Method of moments for $B \to K^{*}\mu\mu$

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1. Introduction
2. Method of Moments - Theory
3. Moments of Ss
4. Toy MC study
5. 2 am discovery

Plan

Why method of moments:

1. Complementary approach in performing the fit.
2. Allows to extract info measuring quantities in event basis depending on the angular distribution.
3. Used in $B \to \rho \ell \nu$(SLAC-386 UC-414), $J/\psi \to KK\gamma$(PRD 71, 032005 (2005)), etc.

Method of moments

Let's assume we have our pdf with k unknown parameters :$PDF(x_i, \alpha)$, $dim(\alpha) = k$. One can calculate k moments, which are the functions of α_i:

$$\mu_i = f(\alpha_1, ..., \alpha_k) = E[W_i] \tag{1}$$

If we have n events in our q^2 bin, we can estimate:

$$\widehat{\mu}_i = \frac{1}{n} \sum_{j=0}^{j=n-1} w_j \tag{2}$$

, where $w_j = g(x_i)$

Trivial example

Lets see how this works in practice:

$$f(x) = \frac{x^{a-1} e^{-x/b}}{b^a \Gamma(a)} \tag{3}$$

we measure the moments:

$$m_1 = \frac{X_1 + X_2 + ... + X_n}{n},$$
$$m_2 = \frac{X_1^2 + X_2^2 + ... + X_n^2}{n}.$$

and calculate them analytically:

$$m_1 = ab,\ m_2 = b^2 a(a+1)$$

So one just needs to solve this and get the answer:

$$a = \frac{m_1^2}{m_2 - m_1^2},\ b = \frac{m_2 - m_1^2}{m_1}$$

Our PDF

The angular terms:

$$\frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = \frac{9}{32\pi}(J_{1s}sin^2\theta_k + J_{1c}cos^2\theta_k + (J_{2s}sin^2\theta_k + J_{2c}cos^2)cos2\theta_l + J_3 sin^2\theta_k sin^2\theta_l cos2\phi + J_4 sin2\theta_k sin\theta_l cos\phi + J_5 sin2\theta_k sin\theta_l cos\phi + (J_{6s}sin^2\theta_k + J_{6c}cos^2\theta_k)cos\theta_l + J_7 sin2\theta_k sin\theta_l sin\phi + J_8 sin2\theta_k sin2\theta_l sinphi + J_9 sin^2\theta_k sin^2\theta_l sin2\phi) \quad (4)$$

Since we are fitting a PDF we need to ensure it is normalized:

$$\int_{-\pi}^{\pi} d\phi \int_{-1}^{1} dcos\theta_l \int_{-1}^{1} dcos\theta_k \frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = 1 \quad (5)$$

Measuring Js

From equation 2 we have the following:

$$\frac{1}{4}(3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}) = 1 \quad (6)$$

For now we will consider the following PDF:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi}(cos\theta_k, cos\theta_l, \phi) \quad (7)$$

Becouse our PDF is not normalized and we are measuring $\Gamma + \bar{\Gamma}$ we are effectively fitting the S_i (aka $J_i \rightarrow S_i$)

Moments for $B \to K^*\mu\mu$

Let's calculate the moments for Ss:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_k sin^2\theta_l cos2\phi = \frac{8S_3}{25} \quad (8)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_k cos\phi = \frac{8S_4}{25} \quad (9)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l cos\phi = \frac{2S_5}{5} \quad (10)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l sin\phi = \frac{2S_7}{5} \quad (11)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_l sin\phi = \frac{8S_8}{25} \quad (12)$$

Moments for $B \to K^*\mu\mu$

- The simplest solution one could imagine.
- We are abusing the fact that the basis is orthogonal.
- Each of the J doesn't know about other.
- Only S_{1s}, S_{2s}, S_{1c}, S_{2c} and S_{6s}, S_{6c} are not orthogonal, but to get the answer you just need to solve a linear equation system so it's not a tragedy.

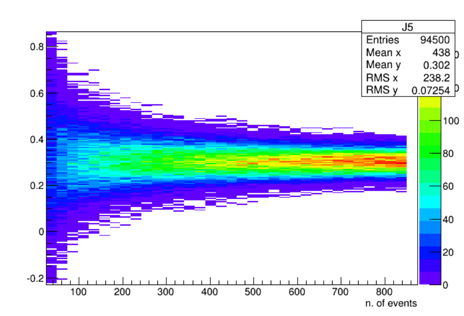
$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_k cos\theta_l = 0.1(S_6c + 4S_6s) \tag{13}$$

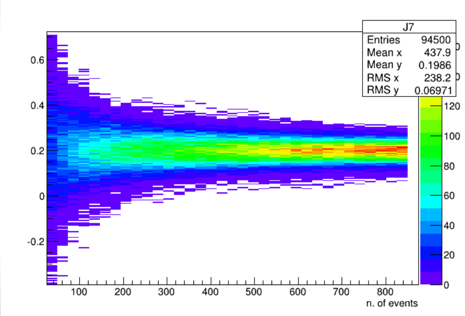
$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} cos\theta_l = 0.25(S_6c + 2S_{6s}) \tag{14}$$

solution: $S_{6c} = 2(4M_{S_{6c}} - 5M_{S_{6s}})$, $S_{6s} = -2M_{S_{6c}} + 5M_{S_{6s}}$

Moments for $B \to K^*\mu\mu$

Lets see if this method actually works. Let's take some random parameters for the PDF and make a toy.





- let's take 300 signal events as a working case.

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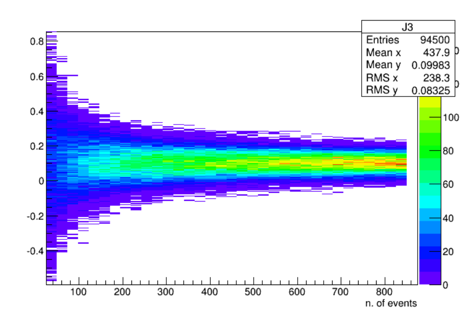

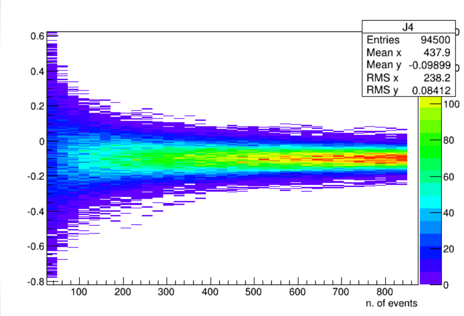

- let's take 300 signal events as a working case... we might still change the binning

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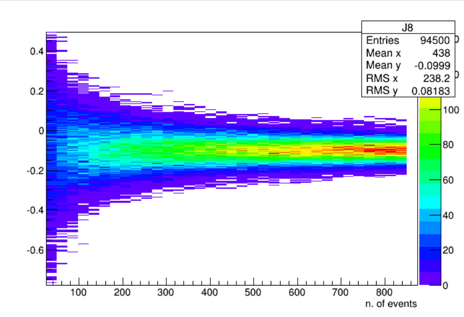
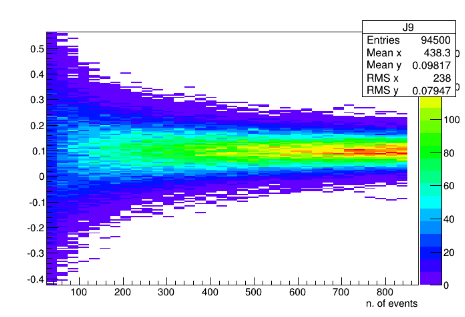
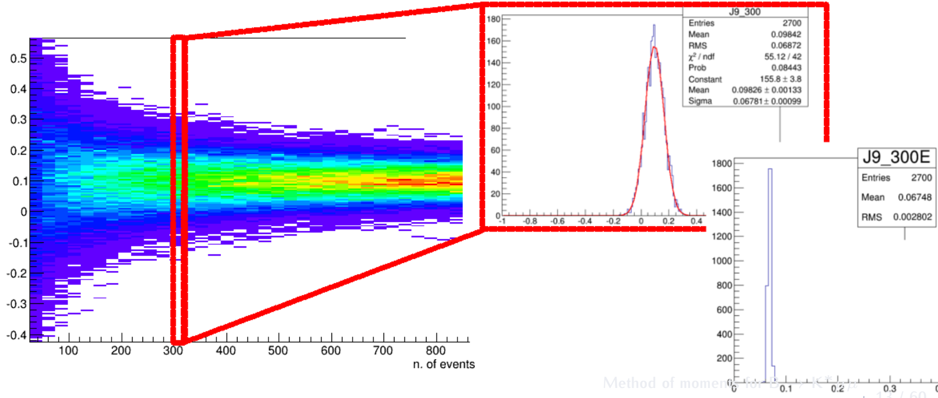




