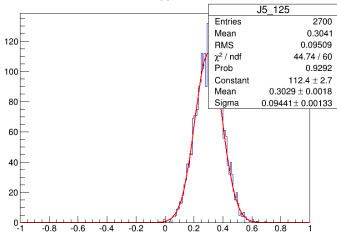
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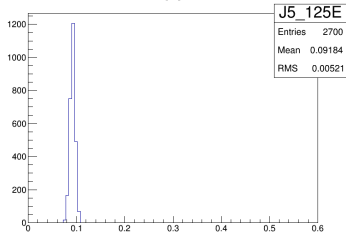
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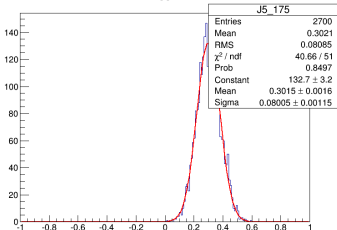
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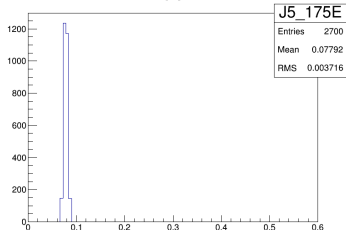
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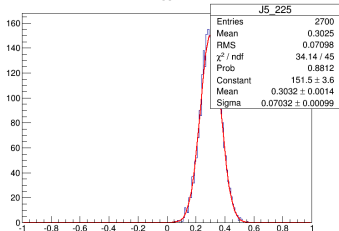
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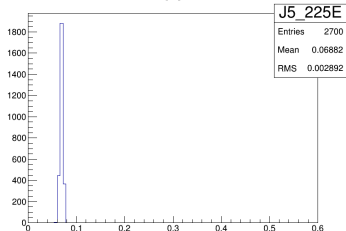
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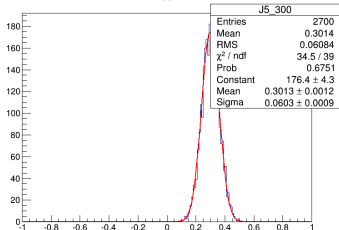
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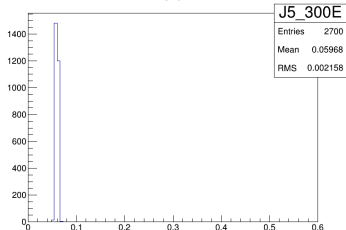
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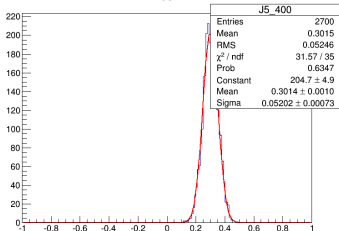
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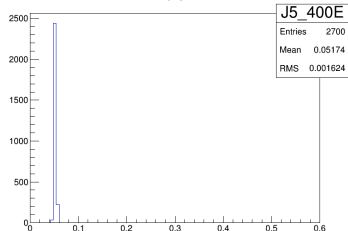
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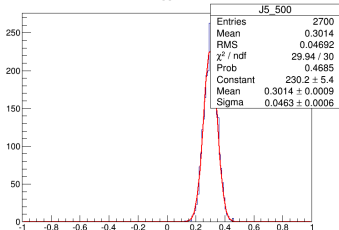
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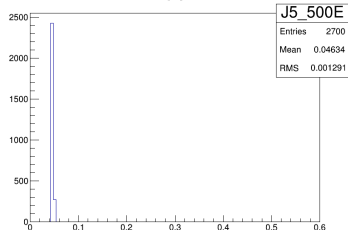
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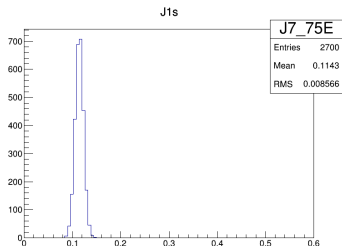
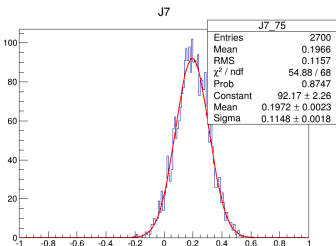
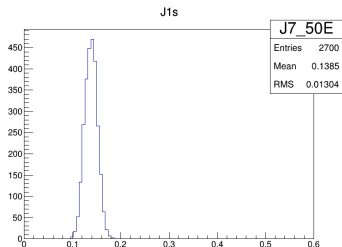
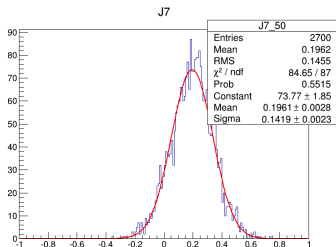


