Jacobian for $B^0 \to K^* \mu^- \mu^+$ proposed solution

$B^0 \to K^* \mu^- \mu^+$ team

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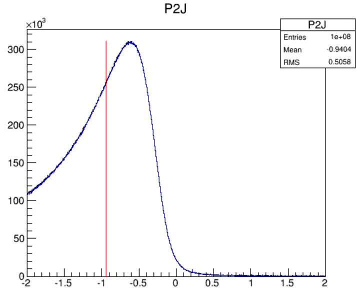
Reminder

- We wanted to calculate the P_i from S_i.
- Both Toy MC error propagation (generating toy experiments based on the covariance matrix) and bootstrapping the data set produces distribution that has a most probable value that is different to the central value in the data (see plot below, most probable value from toys is different then the generated one (red line)).
- As discussed during the referee meeting we considered including the Jacobian the this picture.



Introduction

- Lets write down explicit on what we all agree (I hope at least ;)).
  - Measurement of $\overrightarrow{S} = (F_l,\ S_x)$ is unbiased.
  - Error is also correctly estimated ensuring the correct coverage.
- The questions what I am answering: what is the corresponding confidence and probability distribution in a new space: $\overrightarrow{P} = (F_l,\ P_x)$.
- To put it a bit more simple: I want to map one space on the other one.
- NB: This is a different question than what is the distribution of P measured by the experiments.

Some mathematical theorems assumptions 1

- We have our standard transformation of ($\overrightarrow{S} \rightarrow \overrightarrow{P}$):

$$
\begin{aligned}
F_l &\leftarrow F_l \\
P_1 &\leftarrow 2\frac{S_3}{1-F_{\rm L}} \\
P_2 &\leftarrow \frac{1}{2}\frac{S_6^s}{1-F_{\rm L}} = \frac{2}{3}\frac{A_{\rm FB}}{1-F_{\rm L}} \\
P_3 &\leftarrow -\frac{S_9}{1-F_{\rm L}} \\
P_4' &\leftarrow \frac{S_4}{\sqrt{F_{\rm L}(1-F_{\rm L})}} \\
P_5' &\leftarrow \frac{S_5}{\sqrt{F_{\rm L}(1-F_{\rm L})}} \\
P_6' &\leftarrow \frac{S_7}{\sqrt{F_{\rm L}(1-F_{\rm L})}} \\
P_8' &\leftarrow \frac{S_8}{\sqrt{F_{\rm L}(1-F_{\rm L})}}.
\end{aligned}
$$

Some mathematical theorems assumptions 2

- We know about this transformation:
  - The parameter space is bounded domain (D) ✓
  - The angular PDF is smooth function in the domain ✓
  - There exists 1:1 transformation between $\overrightarrow{S}$ and $\overrightarrow{P}$ ✓
  - Inside the domain the Jacobian is non-zero. ($J \neq 0$) ✓
- Next slide you will know why those assumptions are needed.

Some mathematical theorems assumptions 3

- Now since there is 1:1 correspondence the central point in the $\overrightarrow{P}$ should be derived from the central point of the $\overrightarrow{S}$ basis.
- Now the confidence belt. In the $\overrightarrow{S}$ a 68% confidence belt (D) is:

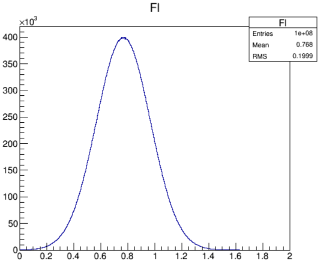
$$\int_D f(\overrightarrow{S})d\overrightarrow{S} = 0.68$$

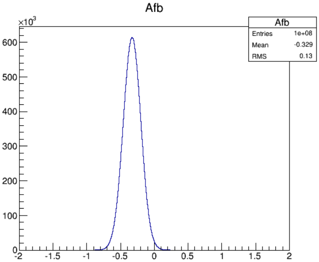
- In this equation our D is effectively the errors that we quote.
- Now form analysis thats to previous slide we can write :

$$\int_D \underbrace{f(\overrightarrow{S})}_{\text{What we simulate/bootstrap}} d\overrightarrow{S} = \int_\Delta \underbrace{f'(\overrightarrow{P})}_{\text{What we get in P}} \times|J|d\overrightarrow{P}$$

Toys

- So to get the integral correct we need to take the Jacobian into account.
- Let's make a toy example calculating P_2. Values used (Gaussian distributed: mean $\pm$ error): $F_l = 0.7679 \pm 0.2$, $A_{FB} = -0.329 \pm 0.13$.
- The Jacobian: $J = \dfrac{2}{3}\dfrac{1}{1 - F_L}$
- Generated F_l and A_{FB}:

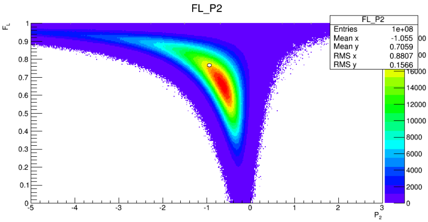




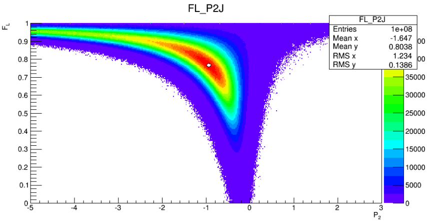
Toys

- Now how does the new space look like.
- Important to take into account the boundary as without all my theorems fall down.
- The white point is the value from which the toy was generated.

Scatter plot $F_L : P_2$, no Jacobian



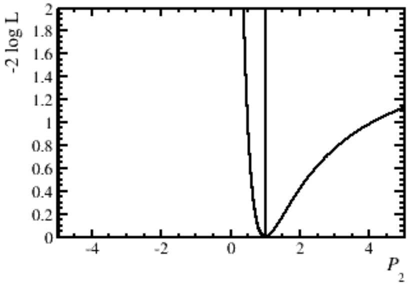
Scatter plot $F_L : P_2$, with Jacobian



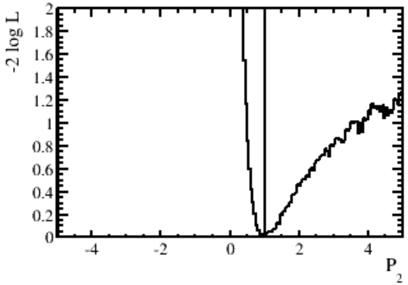
Re parametrization of pdf

- Re parametrization of the pdf gives exactly the same answer as toys taking into account the jacobian:

Profile likelihood from re-parametrised pdf.

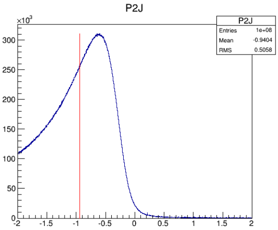


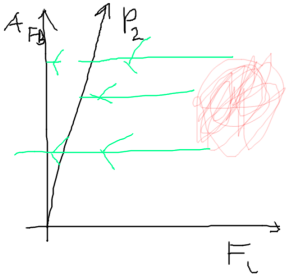
Profile likelihood from toys with Jacobian



Toys Conclusions

- We understand the source of the bias in the most probable value.
- Jacobian gives the same answer as does the parametrization of pdf.
- When we work out the interval on P2 (etc), should we use this Jacobian weighting?
- One should not look just at 1D projections as on them the most probable value is not the correct one:
- Coverage of P_i is ensured by the coverage of S_i.





How to get the errors on the P_i