- let's take 300 signal events as a working case... we might still change the binning

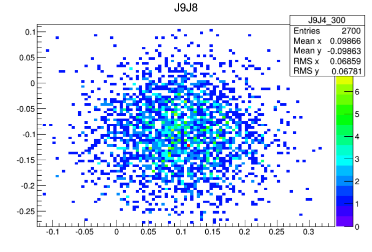
Error Estimation

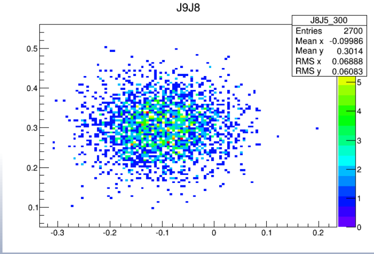
- Since moment is the mean of a given distribution the error can be estimated as *mean*/*RMS*
- use TOY MC to check this assumption



Correlation check

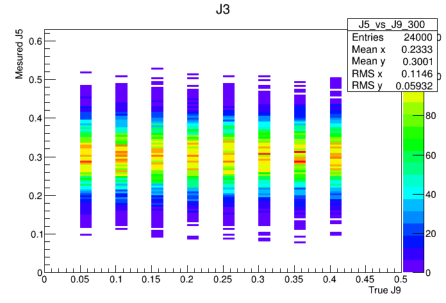
- In theory S_i shouldn't be correlated to S_j in the moment calculation.
- Lets put this to a test.

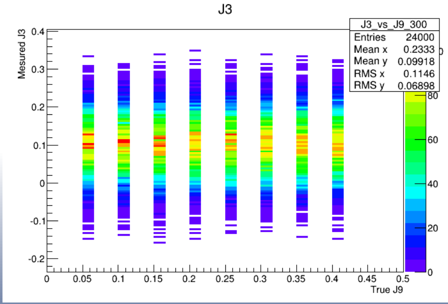




Correlation check 2

- Let's now FIX J_x and simulate different J_y
- Again theory would suggest that one J shouldn't know about the other, so J_x shouldn't change with scanning J_y parameter





What will happen to our problem with an S-wave?

Reminder:

$$\frac{d^4\Gamma}{dq^2 dcos\theta_k dcos\theta_l d\phi} = \frac{9}{32\pi}(J_{1s}sin^2\theta_k + J_{1c}cos^2\theta_k + (J_{2s}sin^2\theta_k + J_{2c}cos^2)cos2\theta_l + J_3 sin^2\theta_k sin^2\theta_l cos2\phi + J_4 sin2\theta_k sin\theta_l cos\phi + J_5 sin2\theta_k sin\theta_l cos\phi + (J_{6s}sin^2\theta_k + J_{6c}cos^2\theta_k)cos\theta_l + J_7 sin2\theta_k sin\theta_l sin\phi + J_8 sin2\theta_k sin2\theta_l sinphi + J_9 sin^2\theta_k sin^2\theta_l sin2\phi) \quad (15)$$

Let's add a very discussing things that keeps us awake at night:

$$W_s = \frac{1}{4\pi}(2I_{1a}sin^2\theta_l + 2I_{1b}sin^2\theta_l cos\theta_k + I_4 sin\theta_k sin2\theta_l cos\phi + I_5 sin\theta_k sin\theta_l cos\phi + I_7 sin\theta_k sin\theta_l + sin\phi + I_8 sin\theta_k sin2\theta_l sin\phi) \quad (16)$$

What will happen to our problem with an S-wave?

So now our PDF is sum of eq. 15 and 16. Of coz we need to require normalization:

$$\frac{1}{12}(32I_{1a} + 9J_{1c} + 18J_{1s} - 3J_{2c} - 6J_{2s}) = 1 \qquad (17)$$

No surprises here. If we have a S-wave it has to enter in Γ. To build up the preasure, what will happen to our Ss?

NOTHING!!!!!!!!

We are completely insensitive to S-wave:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_k sin^2\theta_l cos2\phi = \frac{8S_3}{25} \quad (18)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_k cos\phi = \frac{8S_4}{25} \quad (19)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l cos\phi = \frac{2S_5}{5} \quad (20)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin\theta_l sin\phi = \frac{2S_7}{5} \quad (21)$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin2\theta_k sin2\theta_l sin\phi = \frac{8S_8}{25} \quad (22)$$

Thins get better :)

We can even measure directly the S-wave:

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin^2\theta_l cos\theta_k = \frac{32 I_{1b}}{45} \tag{23}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l cos\phi = \frac{16 I_4}{45} \tag{24}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin\theta_l cos\phi = \frac{4 I_5}{9} \tag{25}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l sin\phi = \frac{4 I_7}{9} \tag{26}$$

$$\frac{d^3\Gamma}{\Gamma dcos\theta_k dcos\theta_l d\phi} sin\theta_k sin2\theta_l sin\phi = \frac{16 S_8}{45} \tag{27}$$

Conclusions on S-wave

- S-wave components are transparent to method of moments.
- If they are orthogonal to others all they toy studies holds for them as well(will reapat for robustness but can bet my house that there is nothing going on there).
-

Conclusions

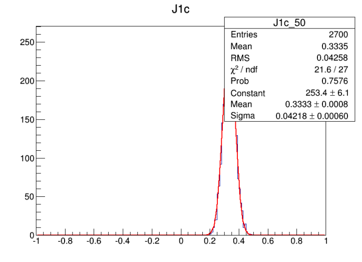
- Implemented moments method for the K*mm and start testing with toy MC
- The method converge fast and works for the "simple case", i.e. signal only.
- Method completely insensitive to S-wave component, thanks to orthogonality.
- Complementary one can measure in-depended S-wave component.

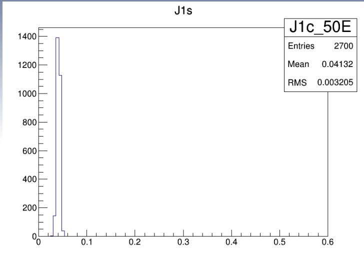
TO DO:

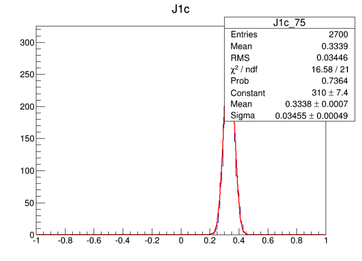
- add realism: backgrounds
- Do the unfolding
- Study binning