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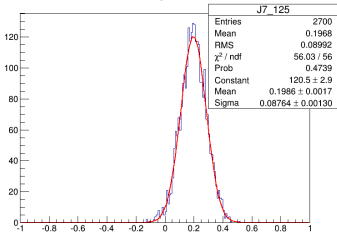


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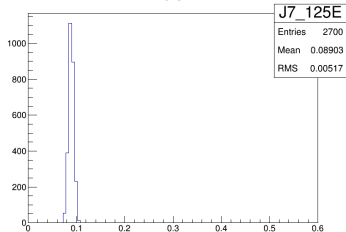
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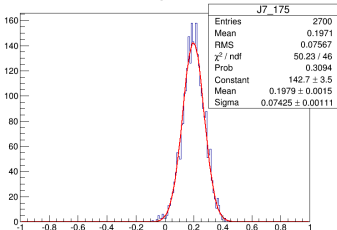
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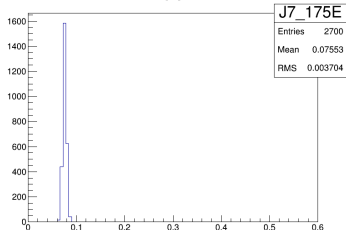
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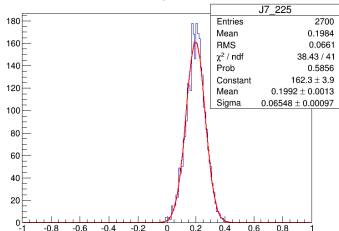
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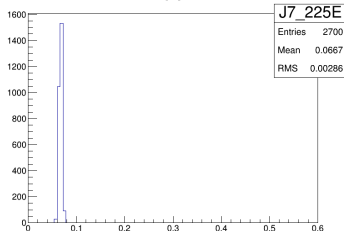
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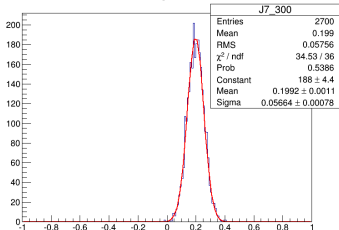
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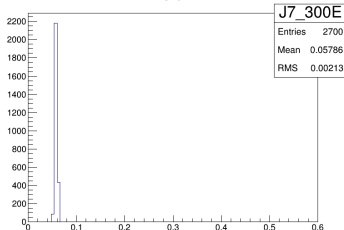
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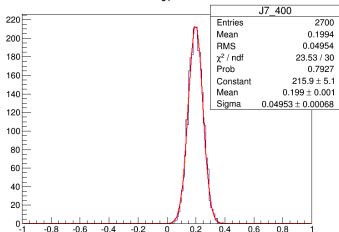
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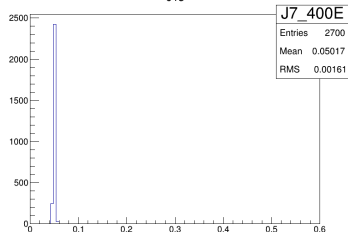
J1c