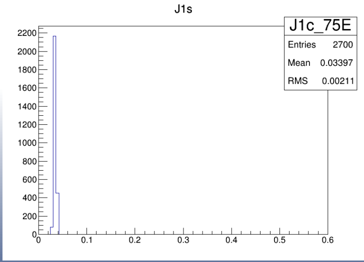
BACKUPS

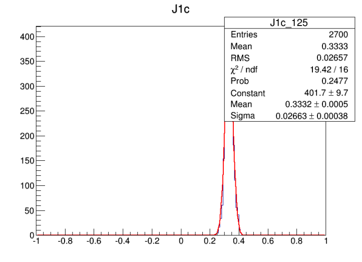
2 am discovery







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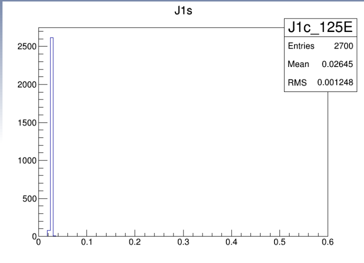
J1c_125
Entries 2700
Mean 0.3333
RMS 0.02657
χ² / ndf 19.42 / 16
Prob 0.2477
Constant 401.7 ± 9.7
Mean 0.3332 ± 0.0005
Sigma 0.02663 ± 0.00038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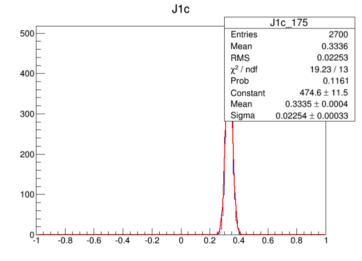
J1c
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Entries 2700
Mean 0.3336
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Prob 0.1161
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Mean 0.3335 ± 0.0004
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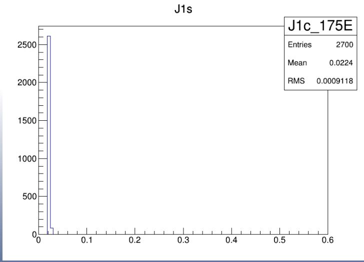
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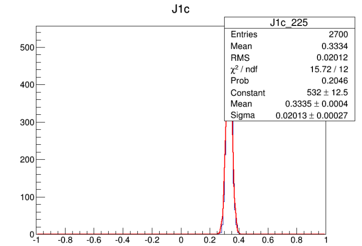
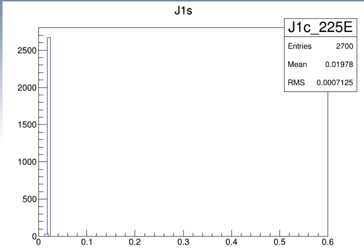
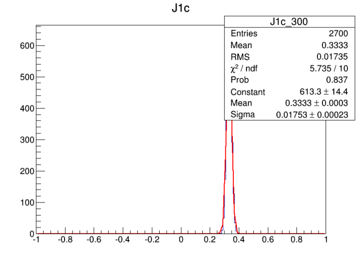
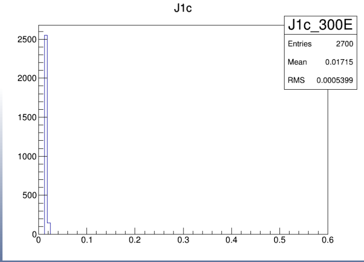
2 am discovery









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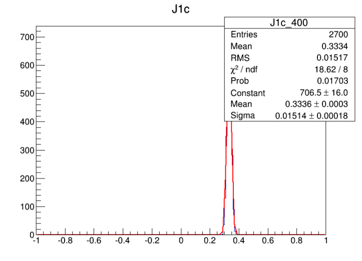

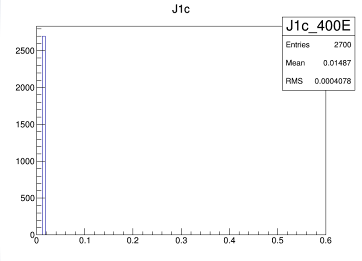

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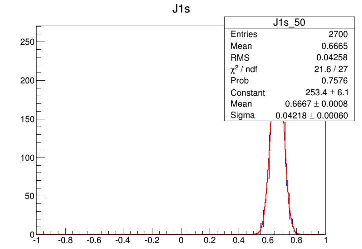

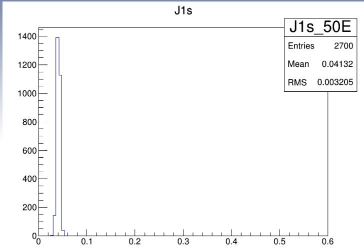

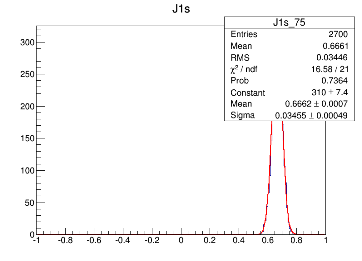

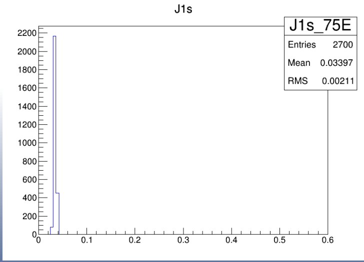

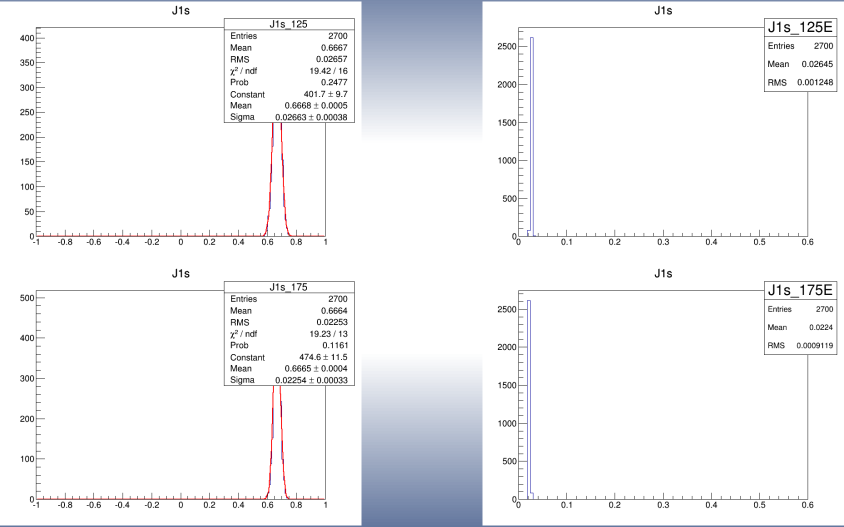
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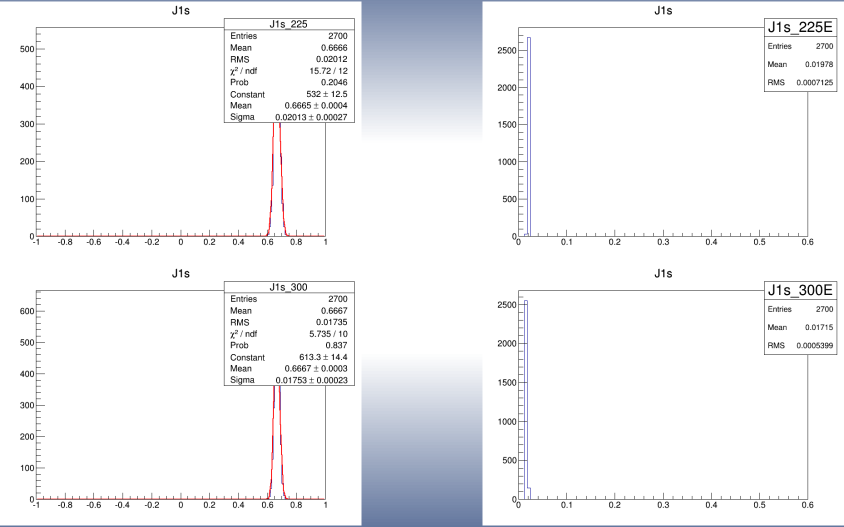

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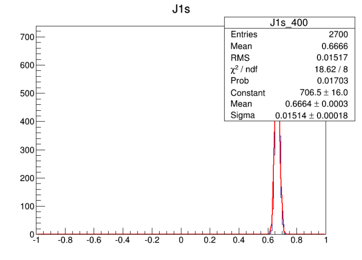

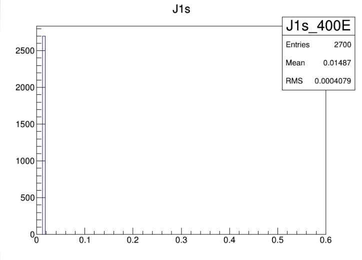

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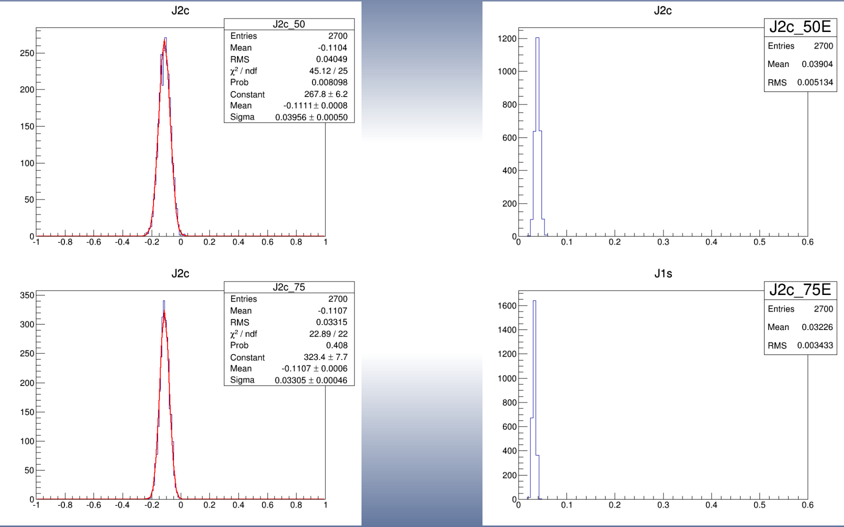

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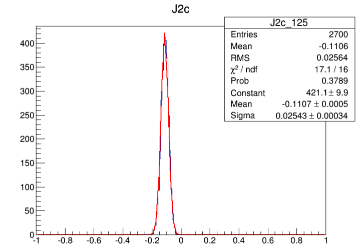

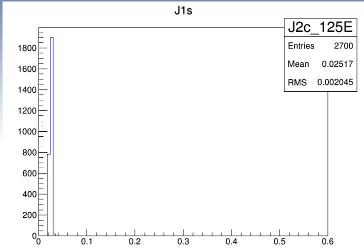

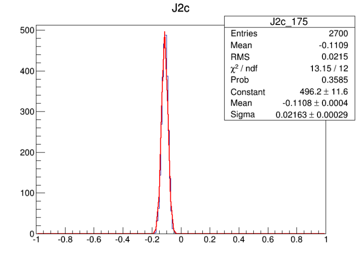

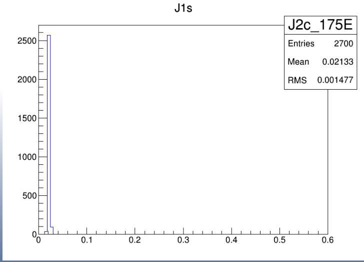

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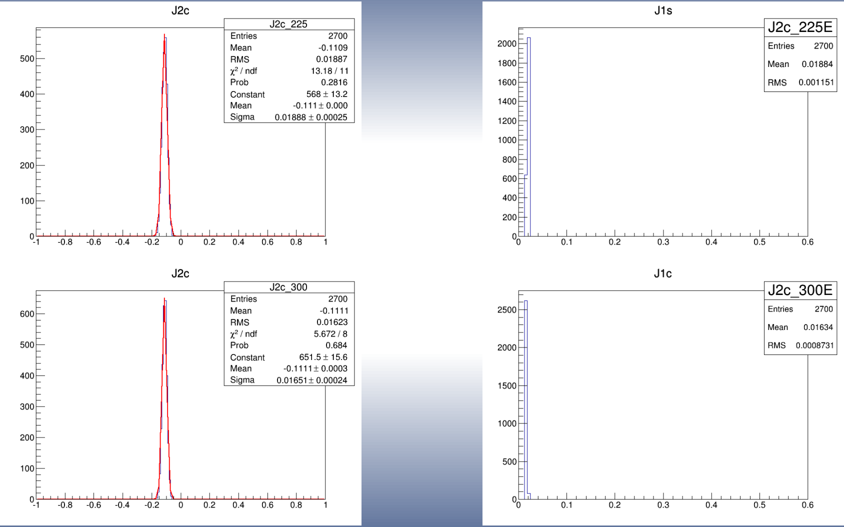

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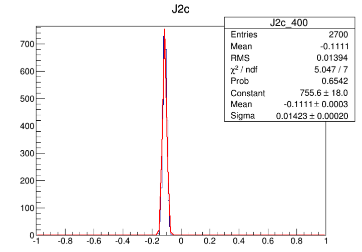

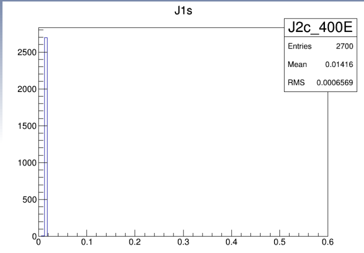

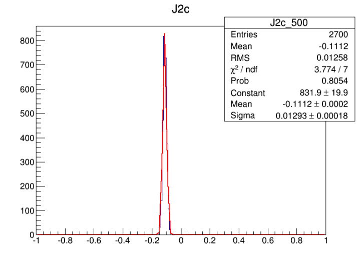

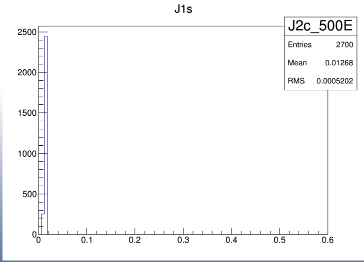

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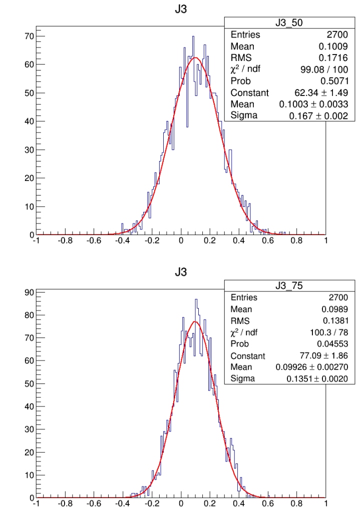

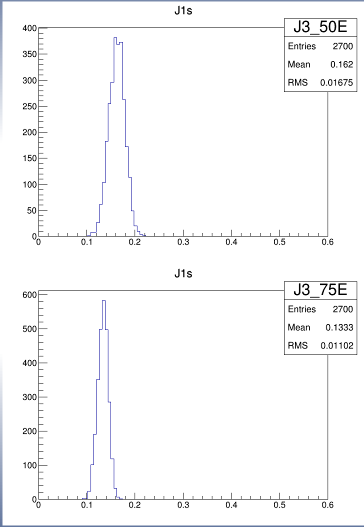

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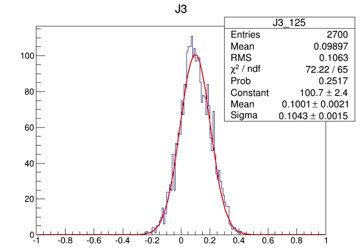