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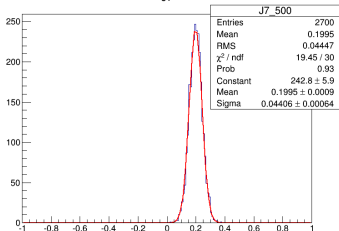
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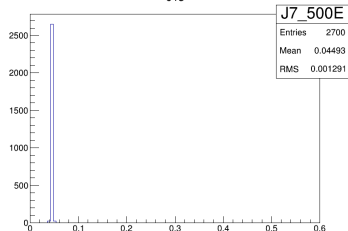
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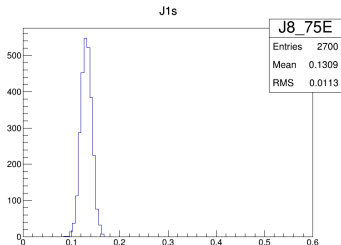
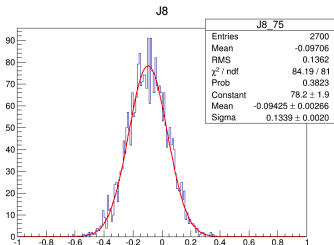
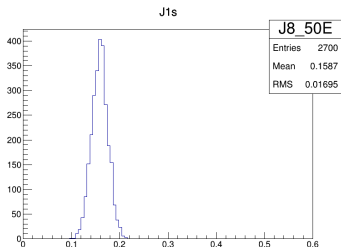
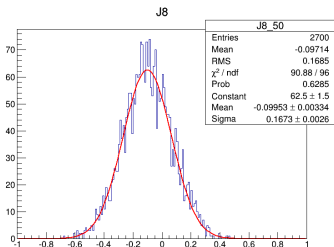


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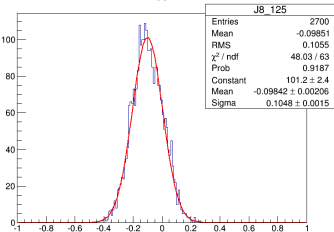


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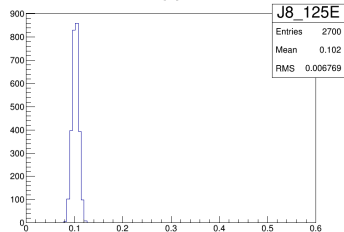
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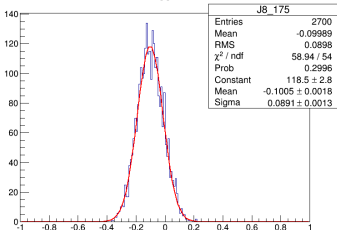
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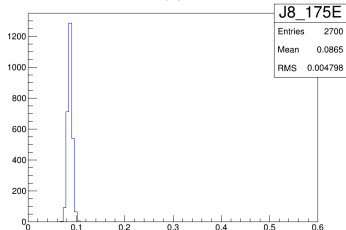
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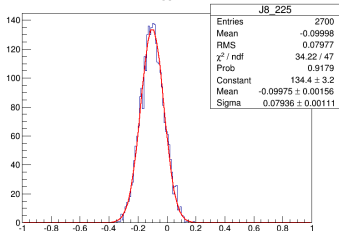
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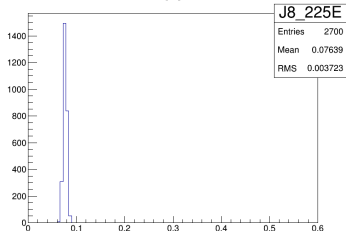
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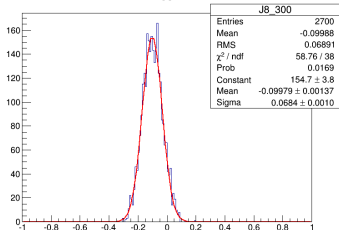
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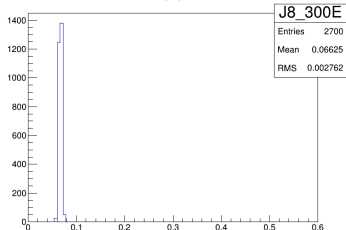
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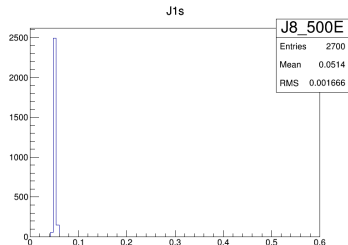
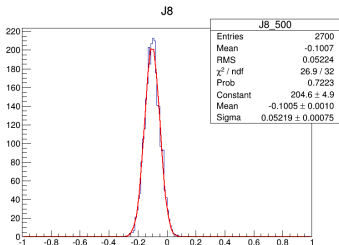
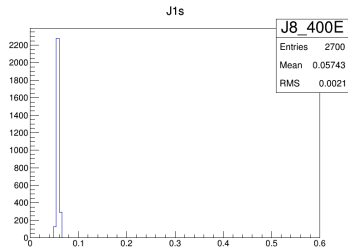
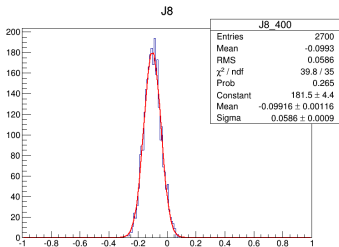