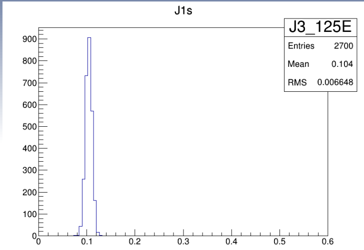

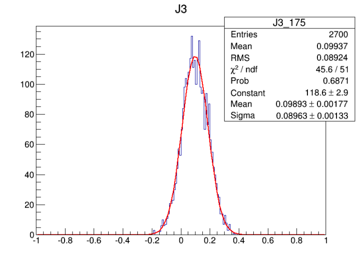

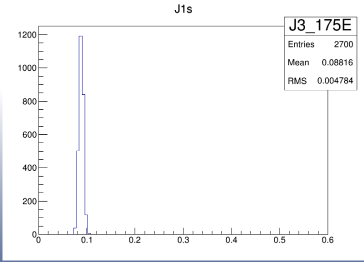

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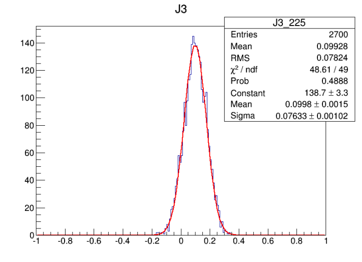

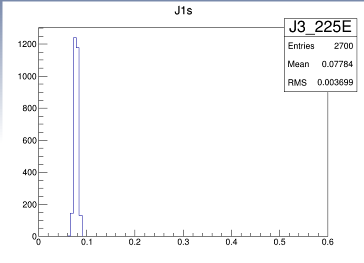

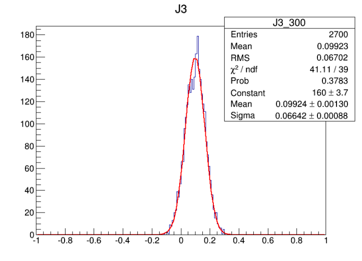

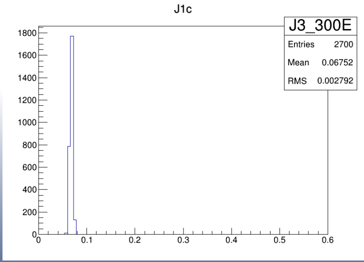

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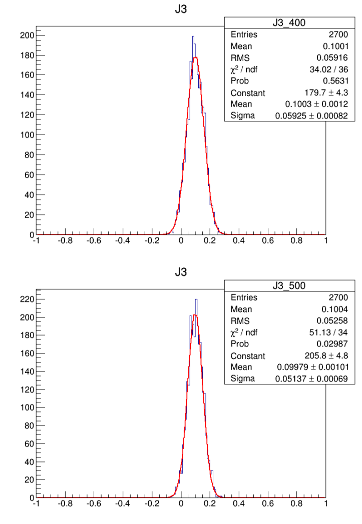

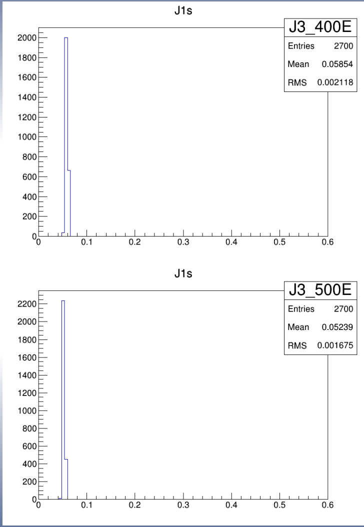

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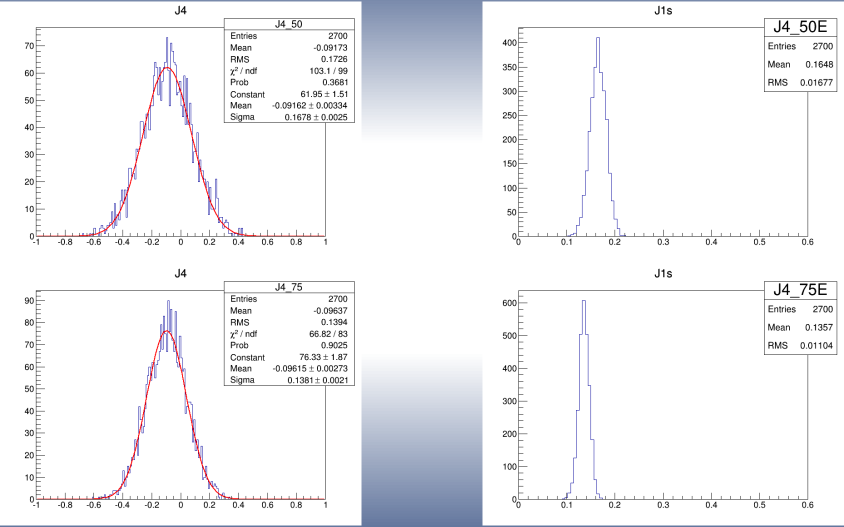

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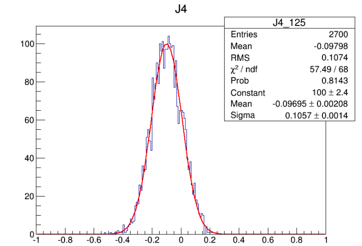

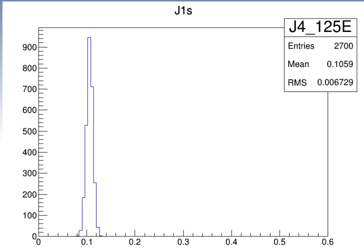

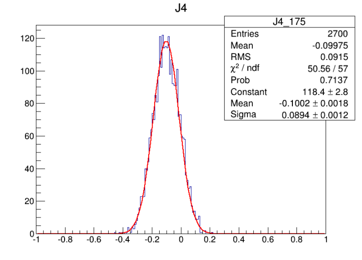

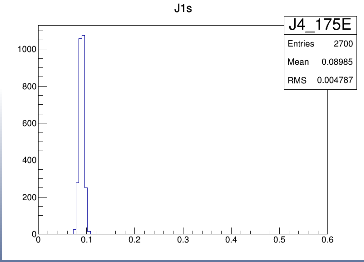

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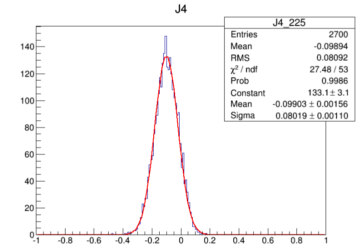
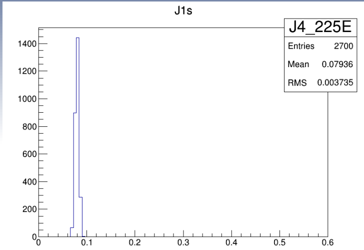
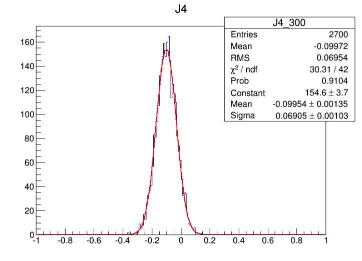
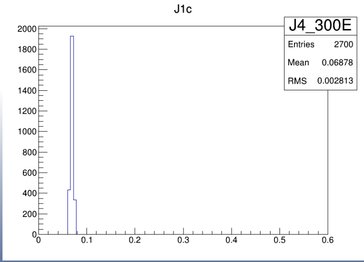
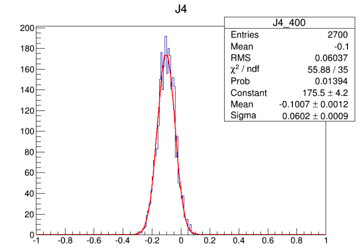
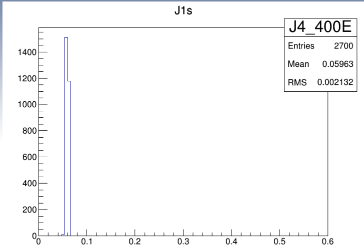


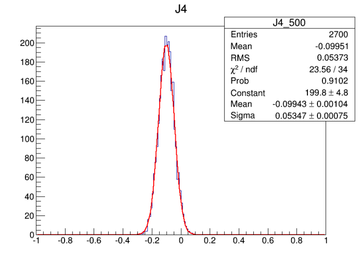


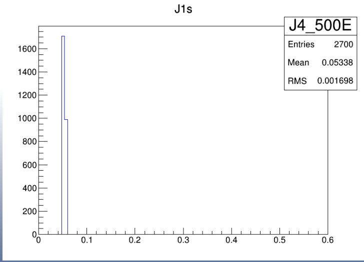




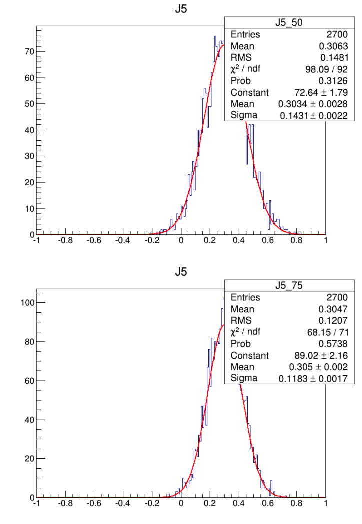

J4
J4_400
Entries 2700
Mean -0.1
RMS 0.06037
χ² / ndf 55.88 / 35
Prob 0.01394
Constant 175.5 ± 4.2
Mean -0.1007 ± 0.0012
Sigma 0.0602 ± 0.0009

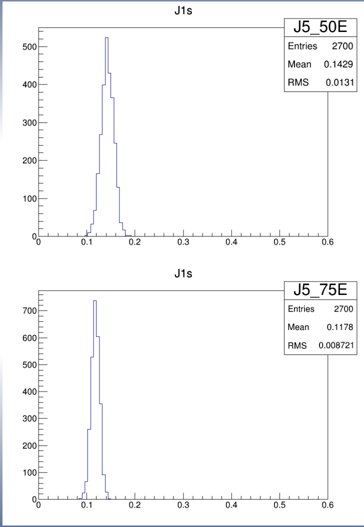

J1s
J4_400E
Entries 2700
Mean 0.05963
RMS 0.002132


J4
J4_500
Entries 2700
Mean -0.09951
RMS 0.05373
χ² / ndf 23.56 / 34
Prob 0.9102
Constant 199.8 ± 4.8
Mean -0.09943 ± 0.00104
Sigma 0.05347 ± 0.00075


J1s
J4_500E
Entries 2700
Mean 0.05338
RMS 0.001698

2 am discovery





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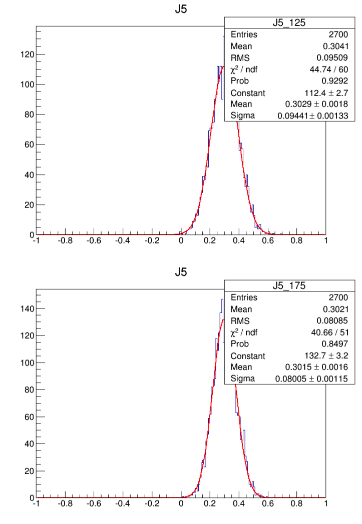
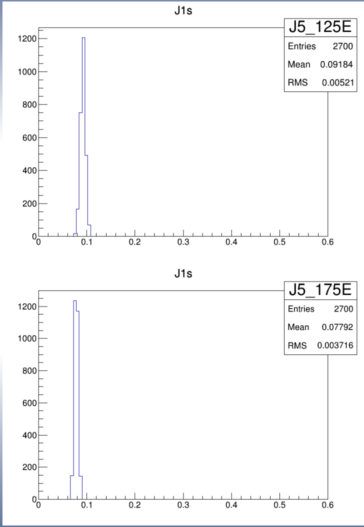
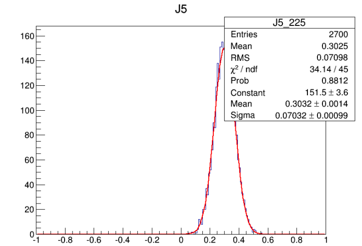
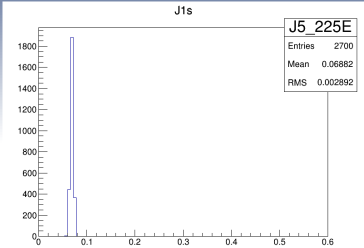
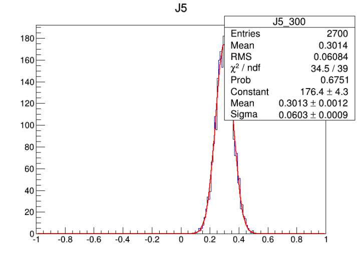
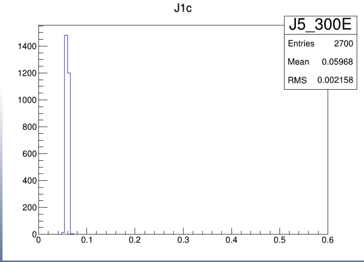




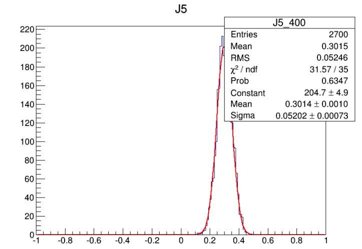

J5
J5_225
Entries 2700
Mean 0.3025
RMS 0.07098
χ^2 / ndf 34.14 / 45
Prob 0.8812
Constant 151.5 ± 3.6
Mean 0.3032 ± 0.0014
Sigma 0.07032 ± 0.00099

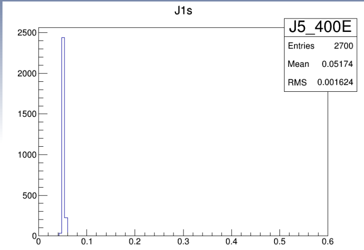

J1s
J5_225E
Entries 2700
Mean 0.06882
RMS 0.002892

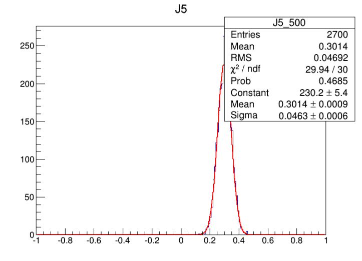

J5
J5_300
Entries 2700
Mean 0.3014
RMS 0.06084
χ^2 / ndf 34.5 / 39
Prob 0.6751
Constant 176.4 ± 4.3
Mean 0.3013 ± 0.0012
Sigma 0.0603 ± 0.0009

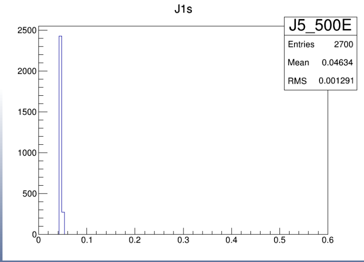

J1c
J5_300E
Entries 2700
Mean 0.05968
RMS 0.002158

2 am discovery









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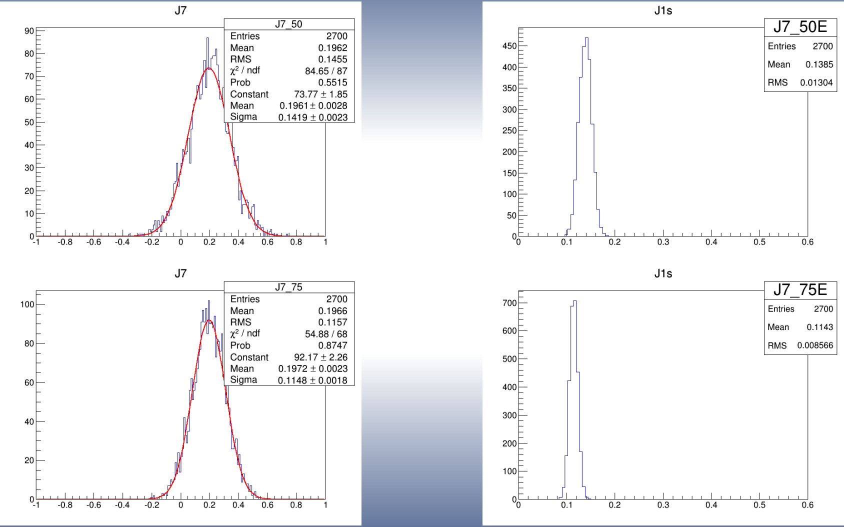