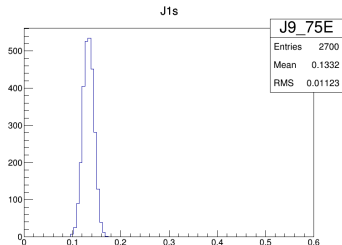
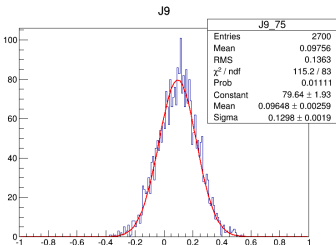
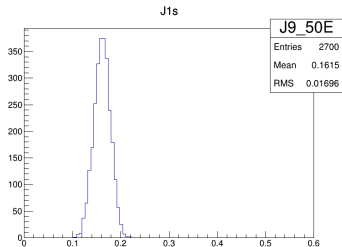
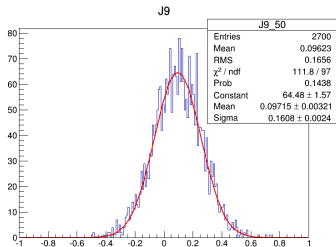
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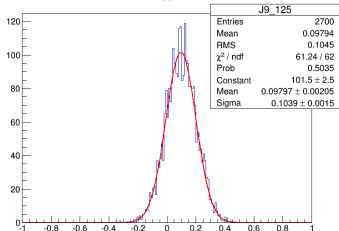
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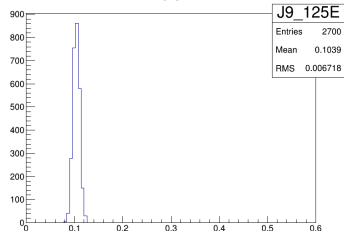