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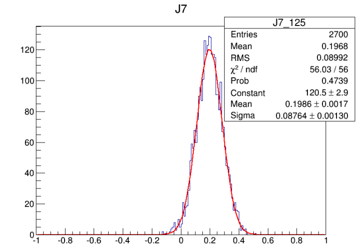

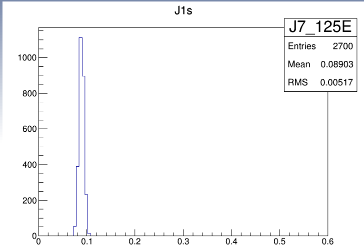

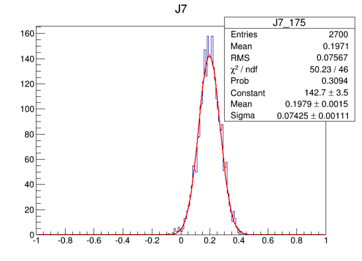

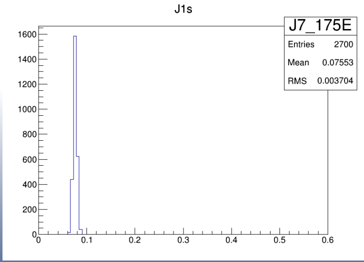

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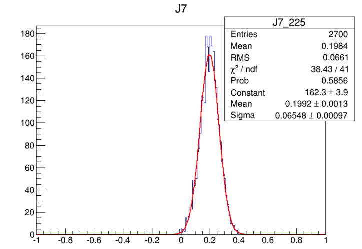

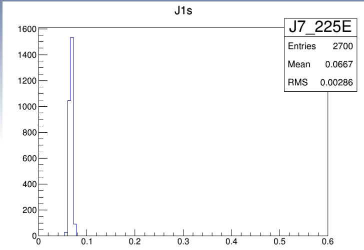

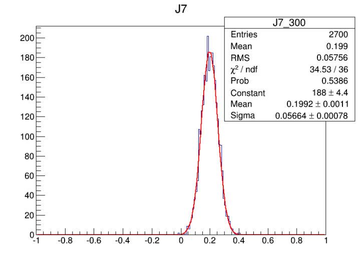

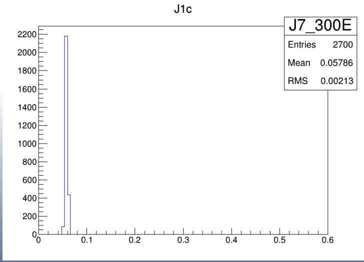

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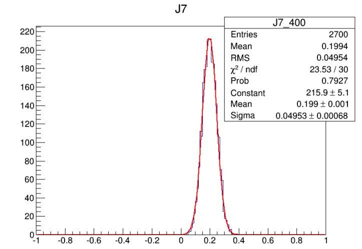

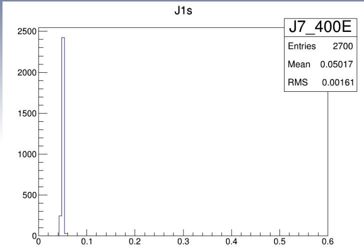

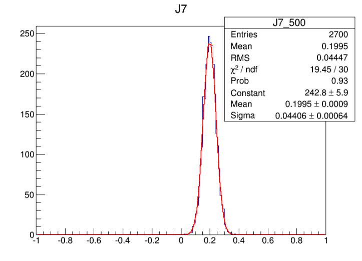

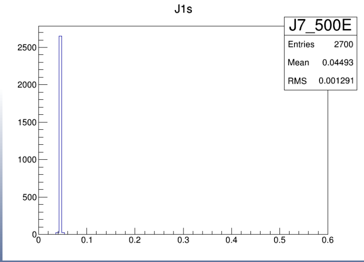

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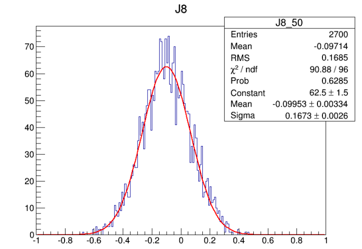

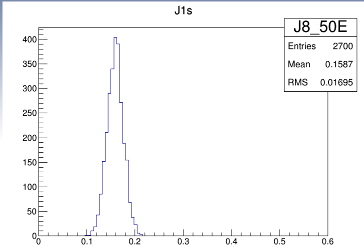

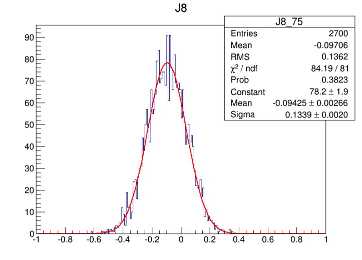

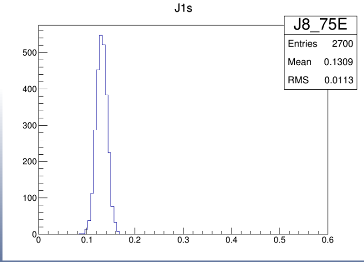

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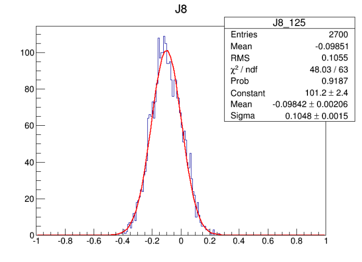

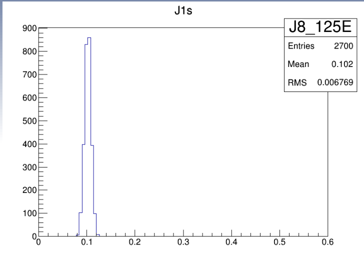

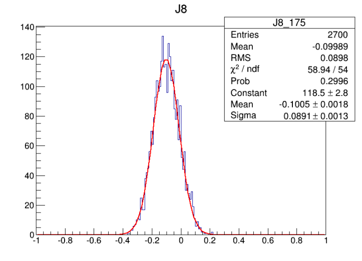

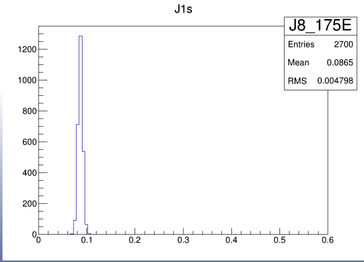

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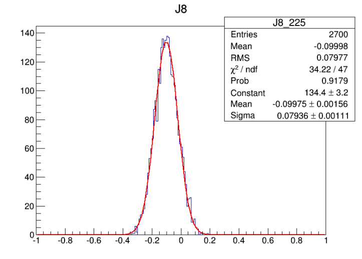

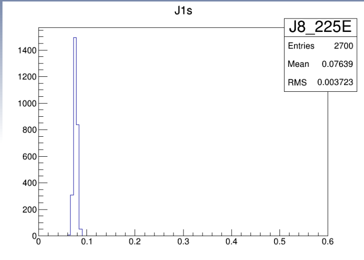

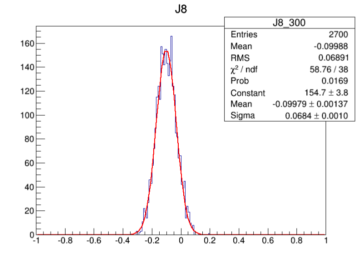

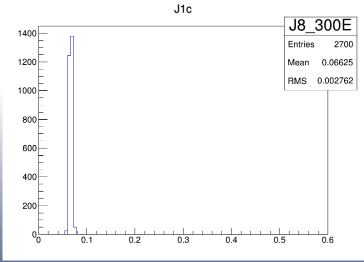

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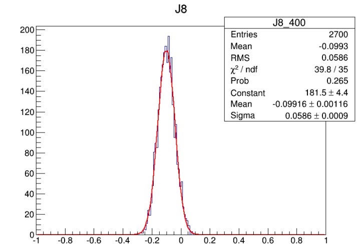

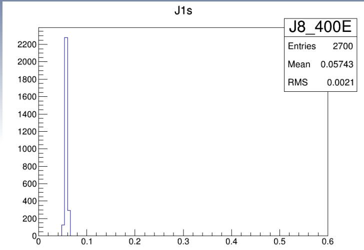

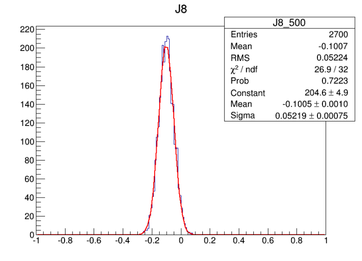

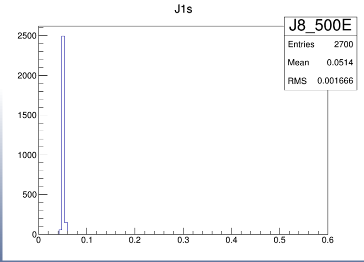

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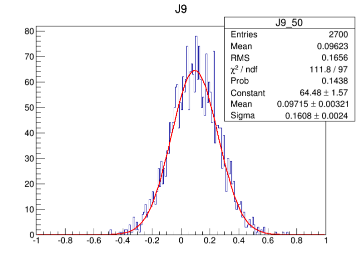

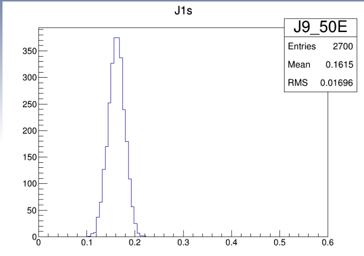

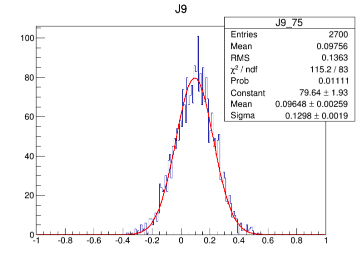

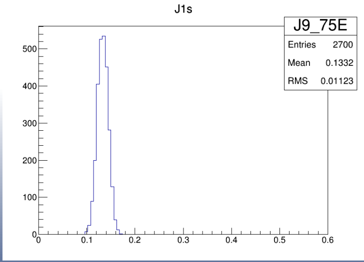

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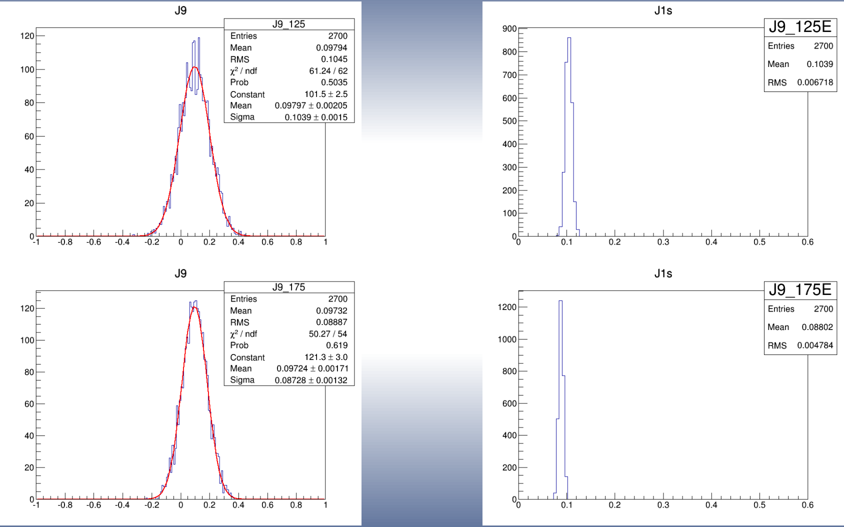

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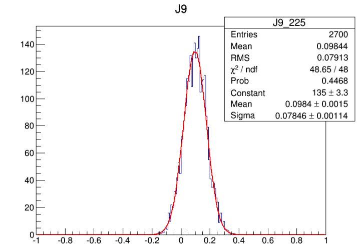

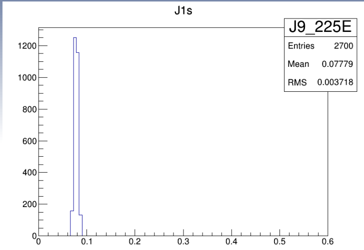

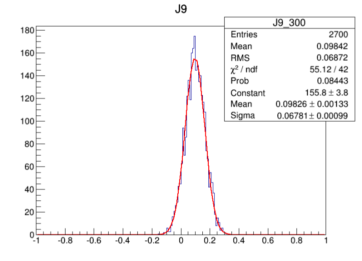

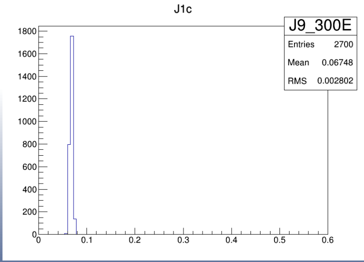

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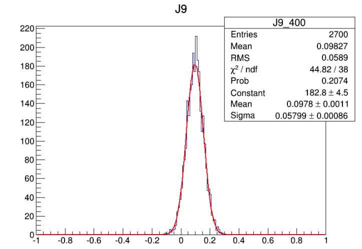

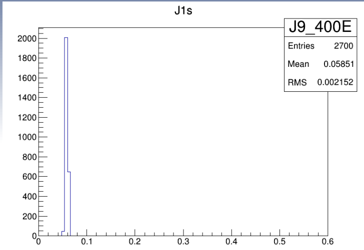

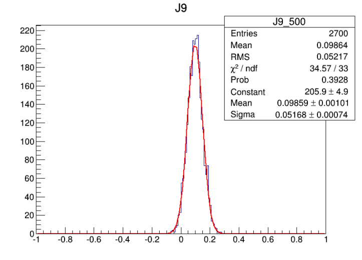

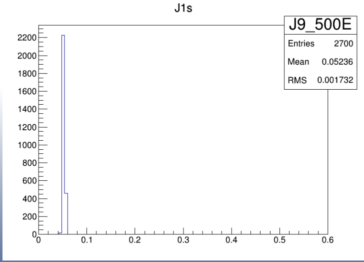

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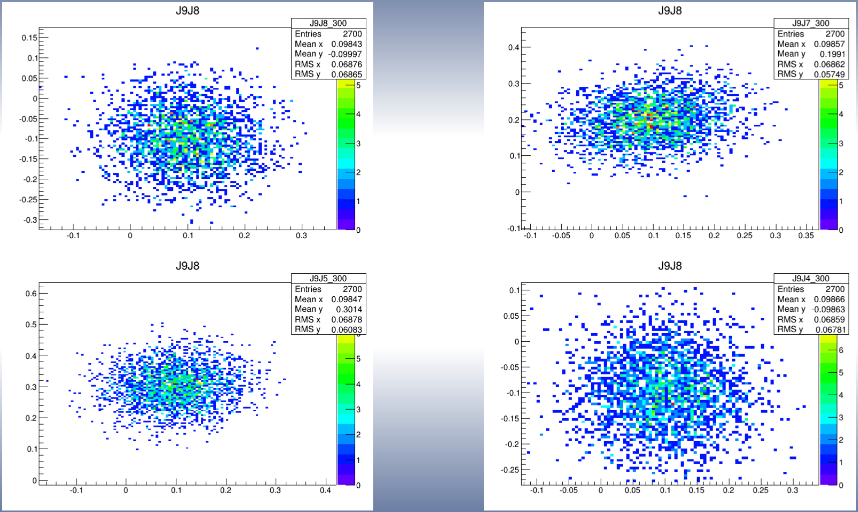

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