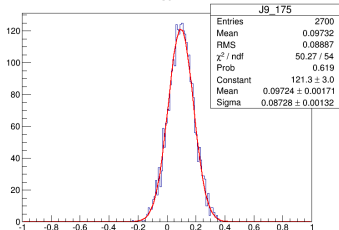
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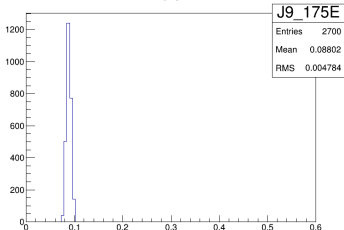
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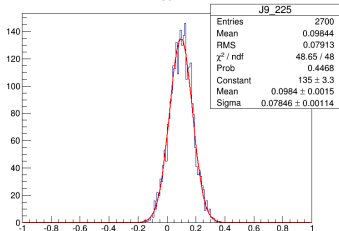
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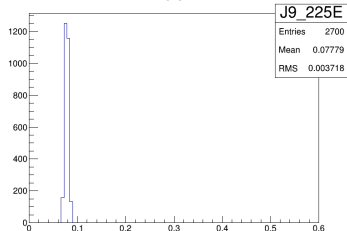
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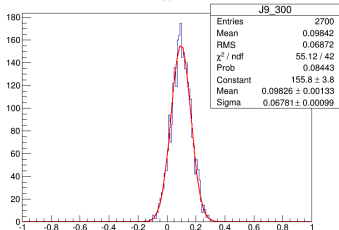
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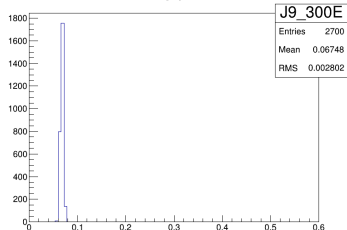
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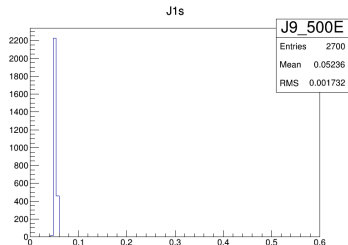
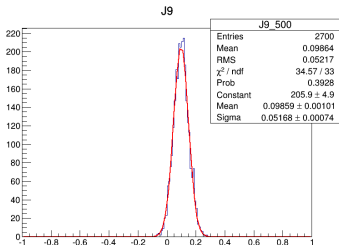
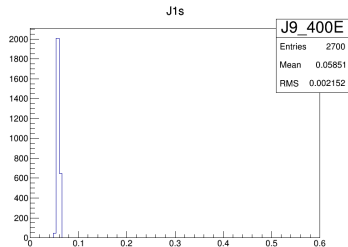
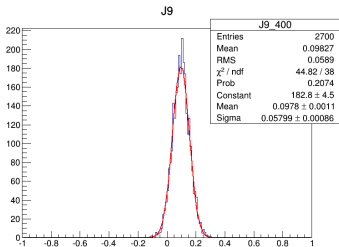


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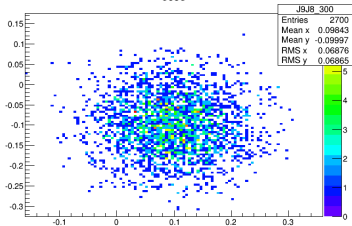


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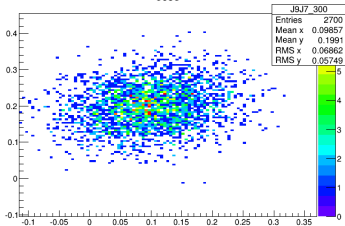




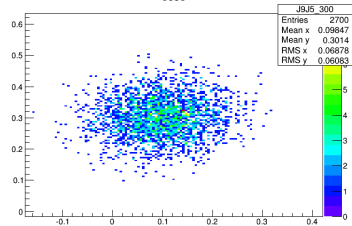
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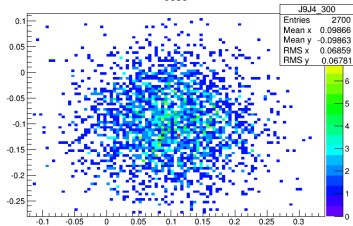
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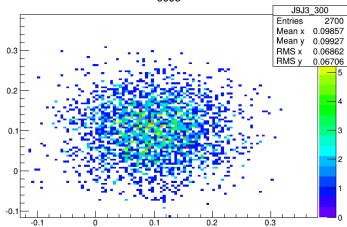
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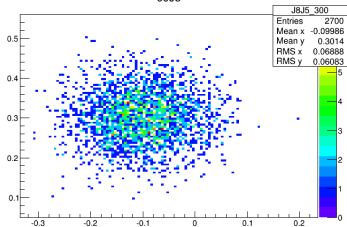
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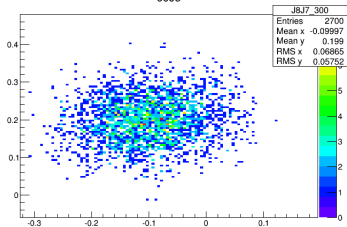
J9J8



J9J8



J9J8



J9J8

